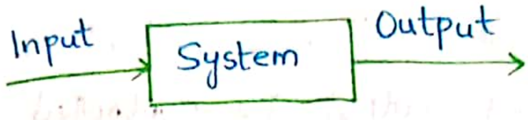
UNIT-1: INTRODUCTION TO CONTROL PROBLEM

System:



. A system is a Combination of different physical Components connected (or) related in such a manner so as to form an entire unit to attain a Certain Objective

(or)

A system is a group of physical Components arranged such that it gives proper output to the given input. (proper output may (or) may not be desired output)

Control:

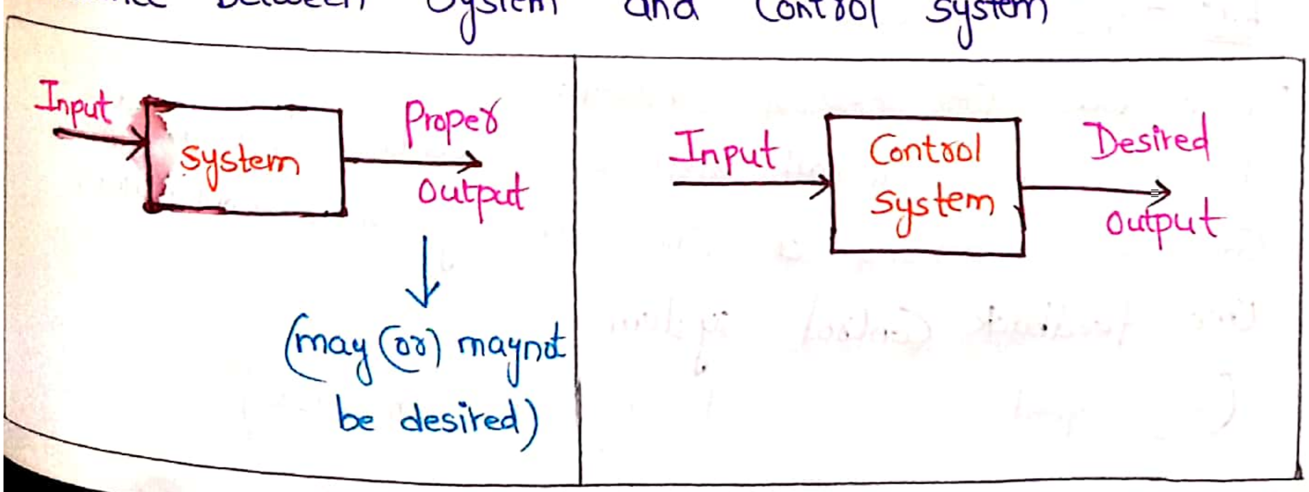
It means to regulate, direct (or) Command a system so that the desired objective is attained.

$$\boxed{\text{System + Control = Control System (cs)}}$$

Control system:

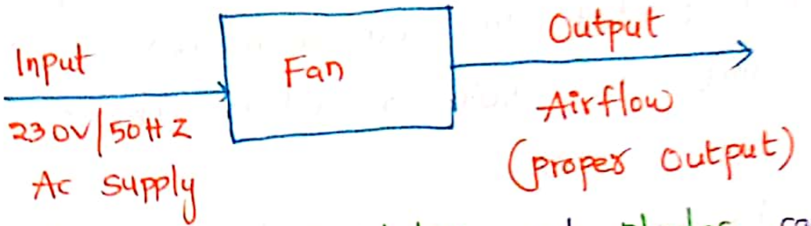
A Control System manages, Commands, directs (or) regulates the behaviour of other devices (or) systems to achieve desired results.

Difference between System and Control system

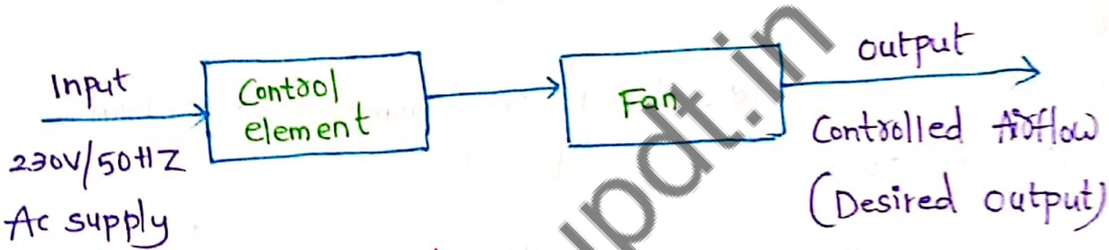


A Fan: can be a system

- A Fan can be a "System" because it can provide a proper output i.e airflow
- But as it cannot become a "Control system". Because it cannot provide the desired output i.e., controlled airflow



- A Fan with a good Regulator and Blades can be a "CONTROL SYSTEM" because it can provide a desired output i.e Controlled Air-flow.



Classification of Control system.

- Natural Control system:
  The system inside a human being (or) biological system.
- Man-made Control system:
  Control system that are designed and developed by man.
- Combinational Control system:
  This Control system is a Combination of natural and man-made Control system.
  Ex: Driving a car.
- Continuous Time Control system:
  If the signals in all parts of a Control system are Continuous functions of time, the system is Continuous time feedback Control system.
  (e.g. speed Control of dc by tachogenerator)

Linear and Non-linear Control systems:

- If a system obeys Superposition principle the system is said to be a linear system
- If it does not obey superposition principle is said to be a Non-linear system

Time-Varying and time invariant Control system:

- Time-Variant Control system: It is a Control system where any one (or) more parameters of the Control system vary with time i.e., driving a vehicle.
- Time-invariant Control system: It is a Control system where none of its parameters vary with time i.e., Control system made up of inductors, capacitors and resistors only.

Lumped and distributed Control system.

- Lumped-parameter Control system: It is a Control system where its mathematical model is represented by ordinary differential equations.
- Distributed-parameter Control system: It is a control system where its mathematical model is represented by an electrical network that is a combination of resistors, Capacitors & inductors

SISO and MIMO Control system.

- SISO Control system: It is a Control system that has only one input and one output
- MIMO Control system: It is a Control system that has only more than one input and more than one output.

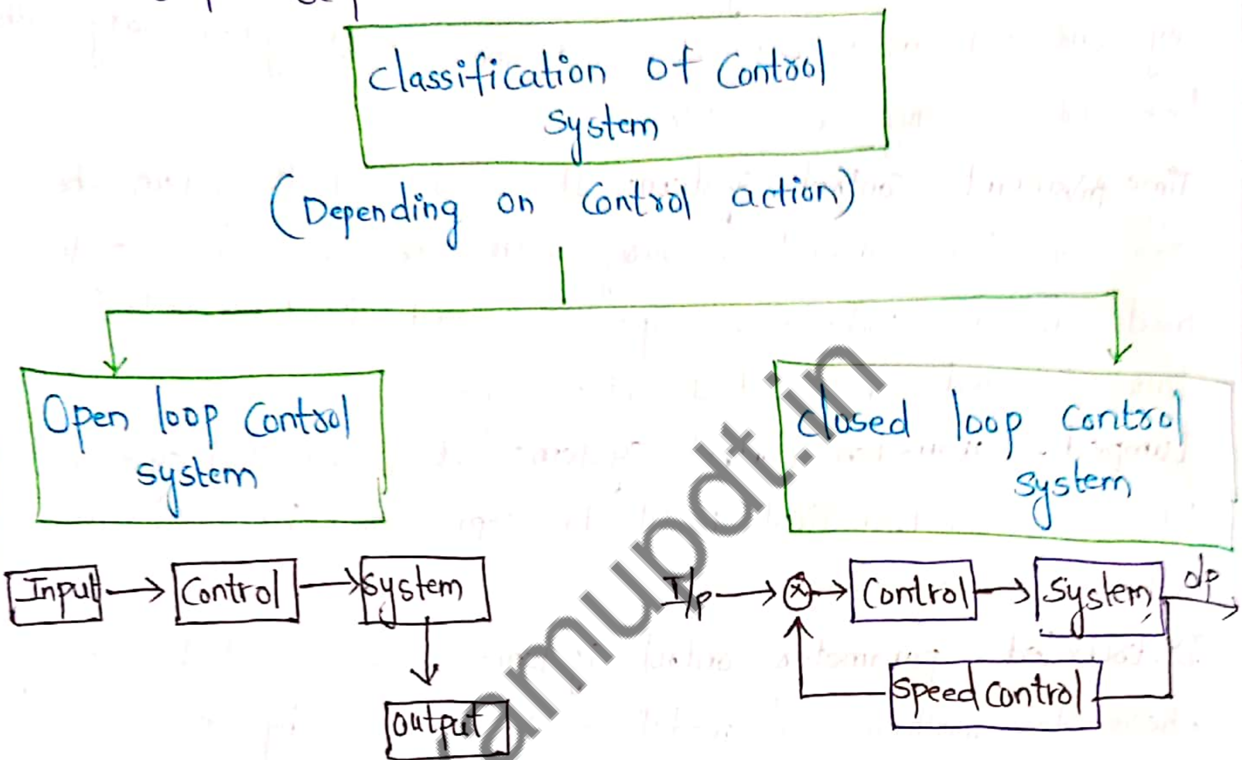
1 I/p - 1 O/p ⟶ SISO (single input single output)

1 I/p - n O/p ⟶ SIMO (single input multiple output)

n I/p - 1 O/p ⟶ MISO (Multiple input single output)

n I/p - n O/p ⟶ MIMO (Multiple input multiple output)

- Closed loop and Open loop Control system
- Open loop Control system: It is a control system where its Control action only depends on input signal and does not depend on its output response.
- Closed loop Control system: It is a control system where its Control action depends on both of its input signal and Output response.

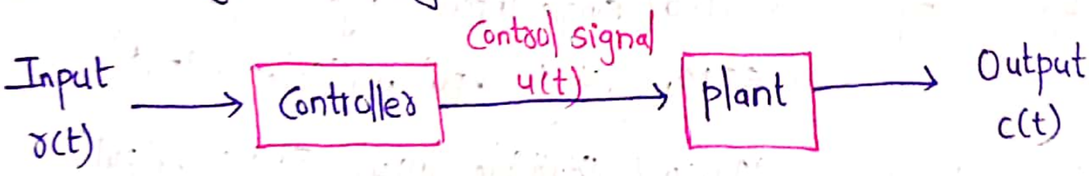


Eg: Garden water sprinkler.

Open Loop Control System

- It is a Control system where its Control action only depends on input signal and does not depend on its output response as shown in figure.

Ex: Traffic light, Washing machine, Bread toaster etc.



Examples of Open Loop Control system

We use open loop Control systems in many applications of our day to day life. Some of the systems designed based on the Concept of open loop Control Systems.

- Automatic Washing machine.
- Electric hair drier.
- Time-based Bread toaster.
- Automatic Coffee Vending machine.
- TV remote Control.
- Door lock system.

Advantages and Disadvantages.

Advantages:

- Simple design and easy to construct.
- Economical.
- Easy for maintenance.
- Highly stable operation.

Disadvantages:

- Not accurate and reliable when input (or) system parameters are variable in nature.
- Recalibration of the parameters are required time to time.

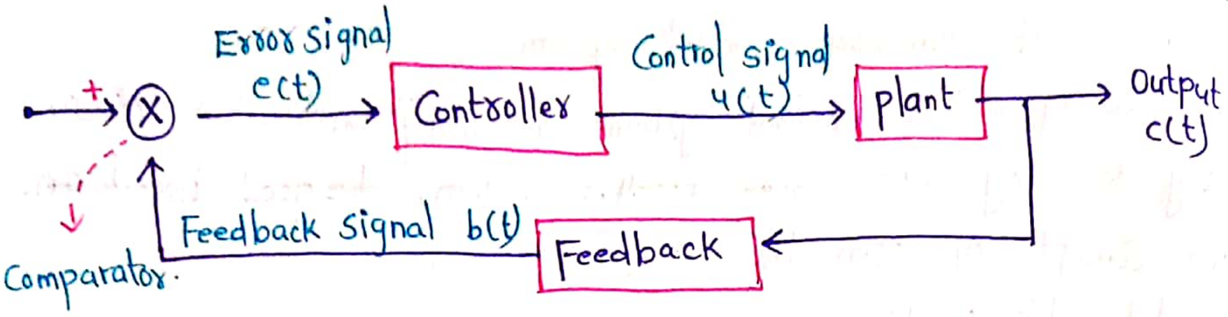
Types of Feedback

- Positive feedback – Oscillators.
- Negative feedback – CLCS

Closed-Loop Control system

- It is a Control system where its control action depends on both of its input signal and output response as shown in figure.

Ex: Automatic electric iron, Missile launcher, Speed Control of Dc motor.



Advantages:

- More accurate operation than that of open-loop Control system.
- Can operate efficiently when input (or) system parameters are Variable in nature.
- Less non-linearity effect of ~~these~~ systems on output response.
- High band width of operation.
- There is facility of automation.
- Time to time recalibration of the parameters are not required

DisadVantages:

- Complex design and difficult to Construct.
- Expensive than that of Open-loop Control system.
- Complicate of maintenance.
- Less stable operation than that of open-loop Control system.

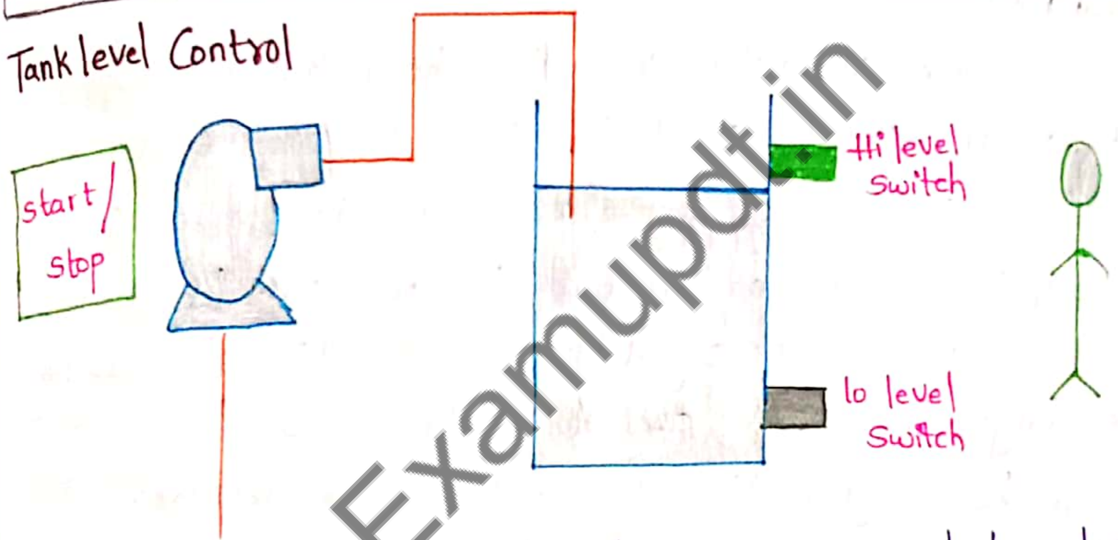
Examples of close Loop Control systems.

- Thermostat Heater.
- Solar System.
- Voltage stabilizer.
- Missile Launcher.
- Auto Engine.
- Ac
- Automatic toaster
- Turbine water Control system at power station.

Open Loop systems Vs Closed loop systems

Open Loop Control Systems	Closed Loop Control Systems
• Control action is independent of the desired output	• Control action is dependent of the desired output.
• Feedback is not present	• Feedback back is present
• They are alo called as non-feedback Control systems	• They are also called as Feedback Control systems
• Easy to design	• Difficult to design.
• They are economical	• They are costlier
• Inaccurate.	• Accurate.

Tank level Control



In open loop Control System we have a process which we have to control and some input to change the process and output. We have an example of tank level control)

Tank level Control example:

• In open loop Control system when we start the pump it will Continue fill the water / fluid in the tank. but when tank reaches (full), it will over flow. But still the pump will not stop i.e., in open loop Control system we don't have any feedback

• Hence we have to keep a level monitor.

• High level (Red) Switch glow - stop the pump. low level (Green)

Switch glow - start the pump

- Which becomes closed loop control systems.

Basic Components of Control system.

1. Plant:

- The portion of a system which is to be controlled (or) regulated is called as plant (or) process.
- It is a unit where actual processing is performed and if we observe in the above figure, the input of the plant is the controlled signal generated by a controller.
- A plant performs necessary actions on a controlled system and produces the desired output.

2. Feedback:

- It is a controlled action in which the output is sampled and a proportional signal is given to the input for automatic correction of any changes in the desired output.
- The output is given as feedback to the input for correction i.e. information about output is given to input for correcting the changes in output due to disturbances.
- The feedback signal is fed to the error detector.
- Negative feedback is preferred as it results in better stability and accuracy. The other disturbances signals are rejected.

3. Error Detector:

- The function of error detector is to compare the reference input with the feedback signal.
- It produces an error signal which is a difference of two inputs which are reference signal and a feedback signal.
- The error signal is fed to the controller for necessary controlled action.

• This error signal is used to correct the output if there is a deviation from the desired value.

4. Controller:

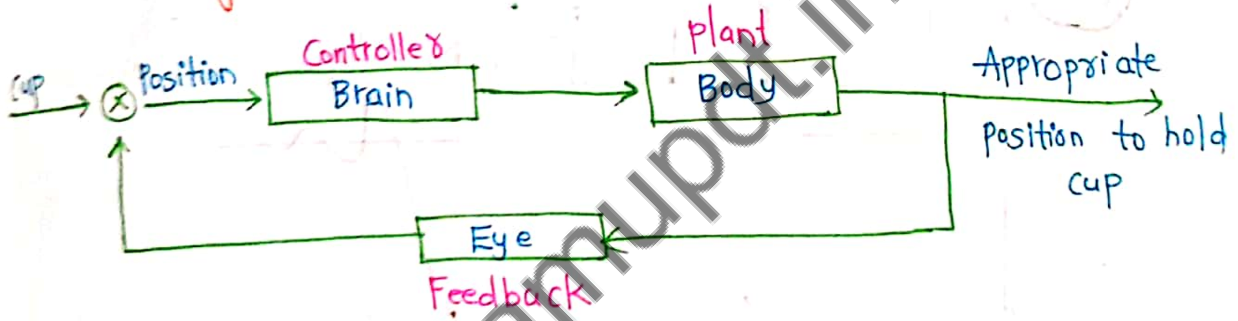
• The element of a system within itself (or) external to the system which controls the plant is called as Controller.

• The error signal will be a weak signal and so it has to be amplified and then modified for better control action.

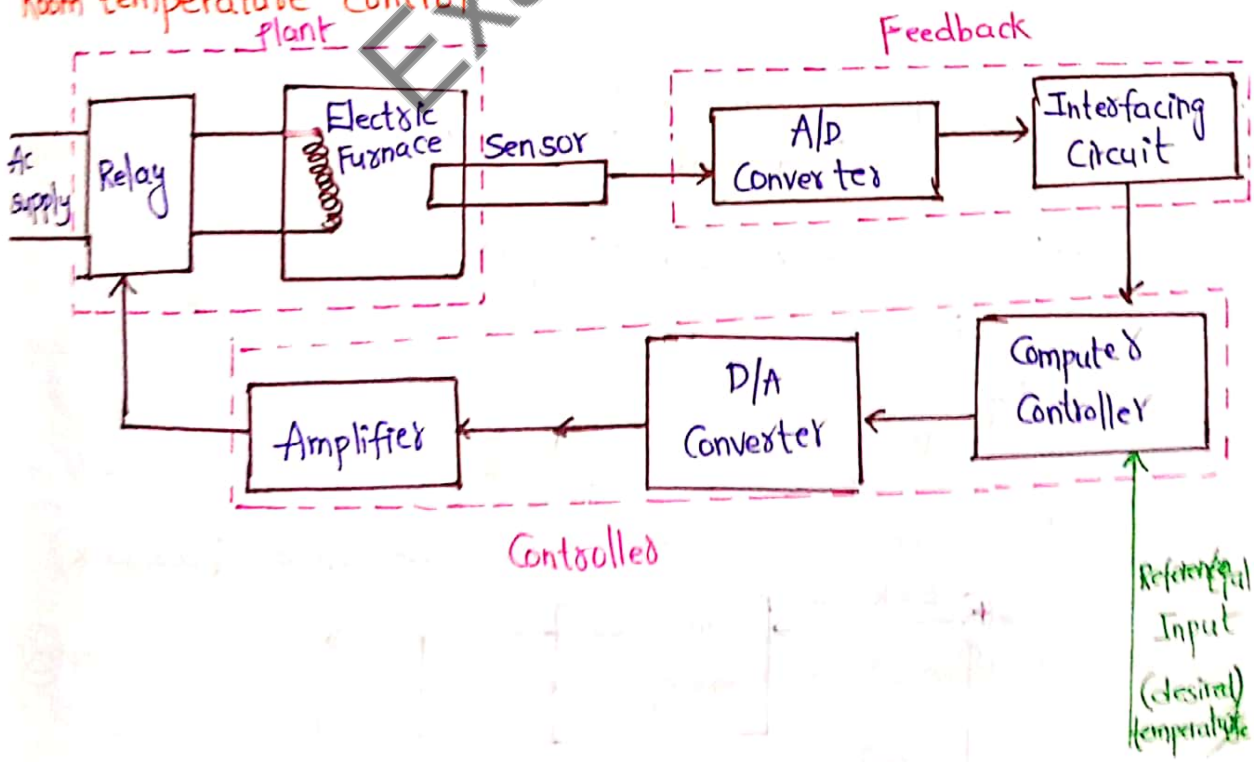
• In most of the systems, the controller itself amplifies the error signal and integrates (or) differentiates to generate a control signal.

• An amplifier is used to amplify the error signals and the controller modifies the error signal.

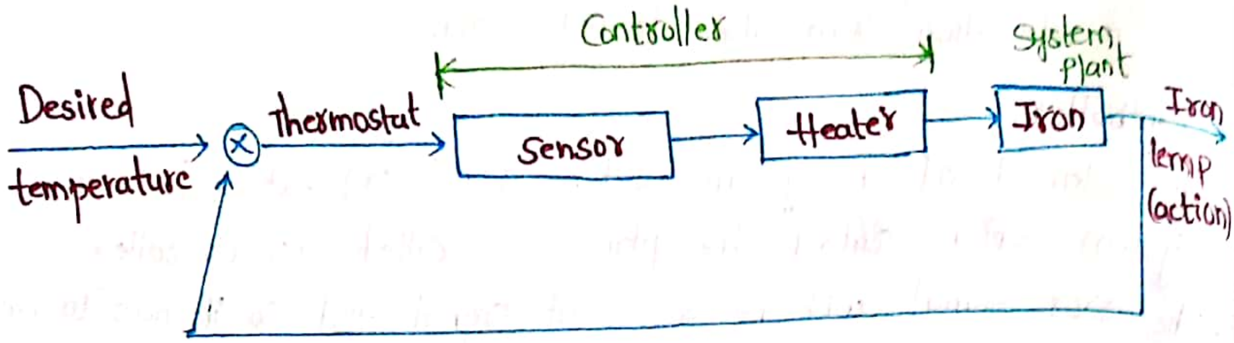
Human Being [closed loop]



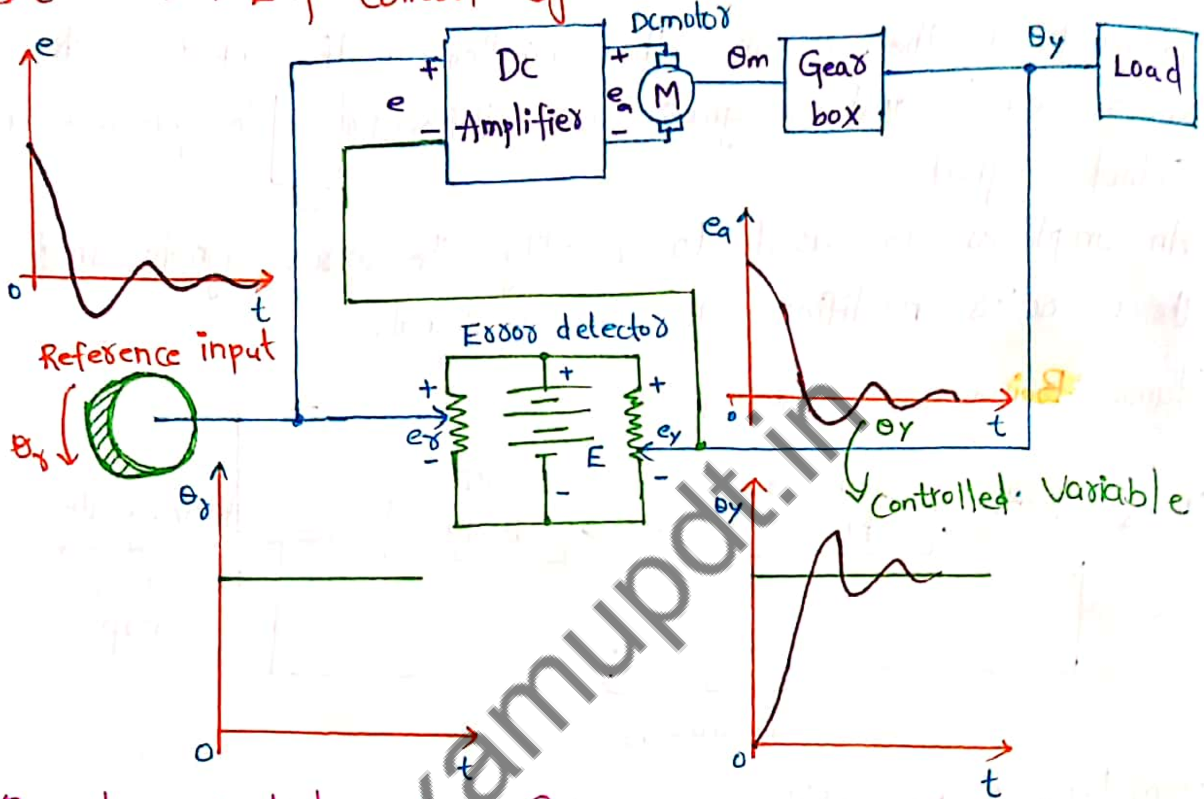
Room temperature Control



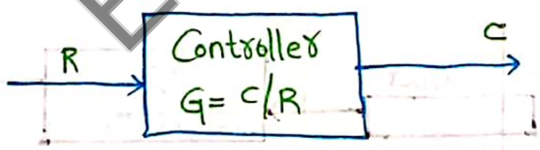
Electric Iron.



Dc closed Loop Control System



Open Loop Control System Gain

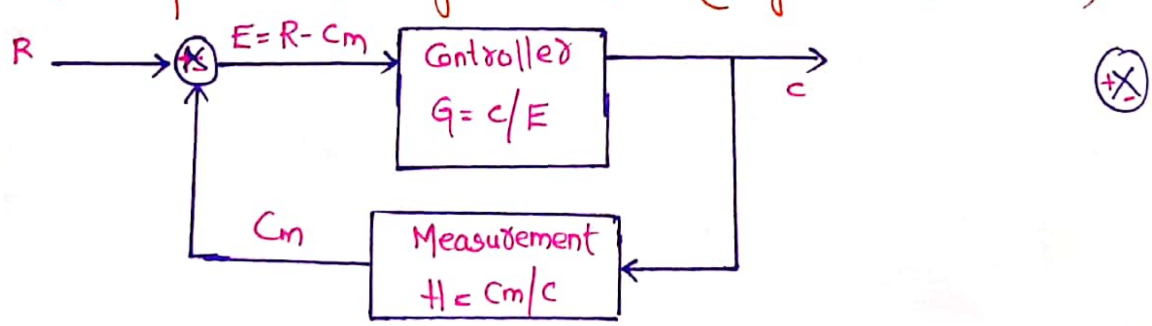


Output = (Controller Gain) × (Input)

$C = G \times R$

$\boxed{G = \frac{C}{R}}$ $\boxed{T = \frac{C}{R} = G}$

Closed Loop Control System Gain (Negative Feedback)



Reference = R

Measured Variable = Cm

Error = E

Controlled Variable = c

$\boxed{O/P - I/P = \text{Transfer function}}$

Find the Overall transfer function $\frac{c}{R}$

Output = Gain X Input

$$\text{Error } (E) = R - cm \quad \text{——} (1)$$

$$\text{Gain } (G) = \frac{c}{E} \Rightarrow c = G \times E \quad \text{——} (2)$$

$$\text{Gain } (H) = \frac{Cm}{c} \Rightarrow Cm = c \times H \quad \text{——} (3)$$

Sub (3) in (1)

$$E = R - cH$$

Sub above equation in eq^n (2)

$$c = G \times (R - cH) = GR - cGH$$

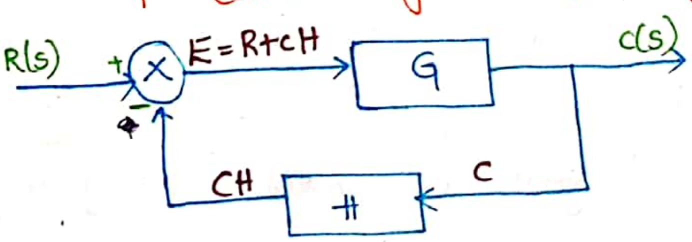
$$c + cHG = GR$$

$$c(1 + HG) = GR$$

$$\frac{c}{R} = \frac{G}{1 + HG}$$

$$\therefore \boxed{T = \frac{c}{R} = \frac{G}{1 + GH}}$$

Closed Loop Control System Gain (Positive Feedback)



Similar analysis to previous Case

$$\boxed{T = \frac{c}{R} = \frac{G}{1 - GH}}$$

Where,

T = Transfer function (or) overall gain of -ve feedback Control system

G = Open loop gain, Which is function of frequency

H = gain of feedback path, which is function of frequency.

$$C = GR + GHC$$

$$C - GHC = GR$$

$$C(1-GH) = GR$$

$$C = \frac{GR}{1-GH}$$

$$\frac{C}{R} = \frac{GR}{1-GH}$$

$$\therefore \boxed{T = \frac{C}{R} = \frac{G}{1-GH}}$$

- Open Loop System Gain $\frac{C}{R} = G$
- Closed Loop System Gain (Negative Feedback) $T = \frac{C}{R} = \frac{G}{1+GH}$
- Closed Loop System Gain (positive Feedback) $T = \frac{C}{R} = \frac{G}{1-GH}$

Effect of Feedback On Overall Gain

- Overall gain in closed loop system depends on $(1+GH)$
- It may increase (or) decrease depending on the value of $(1+GH)$
- If GH value is positive, $(1+GH)$ is greater than 1 and the Overall Gain will decrease.
- If GH value is negative, $(1+GH)$ is less than 1 and the Overall Gain will increase.
- In General, G and H are functions of frequency. so, the feedback will increase the Overall gain in one frequency range and decrease the overall gain in another frequency range.

Effect of feedback On sensitivity:

- Sensitivity of the overall gain of a negative feedback closed loop Control system (T) to the variation in the open loop gain (G) is defined as,

$$\boxed{S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\text{Percentage Change in } T}{\text{Percentage Change in } G}}$$

Where,

$\frac{\partial T}{T}$ is the incremental change in T due to incremental change in G.

$$\boxed{S_G^T = \frac{\partial T}{\partial G} \frac{G}{T}}$$

Doing partial differentiation,

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{(1+GH)1 - G(H)}{(1+GH)^2}$$

$$\boxed{\frac{\partial T}{\partial G} = \frac{1}{(GH+1)^2}}$$

$$\boxed{\frac{G}{T} = 1+GH}$$

$$S_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T} = (1+GH) \times \frac{1}{(1+GH)^2}$$

$$\boxed{S_G^T = \frac{1}{1+GH}}$$

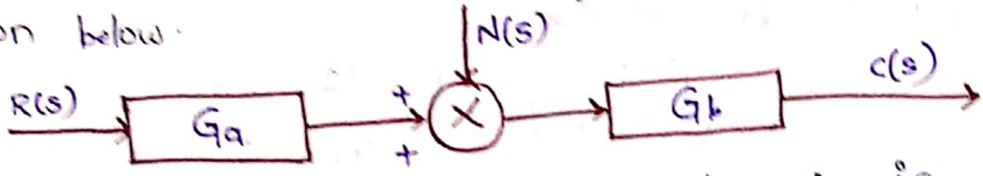
- Sensitivity of the overall gain of the closed loop system is the reciprocal of $(1+GH)$.
- Sensitivity may increase (or) decrease depending on the value of $(1+GH)$.
- If $(1+GH) < 1$, Sensitivity will decrease. GH is negative.
- If $(1+GH) > 1$, Sensitivity will ~~d~~ increase. GH is positive
- Since G and H are functions of frequency, the sensitivity of the overall gain will increase in one frequency range and decrease in another frequency range
- We have to choose G and H such that system is less sensitive (or), GH is positive.
- A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable
- In the transfer function, if the denominator value is zero (i.e., $GH = -1$), then the output of the control system will be

infinite. So, the control system becomes unstable

- Therefore, we have to properly choose the feedback in order to make the control system stable.

Effect of feedback on Noise :

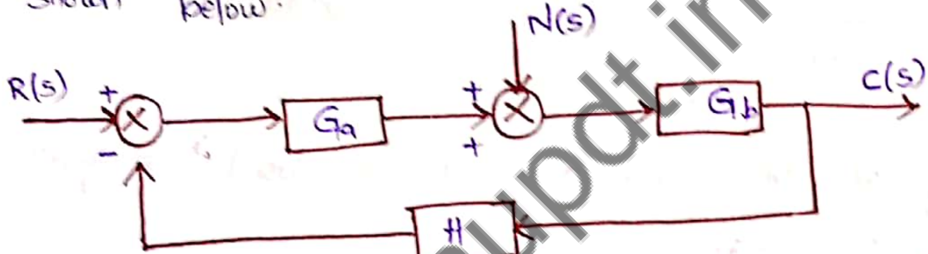
- Consider an open loop control system with noise signal as shown below.



- The open loop transfer function due to noise is

$$\boxed{\frac{C(s)}{N(s)} = G_b}$$

- Obtained by making the other input 'zero'.
- Consider an closed loop control system with noise signal as shown below.



- The closed loop transfer function due to noise is

$$\boxed{\frac{C(s)}{N(s)} = \frac{G_b}{1+G_aG_bH}}$$

- Obtained by making the other input 'zero'
- In the closed loop system, the gain due to the noise signal is decreased by a factor $(1+G_aG_bH)$ provided that the factor $(1+G_aG_bH)$ is greater than one.

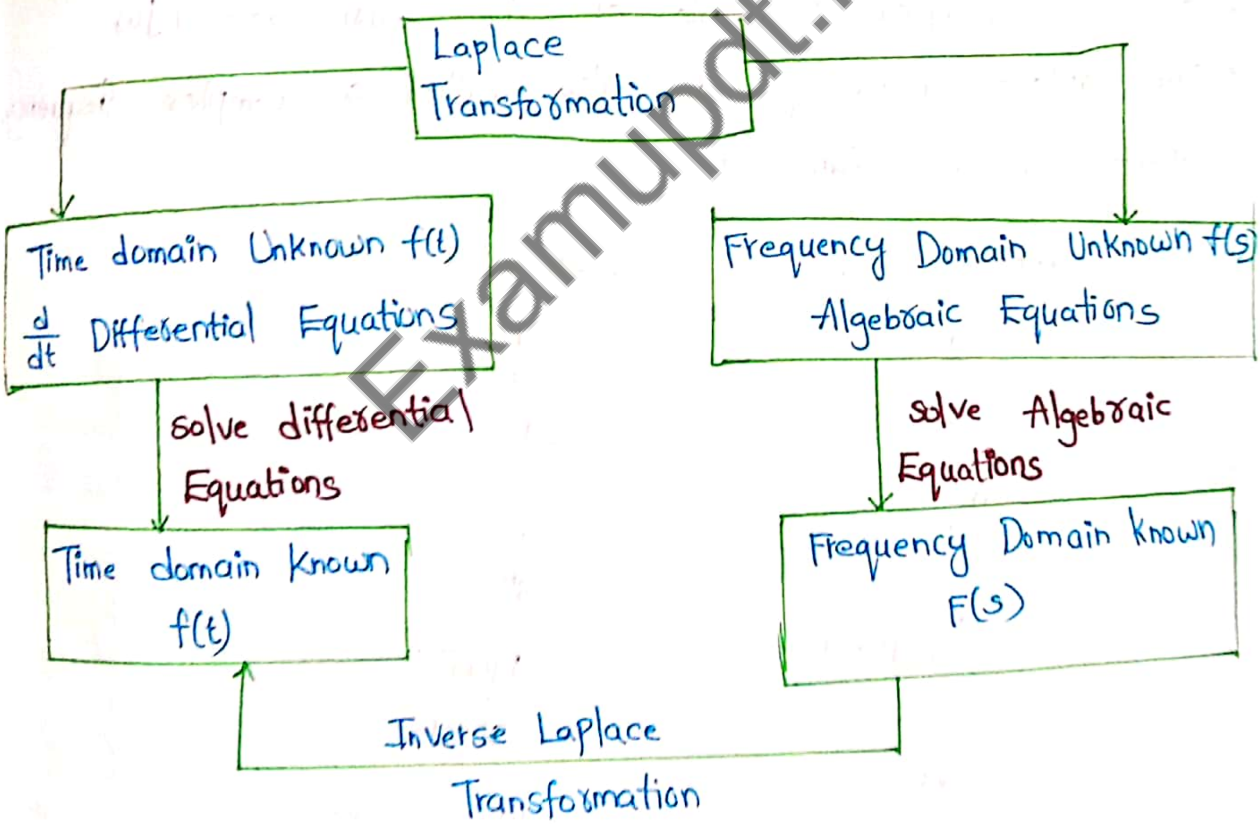
Mathematical Modelling

- The control systems can be represented with a set of mathematical equations known as Mathematical model.
(It is useful for analysis and ~~design~~ design of control system)
- Analysis means finding the output when we know the input and mathematical model.

- Design of Control system means finding the mathematical model when we know the input and the output.
- Different types of mathematical models used are:
  - Differential equation model
  - Transfer function model
  - State space model

Laplace Transform:

- To evaluate the performance of an automatic control system commonly used mathematical tool is "Laplace Transform"
- Laplace transform converts the differential equation into an algebraic equation in 's'.
- Laplace transform exist for almost all signals of practical interest.



Advantages of Laplace Transform:

- Solution of the Integro Differential equations of time can be easily obtained.
- Initial Conditions are automatically incorporated.

- Laplace transform provides an easy & effective solution of many Problems arising in automatic Control systems.
- Laplace transform allows the use of graphical techniques for predicting the system procedure.

Laplace Transform:

- The laplace transform of a function $f(t)$ is defined as

$$\boxed{F(s) = L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\, dt}$$

Where,

$F(s)$ is the symbol for laplace transform

L is the laplace transform operator

$f(t)$ is a function of time t.

- The 'L' operator transforms a time domain function $f(t)$ into a frequency domain function $F(s)$.
- 's' is a Complex Variable of the form $s = \sigma + j\omega$
- The laplace frequency is also called as Complex frequency.

Standard laplace Transforms

$f(t) = L^{-1}F(s)$	$F(s) = L\{f(t)\}$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$t^n,\ n=1,2,3$	$\frac{n!}{s^{n-1}}$
$t^p,\ p>-1$	$\frac{\Gamma p+1}{s^{p+1}}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{1/2}}$
$t^{n-1/2},\ n=1,2,3$	$\frac{1\cdot 3\cdot 5\ldots(2n-1)\sqrt{\pi}}{2^n\cdot s^{n+1/2}}$
$\sin at$	$\frac{a}{s^2+a^2}$

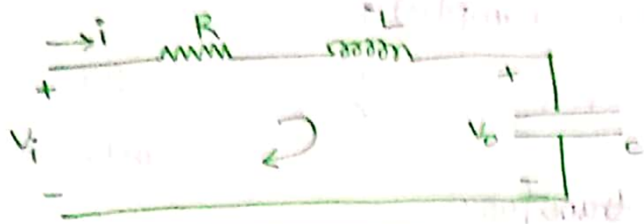
$\cos at$	$\frac{s}{s^2+a^2}$
$t \sin at$	$\frac{2as}{(s^2+a^2)^2}$
$t \cos at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
$\sin at - at\cos at$	$\frac{2a^3}{(s^2+a^2)^2}$
$\sin at + at\cos at$	$\frac{2as^2}{(s^2+a^2)^2}$
$\cos at - at\sin at$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$
$\cos at + at\sin at$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
$\sin(a+b)$	$\frac{s\sin(b)+a\cos b}{s^2+a^2}$
$\cos(a+b)$	$\frac{s\cos b - a\sin b}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at}\sinh bt$	$\frac{b}{(s-a)^2-b^2}$
$e^{at}\cosh bt$	$\frac{s-a}{(s-a)^2-b^2}$

$e^{at}\sinh bt$	$\frac{b}{(s-a)^2 + (-b^2)}$
$e^{at}\cosh bt$	$\frac{s-a}{(s-a)^2 - b^2}$
$t^n e^{at}$, $n = 1,2,3$	$\frac{n!}{(s-a)^{n+1}}$
$f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
$u_c(t) = u(t-c)$	$\frac{e^{-cs}}{s}$
$\delta(t-c)$	e^{-cs}
$u_c(t)\, f(t-c)$	$e^{-cs} F(s)$
$u_c(t)\, g(t)$	$e^{-cs} L\{g(t+c)\}$
$e^{ct} f(t)$	$F(s-c)$
$t^n f(t)$, $n = 1,2,3$	$(-1)^n F^{(n)}(s)$

Mathematical model of Differential Equation model

- It is a time domain mathematical model of Control systems.
- Follow these steps to obtain Differential Equation model
  * Apply basic laws of the given System
  * Get the differential equation in terms of the input and Output by eliminating the Intermediate Variables.
- First and Second order differential equations can be solved but solving higher order differential equations is tedious Process. Not very popular

Example :



$$\boxed{V_i = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int i\,dt} \quad \text{—— (1)}$$

$$V_0 = \frac{1}{C}\int i\,dt$$

$$\boxed{i(t) = C\frac{dV_0}{dt}} \quad \text{—— (2)}$$

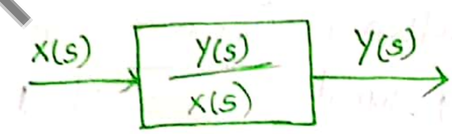
$$V_i(t) = RC\frac{dV_0}{dt} + LC\frac{d^2V_0}{dt^2} + V_0$$

$$LC\frac{d^2V_0}{dt^2} + RC\frac{dV_0}{dt} + V_0 = V_i(t)$$

$$\boxed{\frac{d^2V_0}{dt^2} + \frac{R}{L}\frac{dV_0}{dt} + \frac{1}{LC}V_0 = \frac{1}{LC}V_i(t)}$$

Transfer Function Model.

- Transfer function model is an S-domain mathematical model of Control systems.



- The transfer function of a linear time Invariant (LTI) system is defined as the ratio of laplace transform of output and laplace transform of input by assuming all the initial Conditions are Zero.
- If $x(t)$ and $y(t)$ are the input and Output of an LTI System, then the Corresponding laplace transforms are $X(s)$ & $Y(s)$
- The transfer function of the LTI system is

$$\boxed{\text{Transfer function} = \frac{Y(s)}{X(s)}}$$

Example (previous example):

$$\frac{d^2V_0}{dt^2} + \frac{R}{L}\frac{dV_0}{dt} + \frac{1}{LC}V_0 = \frac{1}{LC}V_i(t)$$

$V_0(t) \longleftrightarrow V_0(s)$

$V_i(t) \longleftrightarrow V_i(s)$

Taking laplace transform,

$$s^2V_0(s) + \frac{R}{L}sV_0(s) + \frac{1}{LC}V_0(s) = \frac{1}{LC}V_i(s)$$

$$\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)V_0(s) = \frac{1}{LC}V_i(s)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{Rs}{L} + \frac{1}{LC}}$$

$$\boxed{\frac{V_0(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}}$$

Mathematical modelling of physical system.

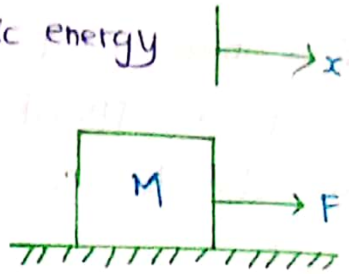
- Mechanical Translation systems.
- Mechanical Rotational systems.
- Electrical Systems.
- Hardware Components of Control systems.
    - * Armature Controlled DC servo motor
    - * Field Controlled DC servo motor
    - * AC servo motor
    - * Synchro Transmitter Receiver Pair

Mathematical modelling of Mechanical Translational Systems

- Translational motion takes place along a straight line. Variables involved in describing the motion – displacement, velocity, acceleration.
- Newtons law of motion govern such systems.
- Fundamental Components of a mechanical translational system – mass, spring, dashpot

Mass:

- Property of a body, which stores kinetic energy



- If the force is applied on a body having mass 'M' then it is opposed by an opposing force due to mass, which is proportional to the acceleration of the body

$$f_m \propto a$$

$$f_m = Ma = M\frac{dv}{dt} = M\frac{d^2x}{dt^2}$$

$$F = F_m$$

Where, F is applied force

F_m is the opposing force due to mass

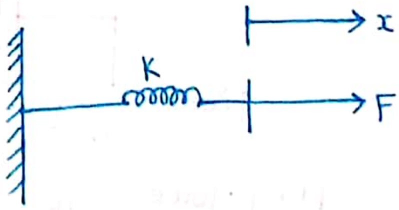
M is the mass

a is the acceleration

V is the velocity and x is displacement

Spring:

- It is an element with stores potential energy



- If a force is applied on the spring, it is opposed by the opposing force due to elasticity, proportional to the displacement of the spring.

$$f \propto x$$

$$\boxed{f = f_K = kx}$$

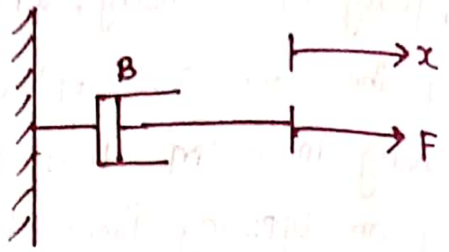
Where, F is the applied force

F_K is the opposing force due to elasticity

K is the spring constant and

x is the displacement.

Dashpot



- If a force is applied on dashpot B, then it is opposed by an opposing force due to friction of the dashpot.
- This opposing force is proportional to the velocity of the body. Assume mass and elasticity are nelgible

$$f_b \propto v$$

$$\boxed{f = f_b = BV = B\frac{dx}{dt}}$$

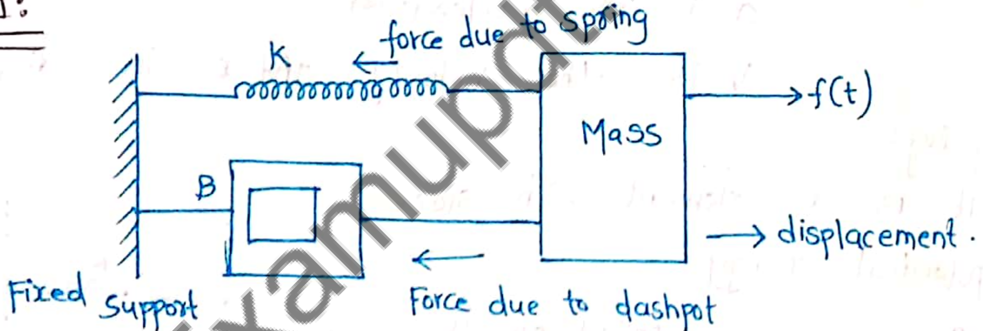
Where, f_b is the opposing force due to friction of dashpot

B is the frictional coefficient

V is the velocity and

x is the displacement.

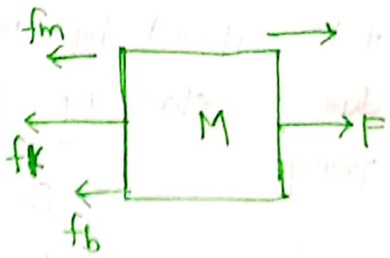
Example-1:



Spring force, $f_k = Kx$

dashpot force, $f_b = BV = B\frac{dx}{dt}$

Mass force, $f_m = Ma = M\frac{d^2x}{dt^2}$



$$f(t) = Kx + B\frac{dx}{dt} + M\frac{d^2x}{dt^2}$$

$$\boxed{M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f(t)}$$

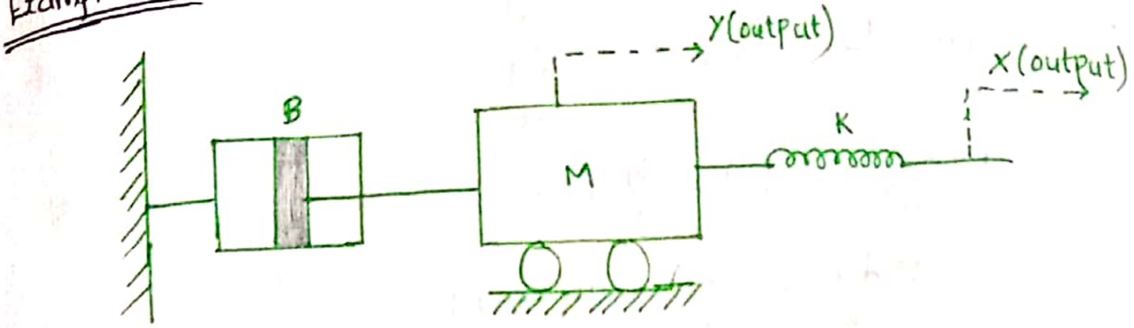
Taking laplace transform,

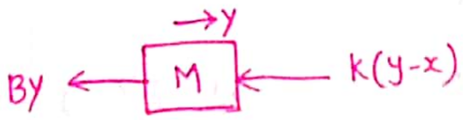
$$Ms^2X(s) + BsX(s) + KX(s) = f(s)$$

$$(Ms^2 + Bs + K)X(s) = f(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2+Bs+K}}$$

Example-2:





External force = 0

Force due to spring = $K(y-x)$

Force due to dashpot = $B\frac{dy}{dt}$

Force due to mass = $M\frac{d^2y}{dt^2}$

$$M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + K(y-x) = 0$$

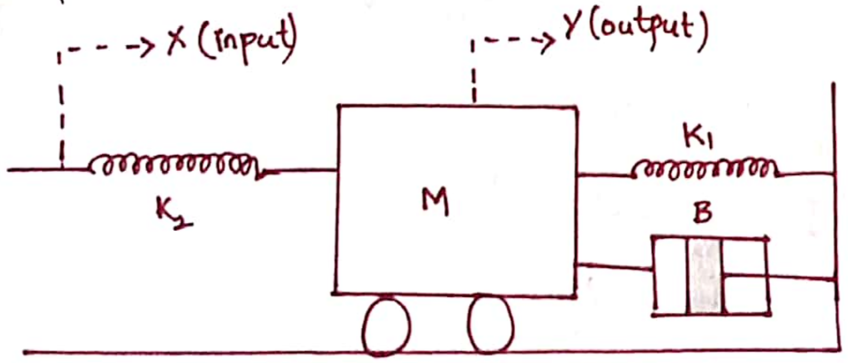
$$\boxed{M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Ky = Kx}$$

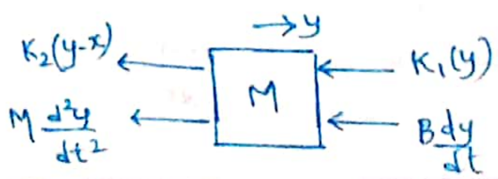
$$Ms^2Y(s) + Bs(Y(s)) + KY(s) = KX(s)$$

$$(Ms^2+Bs+K)\,Y(s) = KX(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{K}{Ms^2+Bs+K}}$$

Example-3:





Force due to spring 1 $= K_1 y$

Force due to spring 2 $= K_2(y-x)$

Force due to dashpot $= B\dfrac{dy}{dt}$

Force due to Mass $= M\dfrac{d^2y}{dt^2}$

$$M\frac{d^2y}{dt^2} + K_1 y + B\frac{dy}{dt} + K_2(y-x) = 0$$

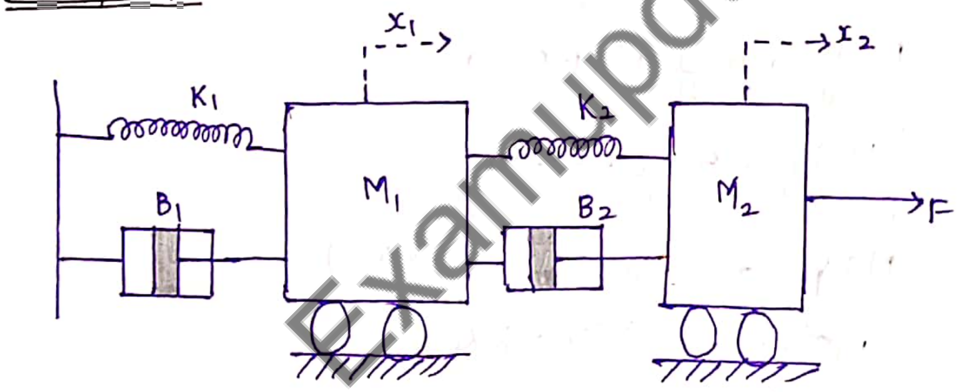
$$\boxed{M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + (K_1+K_2)y = K_2 x}$$

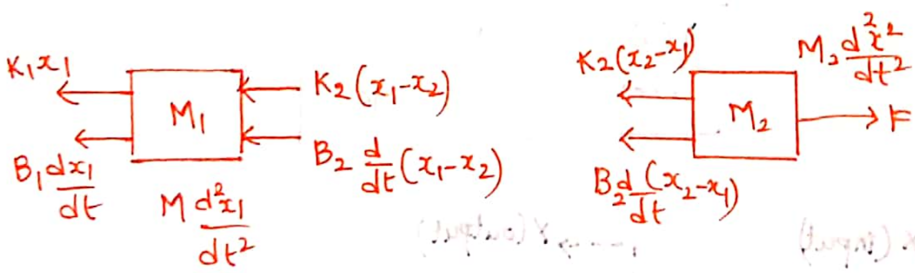
$$Ms^2Y(s) + BsY(s) + (K_1+K_2)Y(s) = K_2X(s)$$

$$\left[Ms^2 + Bs + (K_1+K_2)\right]Y(s) = K_2X(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{K_2}{Ms^2+Bs+K_1+K_2}}$$

Example-4:





Force by spring 1 $= K_1x_1$

Force by spring 2 $= K_2(x_1-x_2)$

Force by dashpot 1 $= B_1\dfrac{dx_1}{dt}$

Force by dashpot 2 $= B_2\dfrac{dx_2}{dt}$

Force of dashpot of 2 on 1 $= B_2\dfrac{dx_1}{dt}$

Force of mass 1 = $M_1 \frac{d^2x_1}{dt^2}$

Force of mass 2 = $M_2 \frac{d^2x_2}{dt^2}$

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_2 \frac{dx_1}{dt} - B_2 \frac{dx_2}{dt} + k_1x_1 + k_2(x_1 - x_2) = 0$$

$$M_1 \frac{d^2x_1}{dt^2} + (B_1+B_2)\frac{dx_1}{dt} + (k_1+k_2)x_1 = B_2 \frac{dx_2}{dt} + k_2x_2 \quad \text{--- (1)}$$

$$M_2 \frac{d^2x_2}{dt^2} + \left(B_2 \frac{dx_2}{dt}\right) - B_1 \frac{dx_1}{dt} + k_2x_2 - k_2x_1 = f(t) \quad \text{--- (2)}$$

$$f(s) \Leftrightarrow x_1(s)$$

$$\left[M_1S^2 + (B_1+B_2)S + (k_1+k_2)\right] X_1(s) = (B_2S + k_2S)\, X_2(s) \quad \text{--- (3)}$$

$$\left[M_2S^2 + B_2S + k_2\right] X_2(s) - (B_1S + k_1)X_1(s) = F(s) \quad \text{--- (4)}$$

From equation (3),

$$X_2(s) = \frac{M_1S^2 + (B_1+B_2)S + (K_1+K_2)}{B_2S+k_2} X_1(s) \quad \text{--- (5)}$$

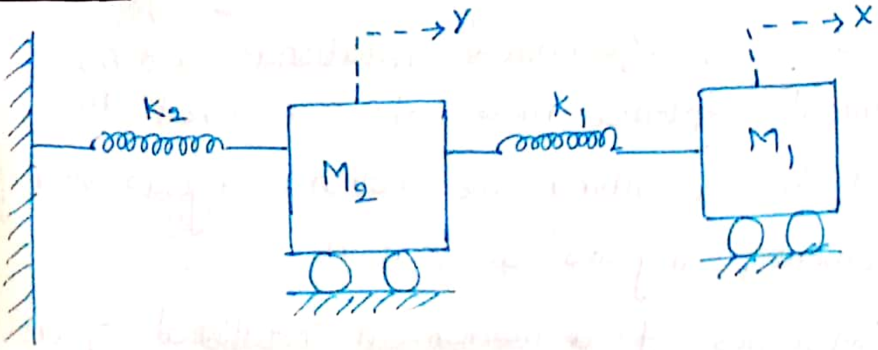
Equation (4),

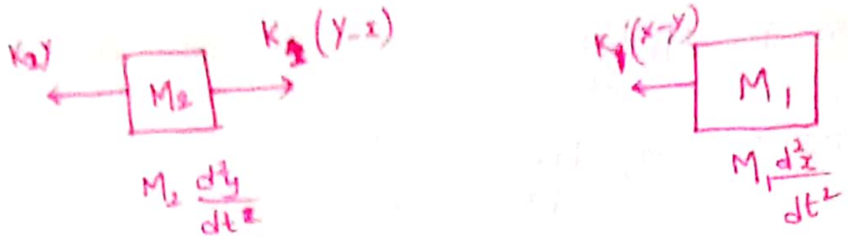
$$\left[M_2S^2 + B_2S + k_2\right] X_2(s) - (B_1S + k_1)\, X_1(s) = f(s)$$

$$\left[(M_2S^2 + B_2S + k_2)\left(\frac{M_1S^2 + (B_1+B_2)S + (k_1+k_2)}{B_2S+k_2}\right) - (B_1S+k_1)\right] X_1(s) = F(s)$$

$$\boxed{\frac{X_1(s)}{F(s)} = \frac{B_2S+k_2}{(M_2S^2+B_2S+k_2)\left[M_1S^2 + (B_1+B_2)S + (k_1+k_2)\right] - (B_1S+k_1)(B_2S+k_2)}}$$

Example - 5:





$$M_1 \frac{d^2x}{dt^2} + k_1(x-y) = 0 \quad \text{--- (1)}$$

$$M_2 \frac{d^2y}{dt^2} + k_1(y-x) + k_2 y = 0 \quad \text{--- (2)}$$

Taking laplace transform,

$$(M_1 s^2 + k_1)\, X(s) = K_2 Y(s) \quad \text{--- (3)}$$

$$[M_2 s^2 + (k_1+k_2)]\, Y(s) = k_1 X(s) \quad \text{--- (4)}$$

From equation (3) & (4),

$$X(s) = \frac{K_2}{M_1 s^2 + k_1} Y(s) \quad \text{--- (5)}$$

$$Y(s) = \frac{k_1 X(s)}{(M_2 s^2 + k_1 + k_2)} \quad \text{--- (6)}$$

Sub eq^n (5) in eq^n (4)

$$[M_2 s^2 + (k_1 + k_2)]\, Y(s) = k_1 \left(\frac{K_2}{(M_1 s^2 + k_1)}\right) Y(s)$$

$$(M_1 s^2 + k_1)(M_2 s^2 + (k_1 + k_2))\, Y(s) = k_1 k_2 Y(s)$$

$$(M_1 M_2 s^4 + M_1(k_1+k_2)s^2 + M_2 k_1 s^2 + k_1^2 + k_1 k_2)\, Y(s) = k_1 k_2 Y(s)$$

$$\boxed{[M_1 M_2 s^4 + (M_1 k_1 + M_1 k_2 + M_2 k_1)s^2 + k_1^2]\, Y(s) = 0}$$

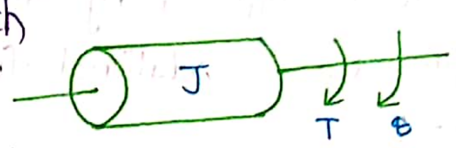
Mathematical modelling of Mechanical Rotational systems

- Rotational mechanical systems move about a fixed axis, Variables involved in describing the motion - angular velocity, angular displacement, angular acceleration.
- Fundamental Components of a mechanical rotational system - Moment of inertia, Torsional spring and Dashpot

- Torque applied to a system is opposed by the opposing torques due to moment of inertia, elasticity and friction of the system.
- Since applied torque and opposing torques are acting in opposite direction, the algebraic sum of the torques acting on the system is zero.

Moment of Inertia

- Property of rotational body, which stores kinetic energy.



- If the torque is applied on a body having moment of Inertia 'J' then it is opposed by an opposing torque due to moment of inertia, which is proportional to the angular acceleration of the body.

$$T_j = J\frac{d^2\theta}{dt^2}\ T$$

$$T_j \propto \alpha$$

$$\boxed{T_j = J\alpha = J\frac{d\omega}{dt} = J\frac{d^2\theta}{dt^2} = T = T_j}$$

Where,

T_j is the opposing torque due to the moment of inertia

T is the applied force

J is the moment of inertia

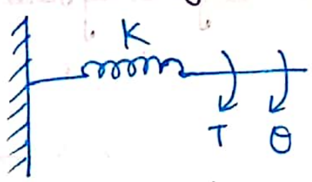
α is the angular acceleration

ω is the angular velocity

θ is the angular displacement

Torsional Spring

- It is an element which stores potential energy.



- If a torque is applied on the torsional spring, it is opposed by the opposing torque due to elasticity, proportional to the angular displacement of the torsional spring

$$T_k \propto \theta$$

$$\boxed{T = T_k = K\theta}$$

Where, T is the applied torque

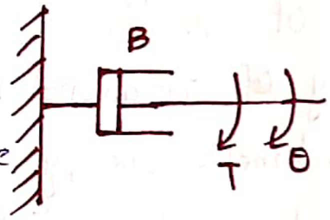
T_K is the opposing torque due to elasticity

K is the spring constant

θ is angular displacement.

Dashpot

- If a torque is applied on dashpot B, then it is opposed by an opposing torque due to rotational friction of the dashpot.



- This opposing torque is proportional to the angular velocity of the body.

$$T_b \propto \omega$$

$$\boxed{T = T_b = B\omega = B\frac{d\theta}{dt}}$$

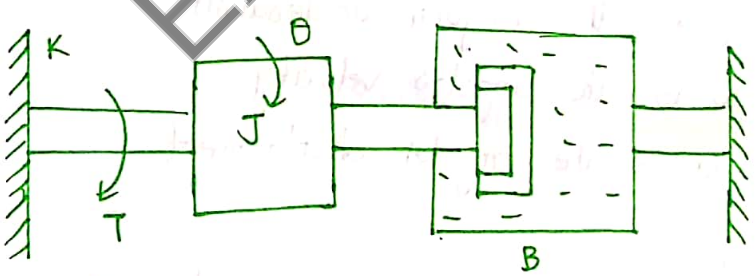
Where, T_b is the opposing torque due to friction of dashpot

B is the frictional coefficient

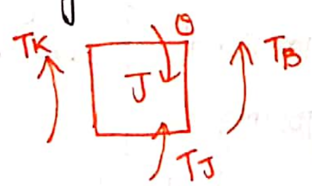
ω is the angular velocity &

θ is the angular displacement

Example-6:



Free Body diagram:



$$T = T_J + T_B + T_K$$

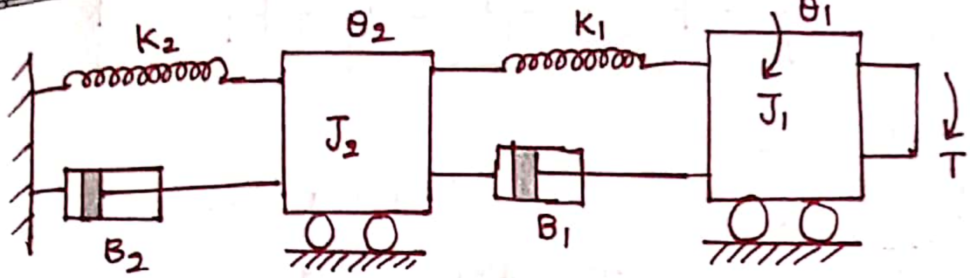
$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

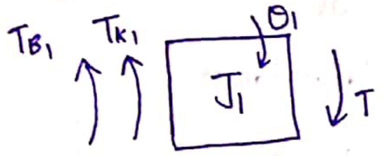
Apply laplace transform

$$T(s) = (Js^2 + Bs + K)\,\theta(s)$$

$$\boxed{\frac{\Theta(s)}{T(s)} = \frac{1}{Js^2+Bs+K}}$$

Example-7:



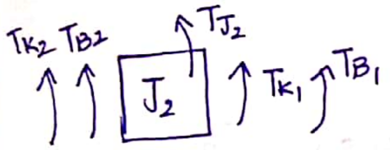


$$T = T_{J_1} + T_{B_1} + T_{K_1}$$

$$T = J_1 \frac{d^2\theta}{dt^2} + B_1 \frac{d}{dt}(\theta_1 - \theta_2) + K_1(\theta_1 - \theta_2)$$

Apply laplace transform

$$\boxed{T(s) = (J_1 s^2 + B_1 s + k_1)\,\theta_1(s) - (B_1 s + k_1)\,\theta_2(s)} \longrightarrow (1)$$



$$K_1(\theta_2 - \theta_1) + B_1 \frac{d}{dt}(\theta_2 - \theta_1) + J_2 \frac{d^2\theta_2}{dt^2} + k_2\theta_2 + B_2 \frac{d\theta_2}{dt} = 0$$

$$(J_2 s^2 + B_1 s + B_2 s + k_1 + k_2)\,\theta_2(s) - (B_1 s + k_1)\,\theta_1(s) = 0$$

$$\boxed{\theta_1(s) = \frac{J_2 s^2 + (B_1 + B_2)s + (k_1 + k_2)}{B_1 s + k_1}\,\theta_2(s)} \longrightarrow (2)$$

From equation (1),

$$T(s) = \left[(J_1 s^2 + B_1 s + k_1)\left(\frac{J_2 s^2 + (B_1 + B_2)s + (K_1 + k_2)}{B_1 s + k_1}\right) - (B_1 s + k_1)\right]\theta_2(s)$$

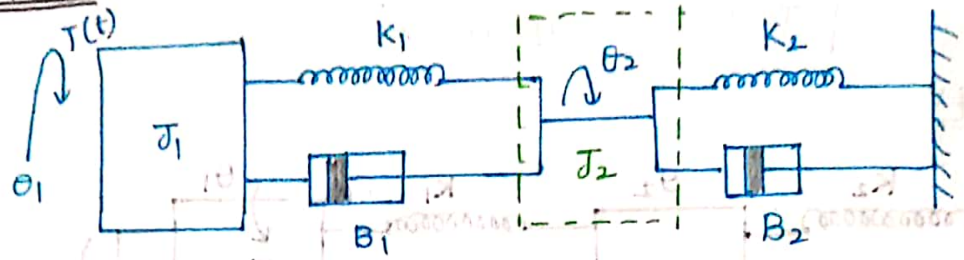
$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{B_1 s + k_1}{(J_1 s^2 + B_1 s + k_1)(J_2 s^2 + (B_1 + B_2)s + (k_1 + k_2)) - (B_1 s + k_1)^2}}$$

From equation (2),

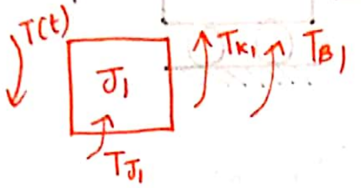
$$\theta_2(s) = \frac{B_1 s + k_1}{J_2 s^2 + (B_1 + B_2)s + (k_1 + k_2)}\,\theta_1(s)$$

Sub this in eqn (1) we get $\frac{\theta_1(s)}{T(s)}$

Example-8 :



Assume J_2



$$T = T_{B_1} + T_{J_1} + T_{K_1}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d}{dt}(\theta_1 - \theta_2) + k_1(\theta_1 - \theta_2)$$

Apply laplace transform

$$\boxed{T(s) = (J_1 s^2 + B_1 s + k_1)\,\theta_1(s) - (B_1 s + k_1)\,\theta_2(s)} \quad (1)$$

$$J_2 \frac{d^2\theta}{dt^2} + B_1 \frac{d}{dt}(\theta_2 - \theta_1) + k_1(\theta_2 - \theta_1) + k_2\theta_2 + B_2 \frac{d\theta_2}{dt} = 0$$

$$\left[J_2 s^2 + (B_1 + B_2)s + (k_1 + k_2)\right]\theta_2(s) = (B_1 s + k_1)\left[\theta_1(s)\right]$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{B_1 s + k_1}{(J_1 s^2 + B_1 s + k_1)(J_2 s^2 + (B_1 + B_2)s + (K_1 K_2)) - (B_1 s + K_1)^2}}$$

Sub $J_1 = 1\ kgm^2$, $J_2 = 0$, $K_1 = K_2 = 1N$, $B_1 = B_2 = 1\ Nm/rad$

$$\frac{\theta_2(s)}{T(s)} = \frac{s+1}{(s^2+s+1)(2s+2) - (s+1)^2}$$

$$= \frac{1}{2s^2+2s+2-s-1}$$

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{1}{2s^2+s+1}}$$

Mathematical modelling of Electrical Systems

Electrical Resistance, Inductance and Capacitance:

1. Resistance

V-I in time domain

$$V_R(t) = i_R(t)\,R$$

V-I in s domain

$$V_R(s) = I_R(s)\,R$$

2. Inductance

V-I in time domain

$$V_L(t) = L\frac{di_L(t)}{dt}$$

V-I in s-domain

$$V_L(s) = sLI_L(s)$$

3. Capacitance

VI in time domain

$$V_c(t) = \frac{1}{c}\int i_c(t)\,dt$$

V-I in s domain

$$V_c(s) = \frac{1}{cs} I_c(s)$$

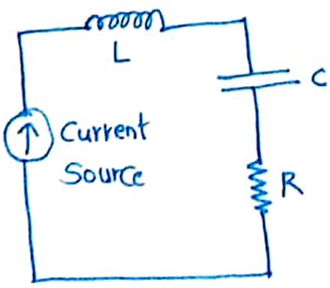
Equivalent Impedance:

$$Z_R(s) = R$$

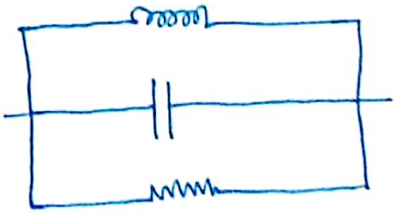
$$Z_L(s) = Ls$$

$$Z_c(s) = \frac{1}{cs}$$

Series/Parallel Impedance.

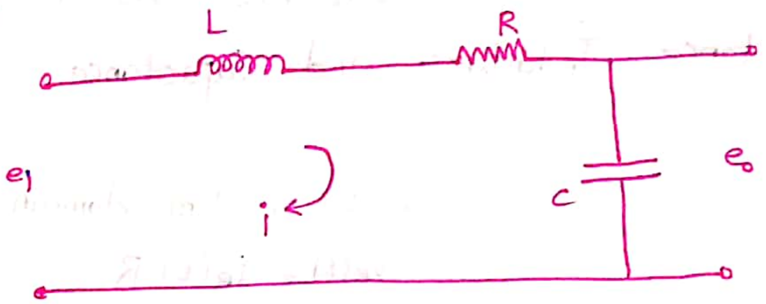


$$Z_T = Z_1 + Z_2 + Z_3$$



$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

Example-9:



$$e_i = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt$$

Taking laplace,

$$E_i(s) = LsI(s) + RI(s) + \frac{1}{Cs}I(s)$$

$$E_i(s) = \left(Ls + R + \frac{1}{Cs}\right)I(s) \quad \text{--- (1)}$$

$$e_o = \frac{1}{C}\int i\,dt$$

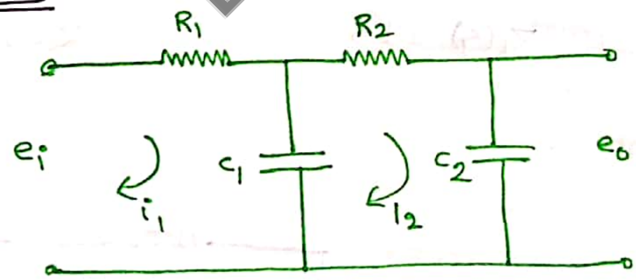
Taking laplace,

$$E_o(s) = \frac{1}{Cs}I(s) \quad \text{--- (2)}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{Cs}I(s)}{\left(Ls + R + \frac{1}{Cs}\right)I(s)}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1}}$$

Example-10:



$$e_i(t) - i_1(t)R_1 + \left[\frac{-1}{C_1}\int (i_1 - i_2)\,dt\right] = 0$$

Taking laplace,

$$E_i(s) - R_1I_1(s) + \left[\frac{-1}{C_1 s}I_1(s) - \frac{1}{C_1 s}I_2(s)\right] = 0$$

$$E_i(s) = \left(R_1 + \frac{1}{C_1 s}\right)I_1(s) - \frac{1}{C_1 s}I_2(s) \quad \text{--- (1)}$$

$$+R_2 i_2(t) + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_1}\int (i_2 - i_1)\,dt = 0$$

$$R_2 I(s) + \frac{1}{C_2 s} I_2(s) + \frac{1}{C_1 s} I_2(s) - \frac{1}{C_1 s} I_1(s) = 0$$

$$\left(R_2 + \frac{1}{C_2 s} + \frac{1}{C_1 s}\right) I_2(s) - \frac{1}{C_1 s}(I_1 s) = 0 \quad \text{--- (2)}$$

$$e_0(t) = \frac{1}{C_2}\int i_2\,dt$$

Taking laplace,

$$E_0(s) = \frac{1}{C_2 S} I_2(s) \quad \text{--- (3)}$$

In equation (1), sub $I_1(s)$ from eqⁿ (2)

$$I_1(s) = C_1 s\left(R_2 + \frac{1}{C_2 S} + \frac{1}{C_1 S}\right) I_2(s)$$

From equation (1),

$$E_i(s) = \left(R_1 + \frac{1}{C_1 s}\right)\left(R_2 + \frac{1}{C_2 S} + \frac{1}{C_1 S}\right)(C_1 s) I_2(s) - \frac{I_2(s)}{C_1 S}$$

$$= \left[\left(R_1 + \frac{1}{C_1 S}\right)\frac{(C_1 C_2 S^2 R_2 + C_1 S + C_2 S)(C_1 S)}{C_1 C_2 S^2} - \frac{1}{C_1 S}\right] I_2(s)$$

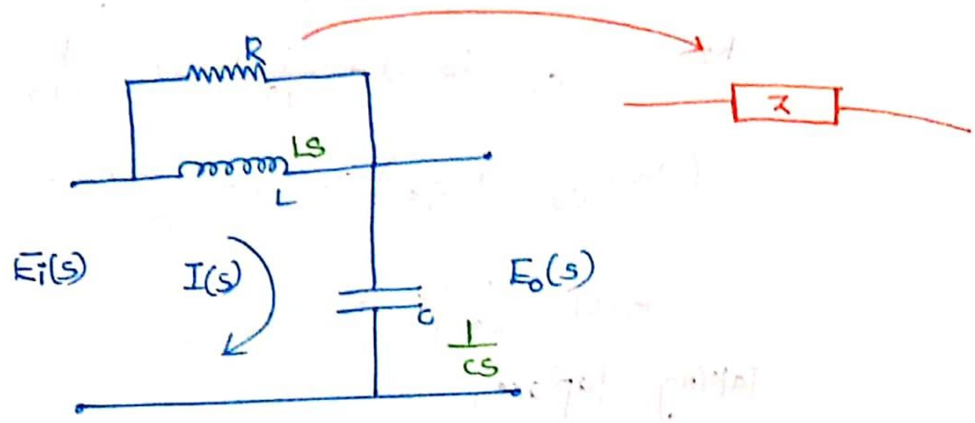
$$= \left[\left(R_1 + \frac{1}{C_1 S}\right)\left(\frac{C_1 C_2 R_2 S^2 + C_1 S + C_2 S}{C_2 S}\right) - \frac{1}{C_1 S}\right] I_2(s)$$

$$E_i(s) = \frac{(R_1 C_1 s + 1)(R_2 C_1 C_2 S^2 + C_1 S + C_2 S) - (C_2 S)}{C_1 C_2 S^2} I_2(s)$$

$$\frac{E_0(s)}{E_i(s)} = \frac{\frac{1}{C_2 s} I_2(s)}{\left[\frac{(R_1 C_1 s + 1)(R_2 C_1 C_2 s^2 + C_1 s + C_2 S) - C_2 S}{C_1 C_2 s^2}\right] I_2(s)}$$

$$\boxed{\frac{E_0(s)}{E_i(s)} = \frac{C_1 S}{\left[(R_1 C_1 S + 1)(R_2 C_1 C_2 S^2 + C_1 S + C_2 S) - C_2 S\right]}}$$

Example-11:



R and Ls are parallel

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{R\,LS}{R+LS}$$

$$E_i(s) = Z_T\, I(s) + \frac{1}{cs} I(s)$$

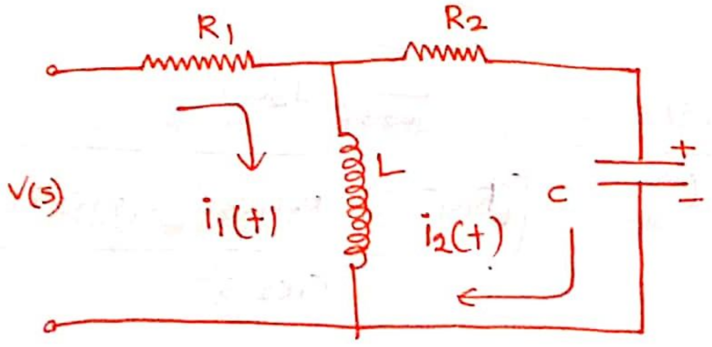
$$= \left[\frac{RLS}{R+LS} + \frac{1}{cs}\right] I(s) \quad \text{--- (1)}$$

$$E_o(s) = \frac{1}{cs} I(s) \quad \text{--- (2)}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{cs}}{\frac{RLCs^2 + R + LS}{(R+LS)\,cs}}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{R+LS}{RLCs^2 + LS + R}}$$

Example-12:



$$V(s) = R_1 I_1(s) + Ls\left[I_1(s) - I_2(s)\right]$$

$$V(s) = (R_1 + LS)\, I_1(s) - LS\, I_2(s) \quad \text{--- (1)}$$

$$R_2 I_2(s) + \frac{1}{Cs} I_2(s) + Ls\left(I_2(s) - I_1(s)\right) = 0$$

$$\left(R_2 + \frac{1}{Cs} + Ls\right) I_2(s) = Ls\, I_1(s)$$

$$\boxed{I_1(s) = \frac{LCs^2 + R_2 Cs + 1}{LCs^2} I_2(s)}$$

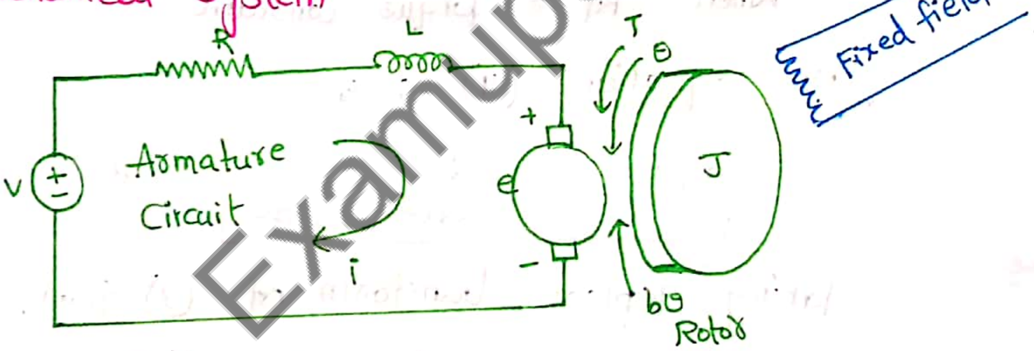
From equation (1),

$$V(s) = (R_1 + Ls)\left(\frac{LCs^2 + R_2 Cs + 1}{LCs^2}\right) I_2(s) - Ls\, I_2(s)$$

$$= \frac{(R_1 + Ls)(LCs^2 + R_2 Cs + 1) - L^2 Cs^3}{LCs^2} I_2(s)$$

$$\boxed{\frac{I_2(s)}{V(s)} = \frac{LCs^2}{(R_1 + Ls)(LCs^2 + R_2 Cs + 1) - L^2 Cs^3}}$$

Electromechanical System



Mathematical modelling DC Motor:

- An actuator, converting electrical energy into rotational mechanical energy.
- For this example, the input of the system is the Voltage Source applied to the motor's armature, while the output is the rotational speed of the shaft θ.

From circuit,

$$V = Ri(t) + L\frac{di}{dt} + e \quad \text{——— (1)}$$

We know that,

$$e = \frac{\phi PNZ}{60} \quad \text{then}$$

$e \propto N$ then

$$e \propto \frac{d\theta}{dt}$$

$$\boxed{e = K_b \frac{d\theta}{dt}} \longrightarrow (2)$$

Where K_b = back emf constant

From equation (1) & (2),

$$\boxed{V = Ri(t) + L\frac{di}{dt} + K_b \frac{d\theta}{dt}} \longrightarrow (3)$$

In mechanical circuit,

$$T = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} \longrightarrow (4)$$

$$T \propto \phi i$$

$$T \propto i$$

$$\boxed{T = K_T\, i(t)} \longrightarrow (5)$$

Where K_T = Torque Constant

From equation (4) & (5),

$$\boxed{K_T\, i(t) = J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt}} \longrightarrow (6)$$

Taking Laplace transform of (3) & (6).

$$V(s) = R\,I(s) + Ls\,I(s) + K_b s\,\theta(s)$$

$$\boxed{V(s) = (R+Ls)\,I(s) + K_b s\,\theta(s)} \longrightarrow (7)$$

$$K_T\, I(s) = Js^2\theta(s) + bs\theta(s)$$

$$\boxed{I(s) = \frac{(Js^2+bs)\,\theta(s)}{K_T}} \longrightarrow (8)$$

Sub eqn (8) in eqn (7)

$$V(s) = (R+Ls)\left(\frac{Js^2+bs}{K_T}\right)\theta(s) + K_b s\,\theta(s)$$

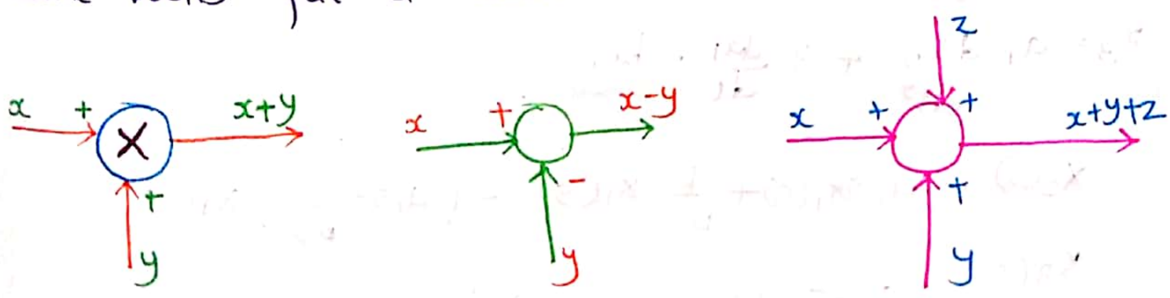
$$V(s) = \frac{[(R+Ls)(Js^2+bs) + K_b K_T s]}{K_T} \theta(s)$$

$$\boxed{\frac{\theta(s)}{V(s)} = \frac{K_T}{(R+Ls)(Js^2+bs) + K_b K_T s}}$$

Block Diagram Techniques:

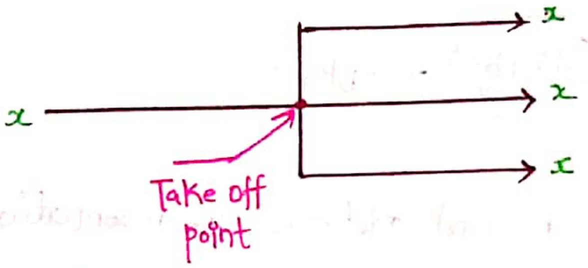
- A Block Diagram is a shorth and pictorial representation of the cause - and - effect relationship of a system.
- The interior of the rectangle representing the block usually contains a description of (or) the name of the element, gain (or) the symbol for the mathematical operation to be performed on the input to yield the output.
- The arrows represent the direction of information (or) signal flow.

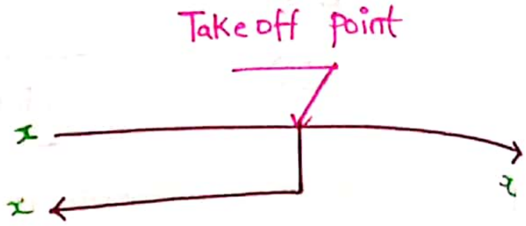
- The operations of addition and Subtraction have a special representation.
- The block ~~bemees~~ becomes a small circle, called a Summing point, with the appropriate plus (or) minus sign associated with the arrows entering the circle.
- The output is the algebraic Sum of the inputs
- Any number of inputs may enter a summing point.
- Some books put a Cross in the circle.



- In order to have the same signal (or) Variable be an input to more than one block (or) Summing point, a takeoff (or) pick off point is used.

- This permits the signal to proceed unaltered along several different paths to several destinations.

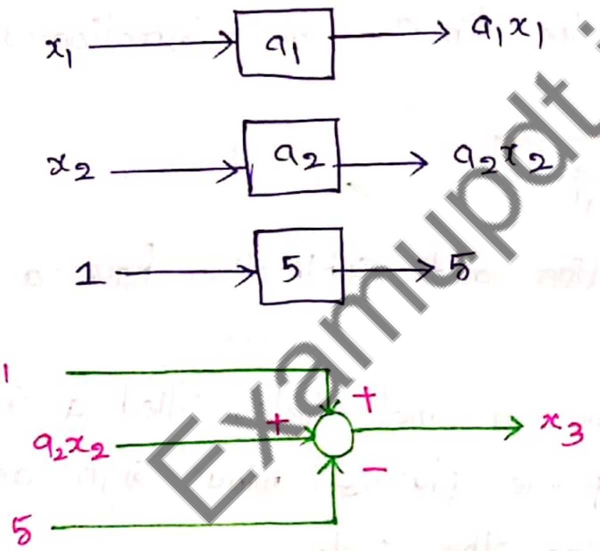




Example-1:

- Consider the following equations in which x_1, x_2, x_3 Variables and a_1, a_2 are general coefficients (or) mathematical operators.

$$x_3 = a_1x_1 + a_2x_2 - 5$$



Example-2:

1. Draw the block diagrams of the following equations

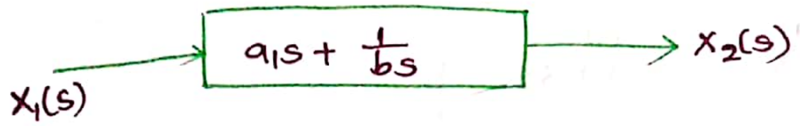
$$x_2 = a_1 \frac{dx_1}{dt} + \frac{1}{b}\int x_1\, dt$$

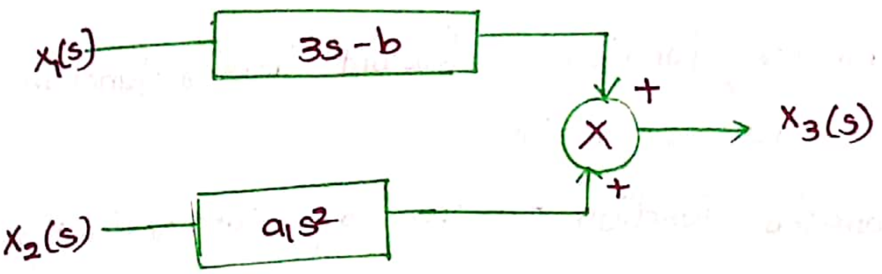
$$x_3 = a_1 \frac{d^2x_2}{dt^2} + 3\frac{dx_1}{dt} - bx_1$$

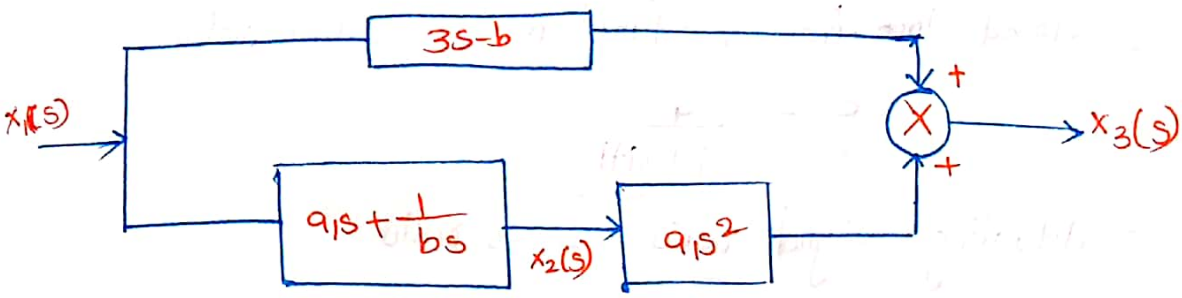
sol:-

$$X_2(s) = a_1 sX_1(s) + \frac{1}{bs} X_1(s) = \left(a_1 s + \frac{1}{bs}\right) X_1(s)$$

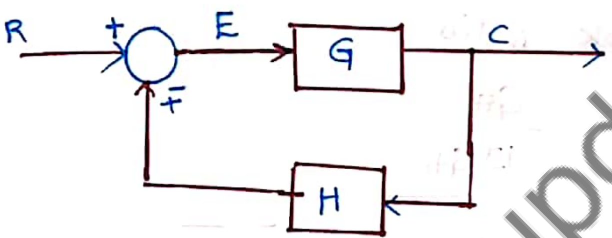
$$X_3(s) = a_1 s^2 X_2(s) + 3sX_1(s) - bX_1(s)$$







Canonical Form of a Feedback Control System:



$C = GE$

$B = HC = HGE$

$E = R \mp B$

$E = R \pm HGE$

$E(1 \pm GH) = R$

$$\boxed{\frac{E}{R} = \frac{1}{1 \pm GH}}$$

$$C = GE = G \times \frac{R}{1 \pm GH}$$

$$\boxed{\frac{C}{R} = \frac{G}{1 \pm GH}}$$

$$C = \frac{B}{H}$$

$$\frac{B/H}{R} = \frac{G}{1 \pm GH}$$

$$\boxed{\frac{B}{R} = \frac{GH}{1 \pm GH}}$$

Where

$G \equiv$ direct transfer function $\equiv$ forward transfer function.

$H \equiv$ feedback transfer function

$GH \equiv$ loop transfer function $\equiv$ open-loop transfer function.

$\frac{C}{R} \equiv$ closed-loop transfer function $\equiv$ control ratio.

$$\boxed{\frac{C}{R} = \frac{G}{1 \pm GH}}$$

$\frac{E}{R}$ = Actuating signal ratio $\equiv$ error ratio.

$$\boxed{\frac{E}{R} \equiv \frac{1}{1 \pm GH}}$$

$\frac{B}{R}$ = primary feedback ratio

$$\boxed{\frac{B}{R} = \frac{GH}{1 \pm GH}}$$

Characteristic Equation:

- The control ratio is the closed loop transfer of the system

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}}$$

- The denominator of closed loop transfer function determines the characteristic equation of the system. which is usually determined as

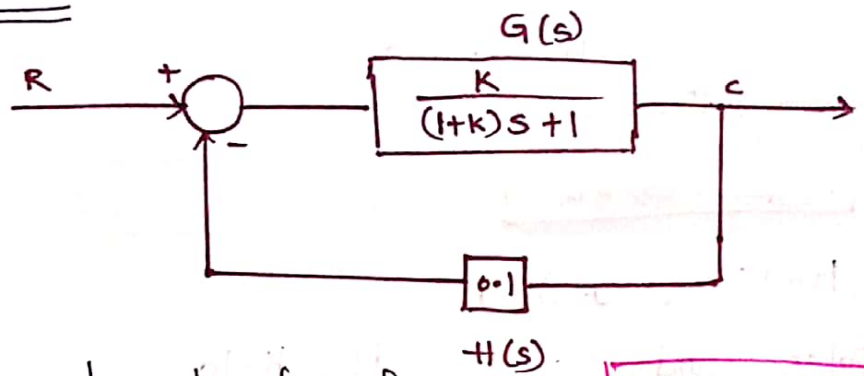
$$\boxed{1 \pm G(s)\,H(s) = 0}$$

Roots of characteristic equation = Poles of the system.

Zero's. ~~of~~ = Roots of numerator polynomial of the Transfer function.

Transfer function = Input/output relationship

Example -3:



1. Open loop transfer function

$$\boxed{\frac{B(s)}{E(s)} = G(s)H(s)}$$

$$\frac{B(s)}{E(s)} = \frac{0.1K}{(1+K)s+1}$$

2. Feed forward Transfer function

$$\boxed{\frac{C(s)}{E(s)} = G(s)}$$

$$\frac{C(s)}{E(s)} = \frac{K}{(1+K)s+1}$$

3. Control ratio.

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(1+K)s+(1+0.1K)}$$

4. Feedback ratio

$$\boxed{\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}}$$

$$\frac{B(s)}{R(s)} = \frac{0.1K}{(1+K)s+(1+0.1K)}$$

5. Error ratio.

$$\boxed{\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}}$$

$$\frac{E(s)}{R(s)} = \frac{(1+K)s+1}{(1+K)s+(1+0.1K)}$$

6. Closed loop transfer function.

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(1+K)s + (1+0.1K)}$$

7. Characteristic equation.

$$\boxed{1+G(s)H(s) = 0}$$

$$(1+K)s + (1+0.1K) = 0$$

8. Open loop poles and zeroes if $K=10$

zeroes = Nil

poles, $(1+K)s+1 = 0$

$(1+10)s+1 = 0$

$$\boxed{s = \frac{-1}{11}}$$

9. Closed loop poles and zero's if $K=10$

zero's = Nil

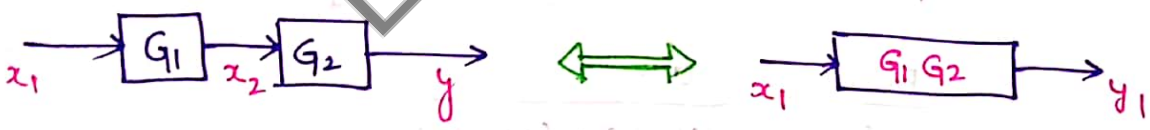
poles, $(1+K)s + 1 + 0.1K = 0$

$(1+10)s + 1 + 0.1(10) = 0$

$$\boxed{s = \frac{-2}{11}}$$

Block Reduction Techniques.

1. Combining blocks in Cascade.



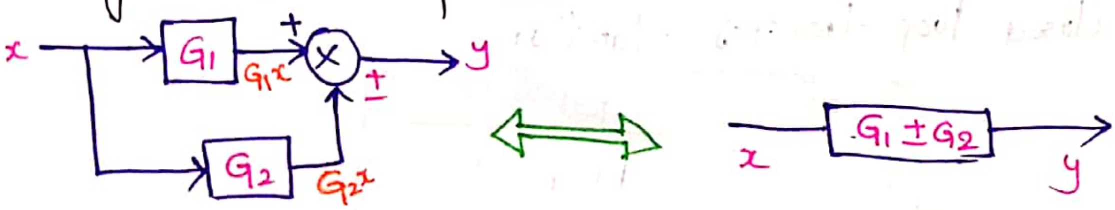
$y = G_2 x_2$

$x_2 = G_1 x_1$

$y = G_1 G_2 x_1$

$$\therefore \boxed{\frac{y}{x_1} = G_1 G_2}$$

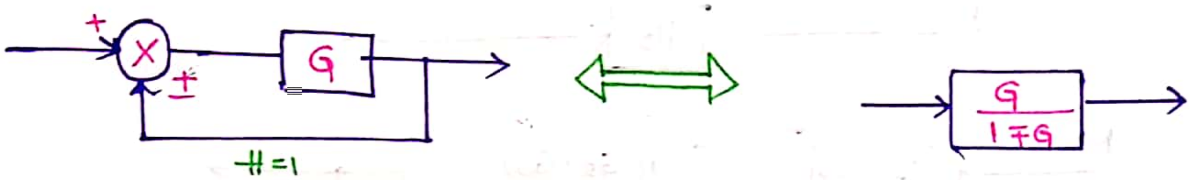
2. Combining blocks in parallel.



$$y = G_1x \pm G_2x = (G_1 \pm G_2)x$$

$$\boxed{\frac{y}{x} = G_1 \pm G_2}$$

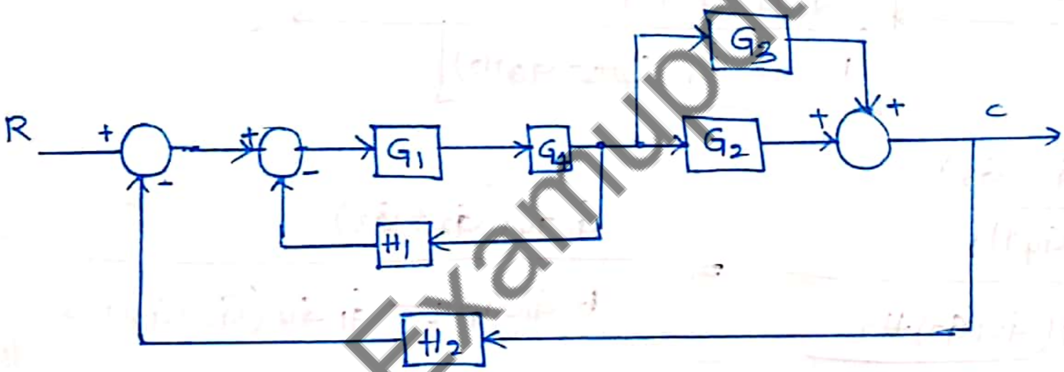
3. Eliminating a feedback loop



Unity Feedback

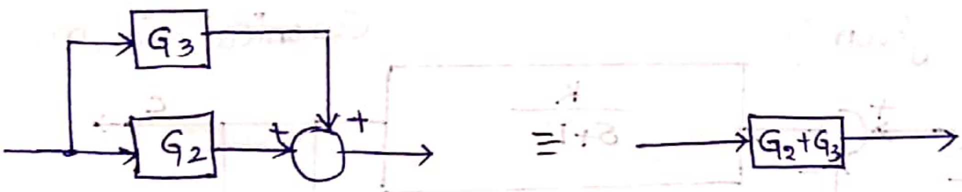
Example-4:

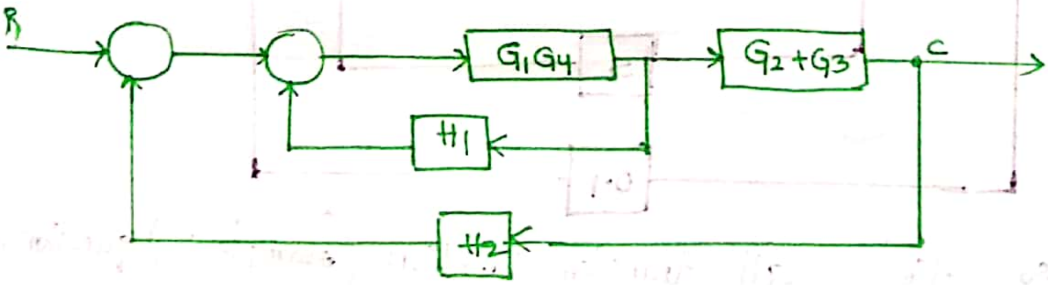
Reduce the block diagram to canonical form.



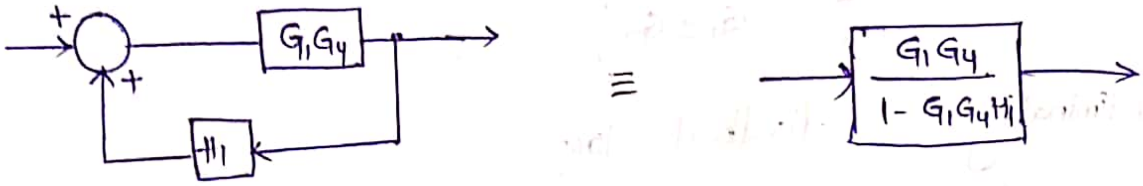
Step-1: Combine all cascade blocks.

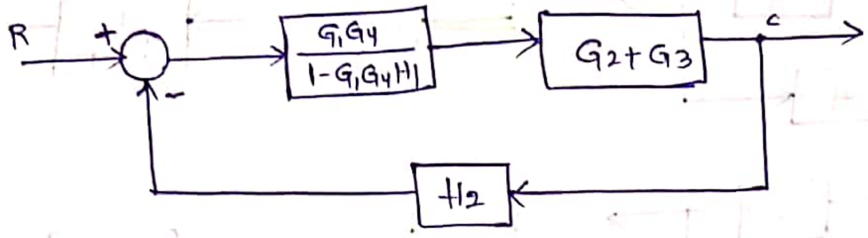
Step-2: Combine all parallel blocks

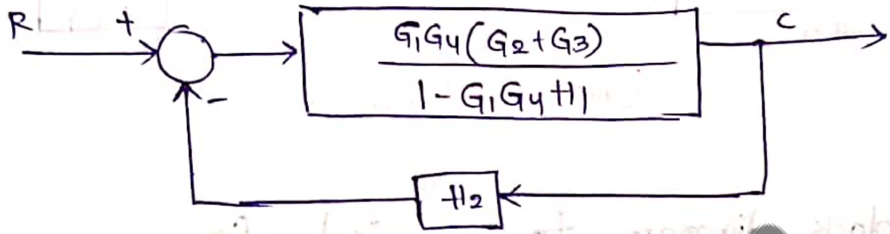


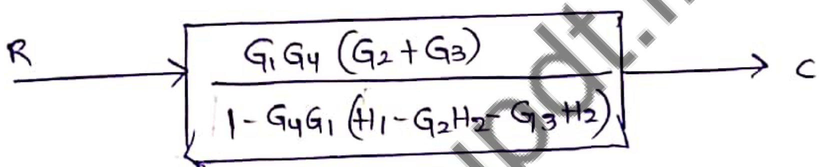


Step-3: Eliminate all minor feedback loops.







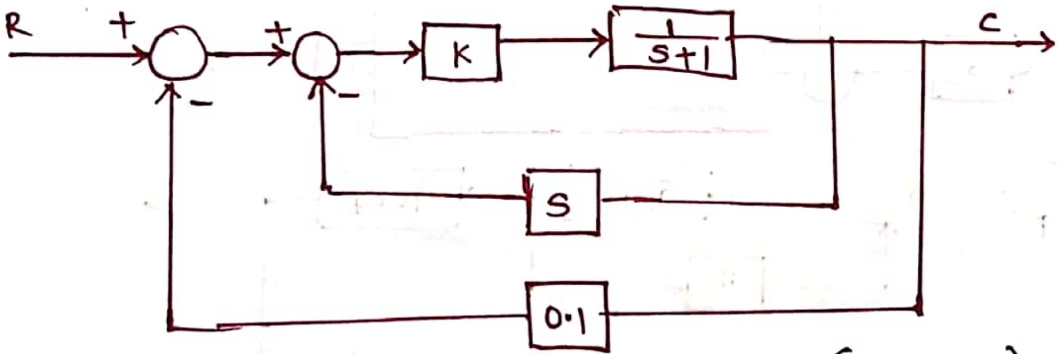


$$\frac{\dfrac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1}}{1-\dfrac{G_1G_4(G_2+G_3)H_2}{1-G_1G_4H_1}} = \frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1-G_1G_4(G_2+G_3)H_2}$$

$$= \frac{G_1G_4(G_2+G_3)}{1-G_1G_4(H_1-G_2H_2-G_3H_2)}$$

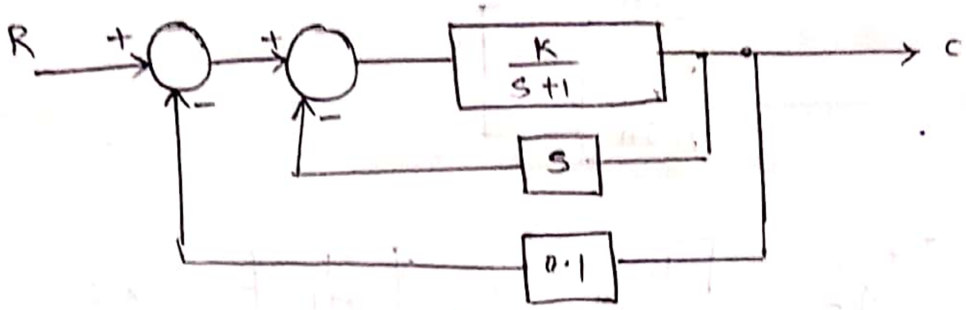
Example-5:

Reduce the given block diagram to Canonical form.

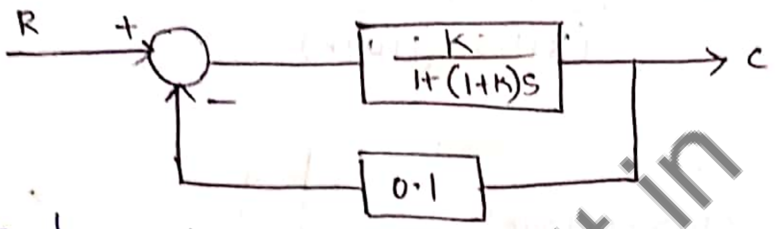


And also find all transfer function (example-3) questions?

Step-1: Combining blocks in cascade:



$$\frac{G}{1+GH} = \frac{\frac{K}{s+1}}{1+\frac{K}{s+1}s} = \frac{K}{1+(1+K)s}$$



1) Open loop transfer function

$$\frac{B(s)}{E(s)} = G(s)H(s) = \frac{0.1K}{(1+K)s+1}$$

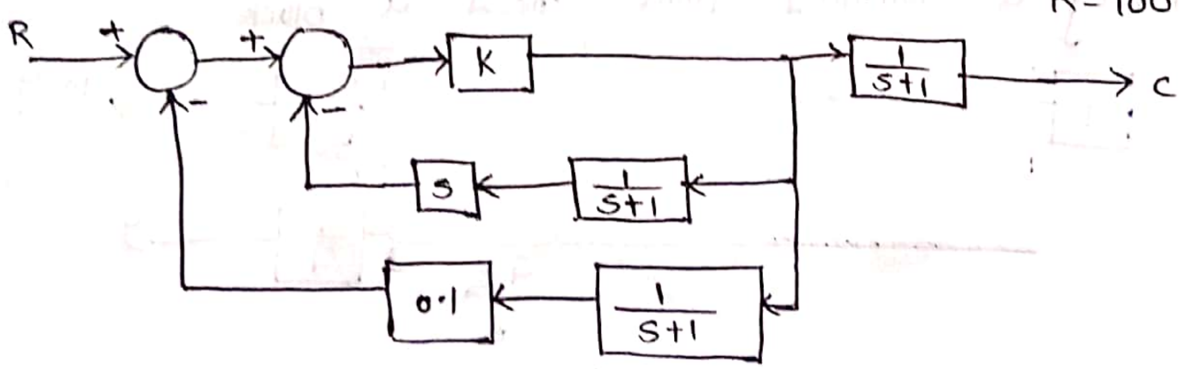
Example-6:

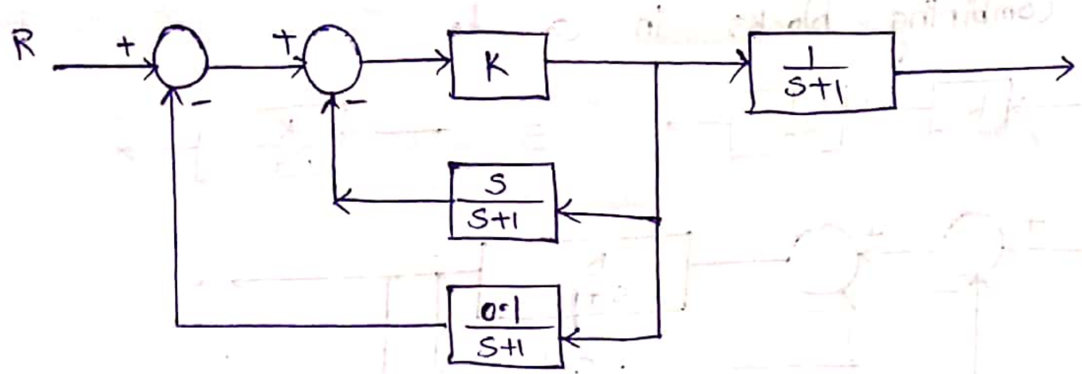
For the system represented by the following block diagram. Determine:

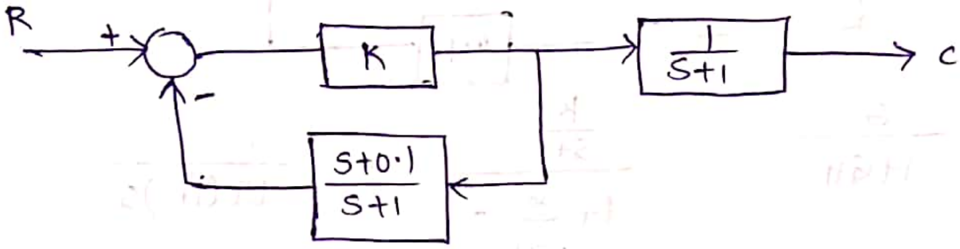
1) open loop T.F 2) Feed forward T.F 3) control ratio.

4) Feed back ratio 5) Error ratio 6) closed loop T.F

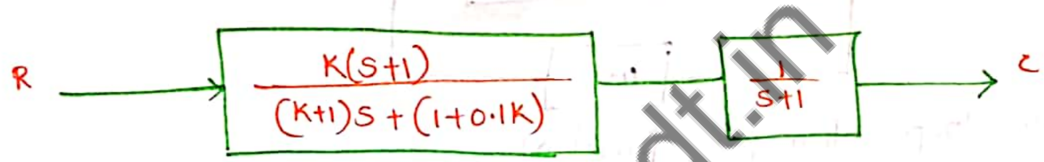
7) characteristic equation 8) closed loop poles and zero's if $K=100$.





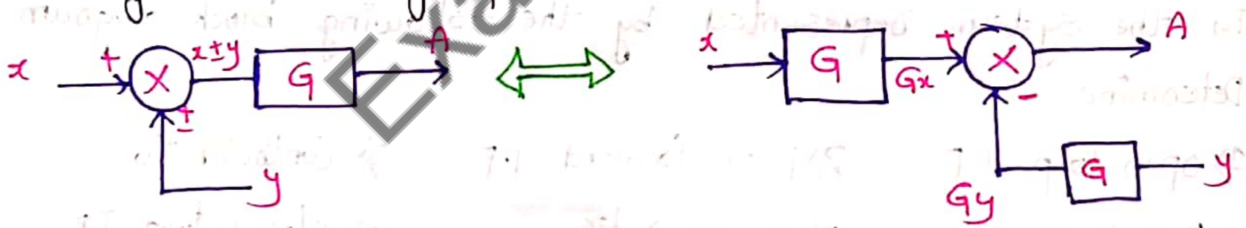


$$\frac{K}{1+K\left(\frac{S+0.1}{S+1}\right)} = \frac{K(S+1)}{(K+1)S+(1+0.1K)}$$



Reduction Techniques:

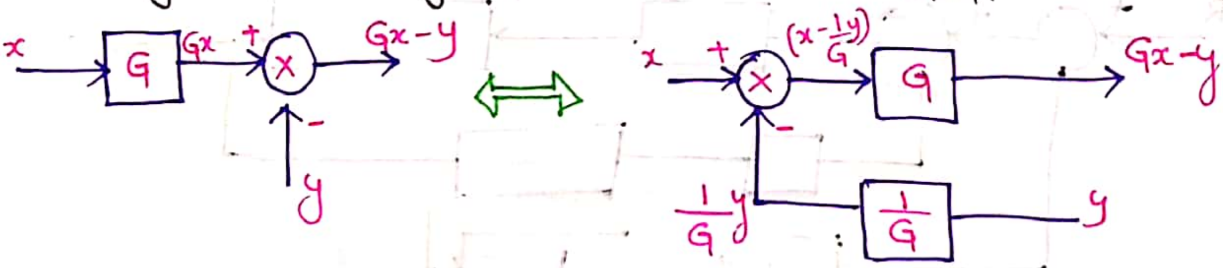
4) Moving a Summing point behind a block



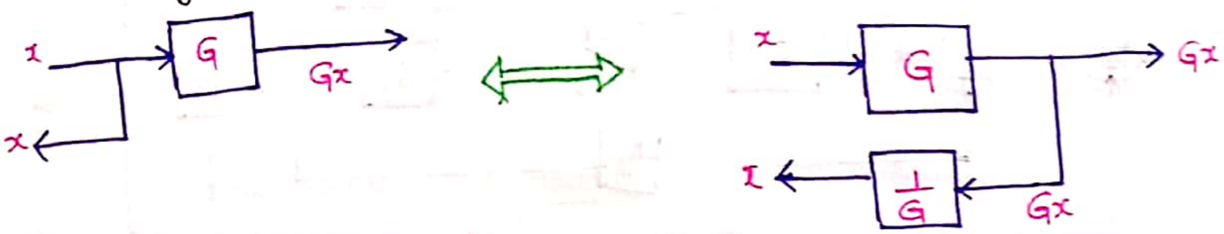
$A = G(x \pm y)$

$A = Gx \pm Gy$

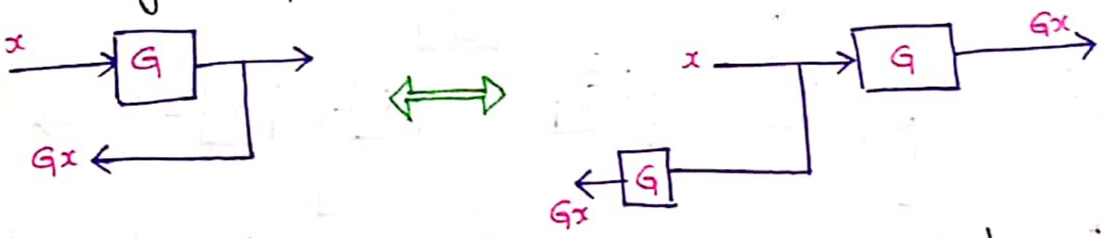
5) Moving a Summing point ahead a block



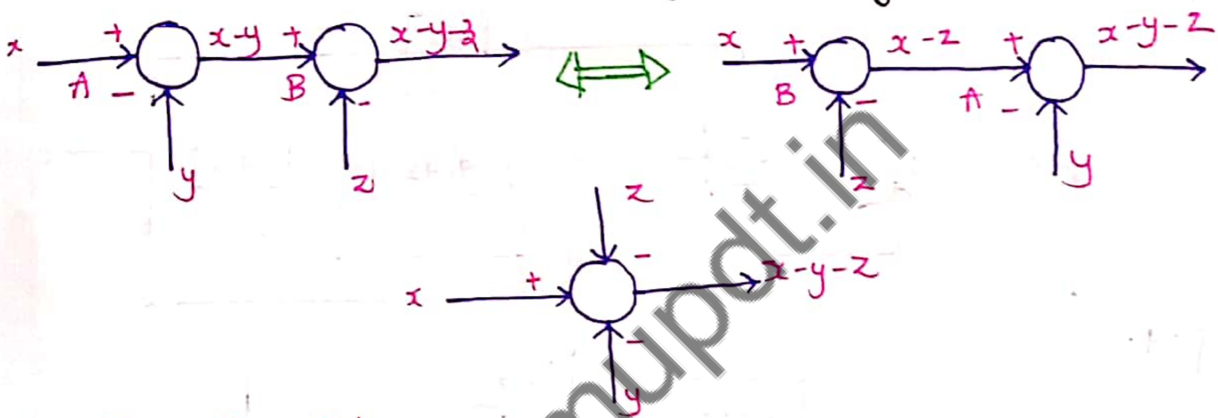
6) Moving a pick off point behind



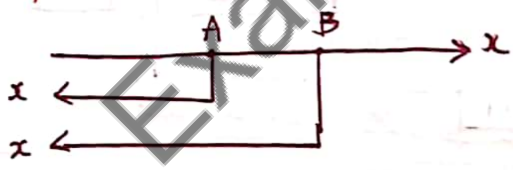
7) Moving a pickoff point after a block.



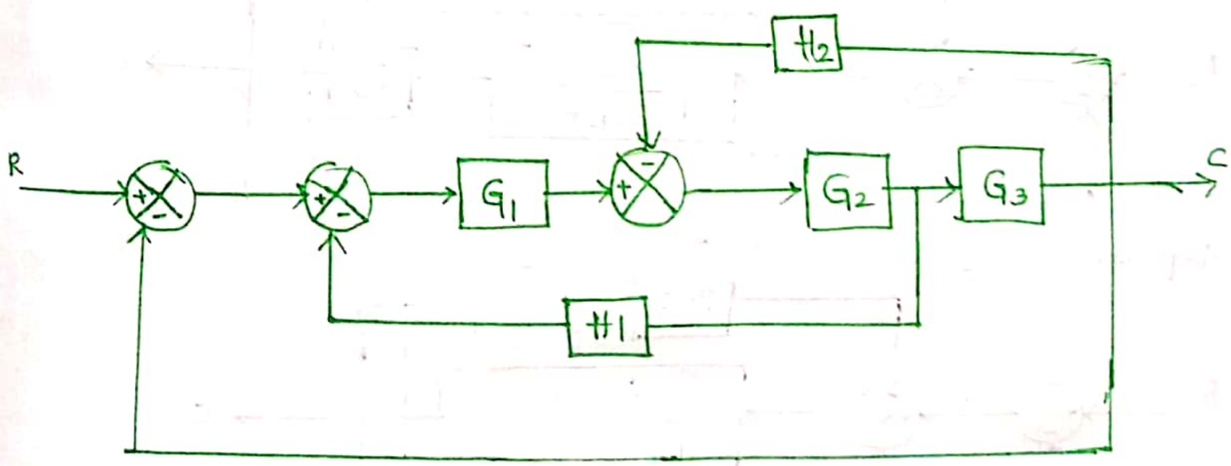
8) Swap with two neighbouring Summing points



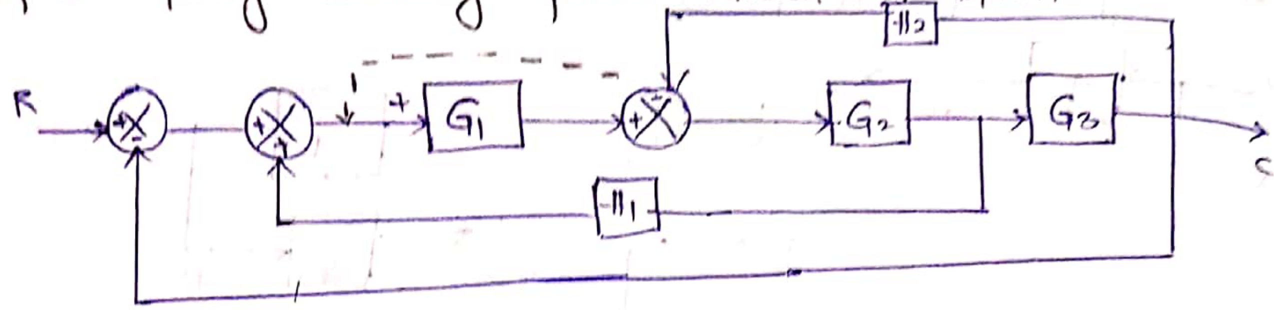
Two Take-off points



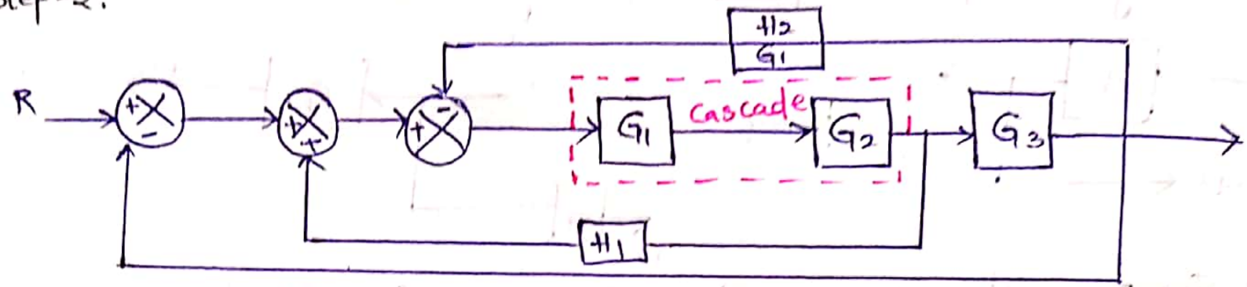
Example-7:



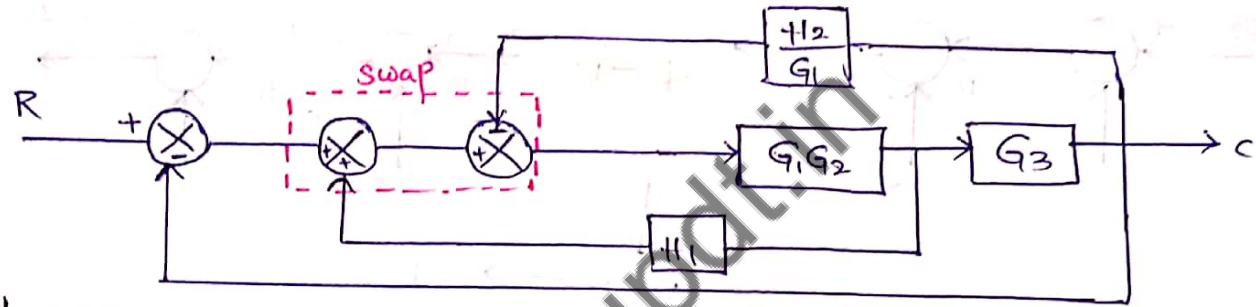
Step-1: Moving Summing point ahead of block.



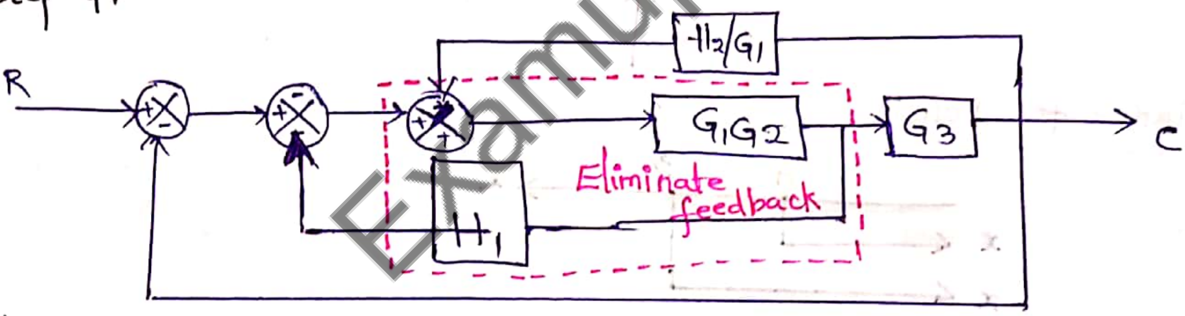
Step-2:



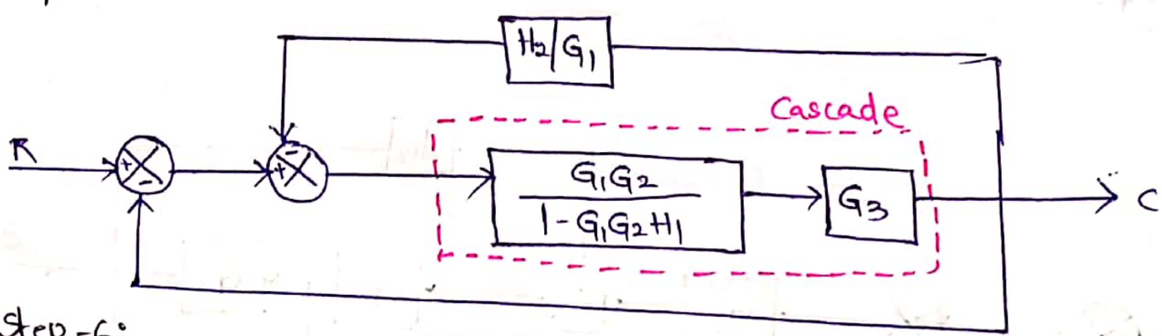
Step-3:



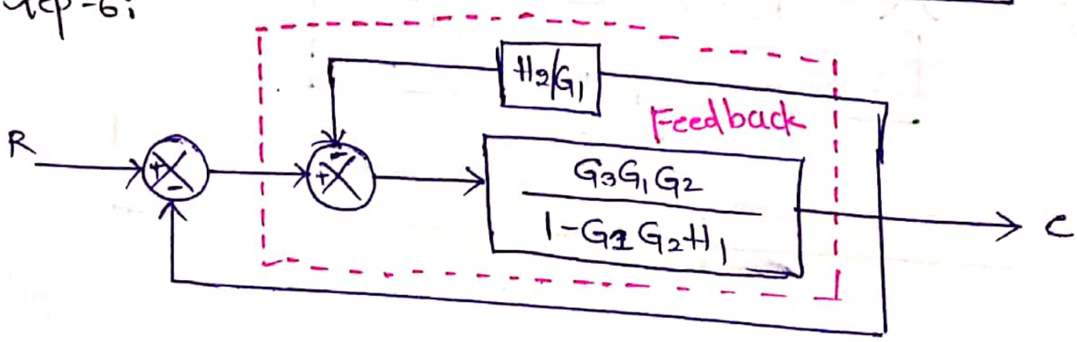
Step-4:



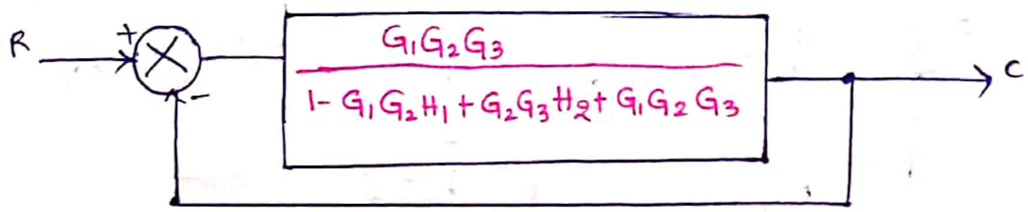
Step-5:



Step-6:



Step-7:

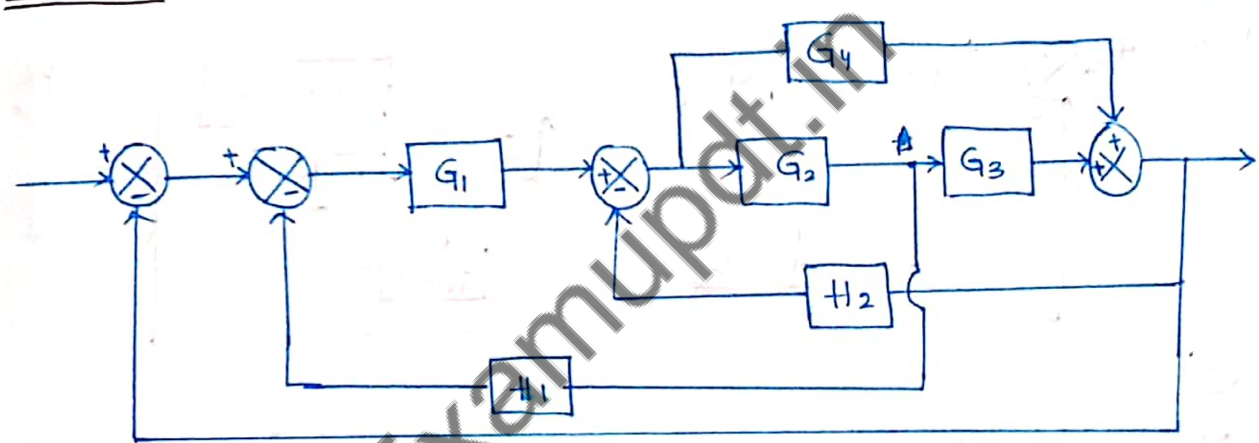


$\frac{G}{1+GH}$ and $H=1$ then

$\frac{G}{1+G}$ and $\frac{C}{R} = \dfrac{\frac{G_1G_2G_3}{\Delta}}{1+\frac{G_1G_2G_3}{\Delta}}$

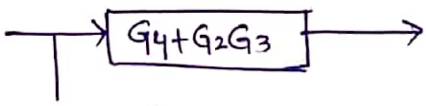
$$\frac{C}{R} = \frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3}$$

Example-8:

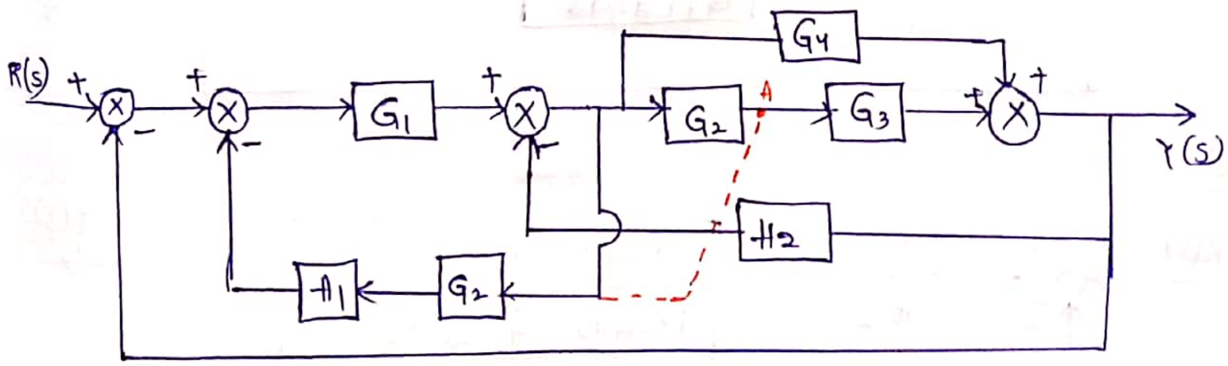


1. Moving pickoff point A ahead of block $\boxed{G_2}$

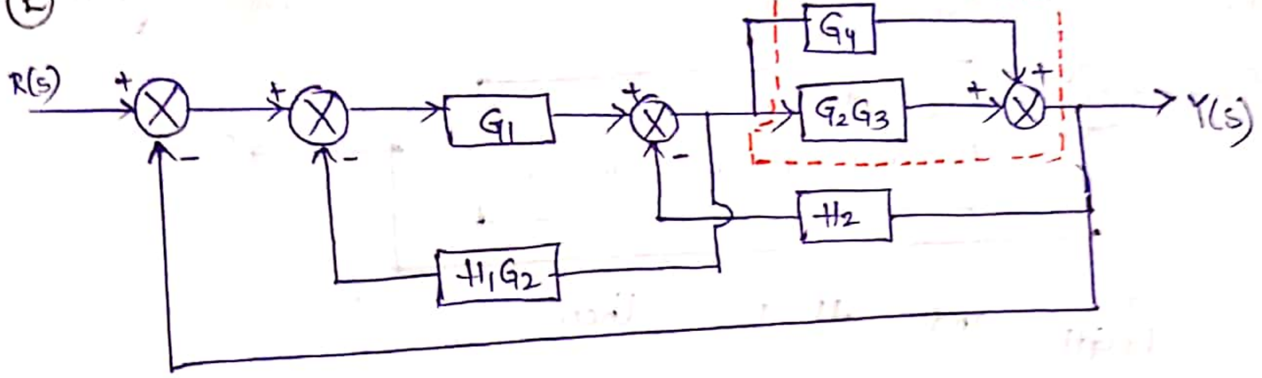
2. Eliminate loop 1 and simplify



(1)

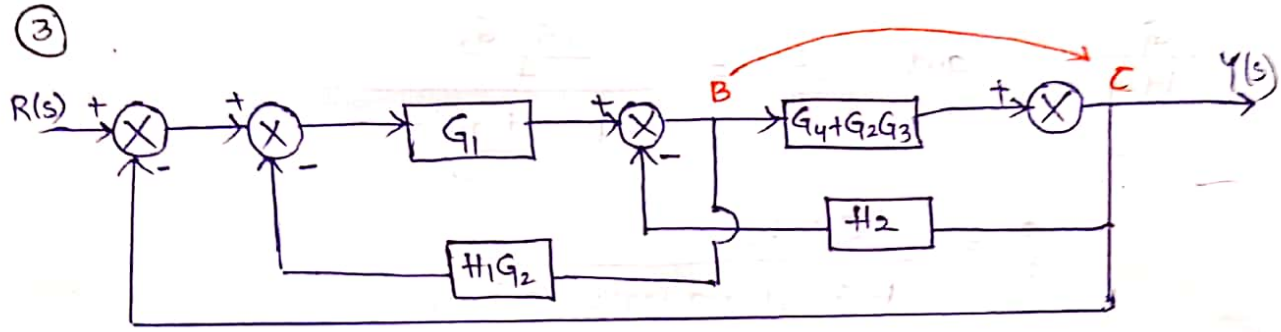


②



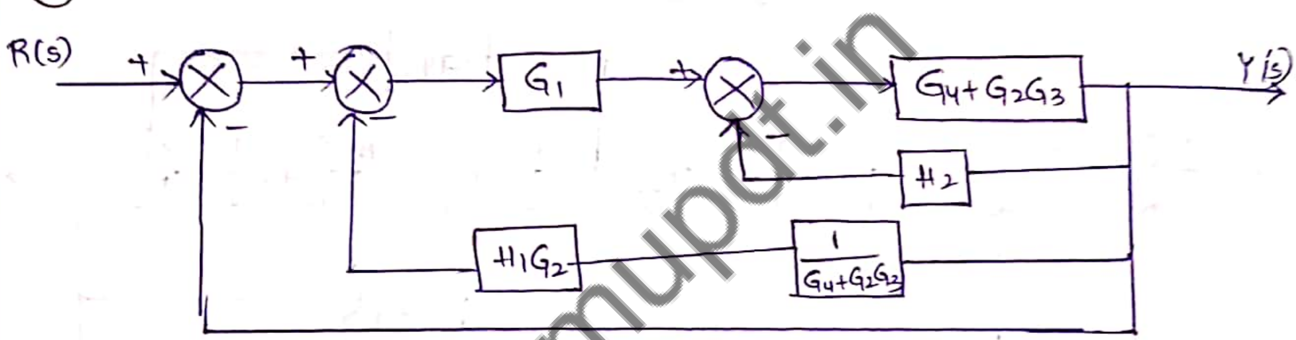
R(s)
G_4
G_2G_3
G_1
H_2
H_1G_2
Y(s)

③



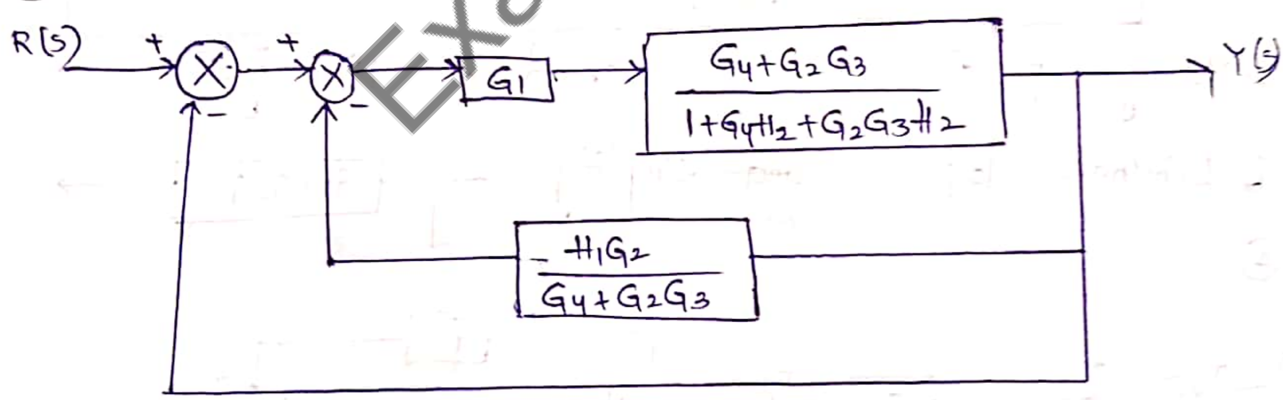
R(s)
G_1
B
$G_4+G_2G_3$
C
H_2
H_1G_2
Y(s)

④



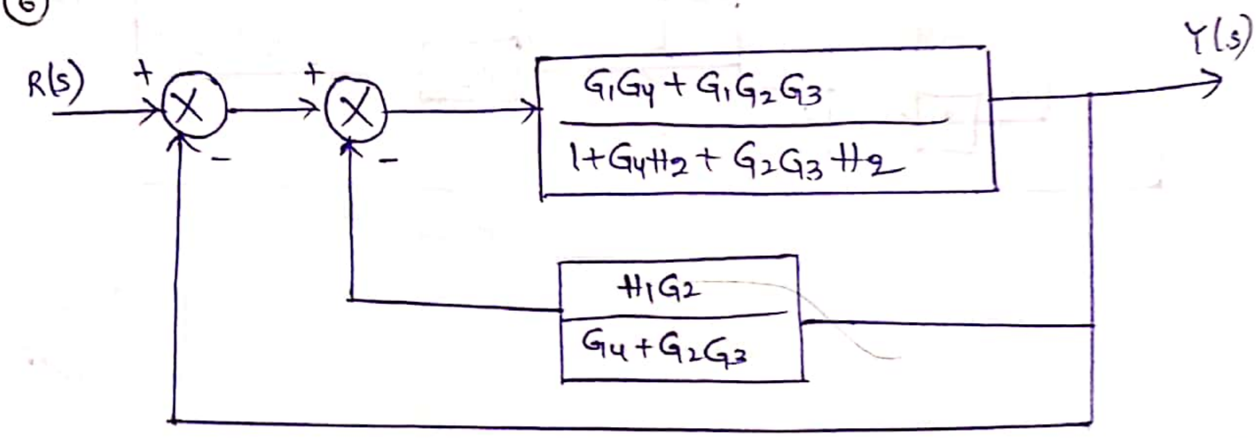
R(s)
G_1
$G_4+G_2G_3$
H_2
H_1G_2
$\frac{1}{G_4+G_2G_3}$
Y(s)

⑤



R(s)
G_1
$\frac{G_4+G_2G_3}{1+G_4H_2+G_2G_3H_2}$
$\frac{H_1G_2}{G_4+G_2G_3}$
Y(s)

⑥

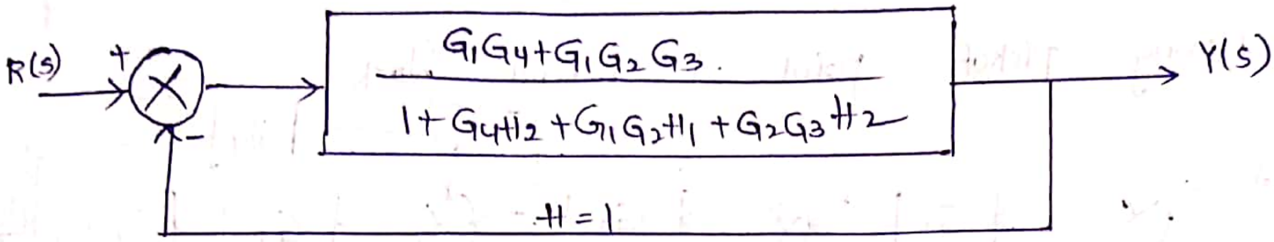


R(s)
$\frac{G_1G_4+G_1G_2G_3}{1+G_4H_2+G_2G_3H_2}$
$\frac{H_1G_2}{G_4+G_2G_3}$
Y(s)

$$\frac{G}{1 \mp GH} = \frac{\dfrac{G_1G_4+G_1G_2G_3}{1+G_4H_2+G_2G_3H_2}}{1+\dfrac{H_1G_2}{G_4+G_2G_3}\left(\dfrac{G_1(G_4+G_2G_3)}{1+G_4H_2+G_2G_3H_2}\right)}$$

$$= \frac{G_1G_4+G_1G_2G_3}{1+G_4H_2+G_1G_2H_1+G_2G_3H_2}$$

(7)

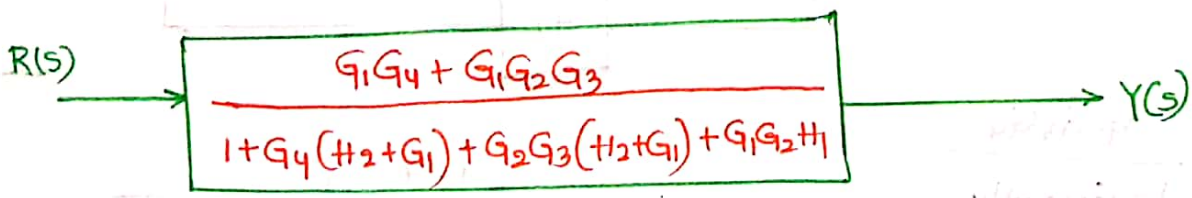


$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_4+G_1G_2G_3}{1+G_4H_2+G_1G_2H_1+G_2G_3H_2}}{1+\dfrac{G_1G_4+G_2G_1G_3}{1+G_4H_2+G_2G_3H_2+G_1G_2H_1}(1)}$$

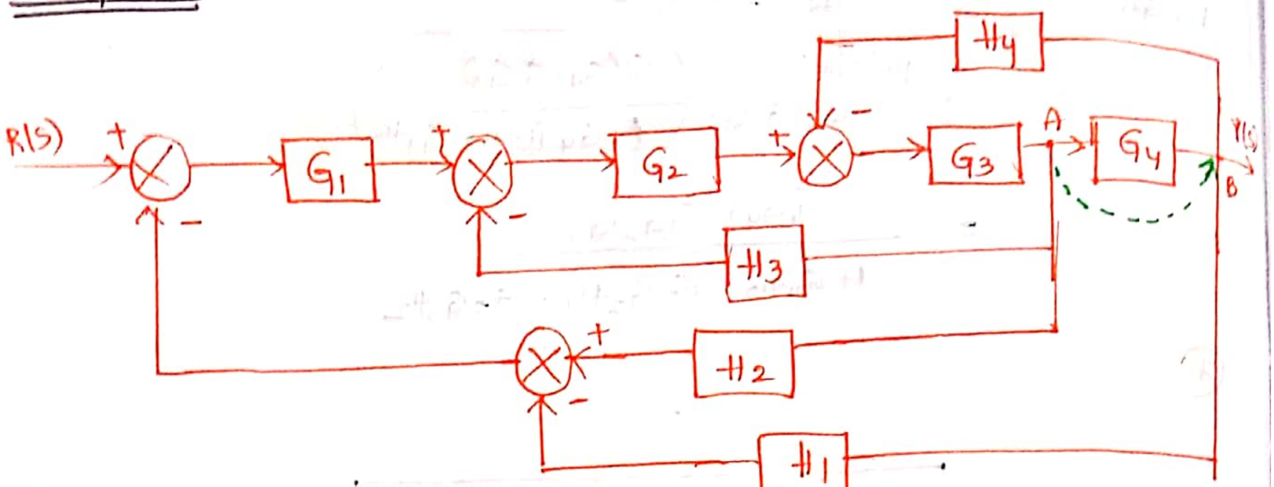
$$= \frac{G_1G_4+G_1G_2G_3}{1+G_4H_2+G_2G_3H_2+G_1G_2H_1+G_1G_4+G_1G_2G_3}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1G_4+G_1G_2G_3}{1+G_4(H_2+G_1)+G_2G_3(H_2+G_1)+G_1G_2H_1}}$$

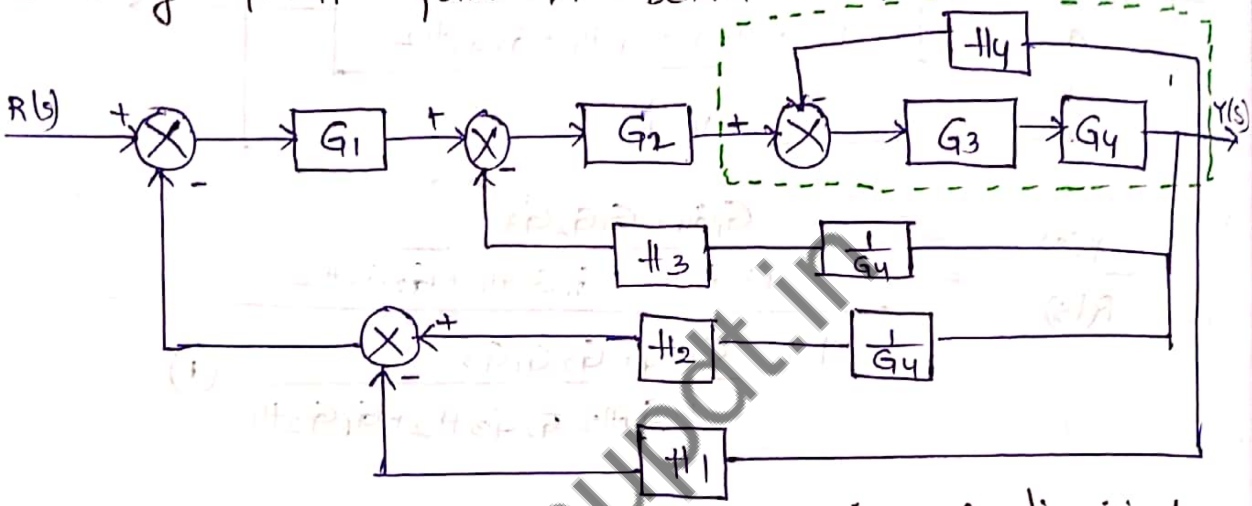
(8)



Example-9:

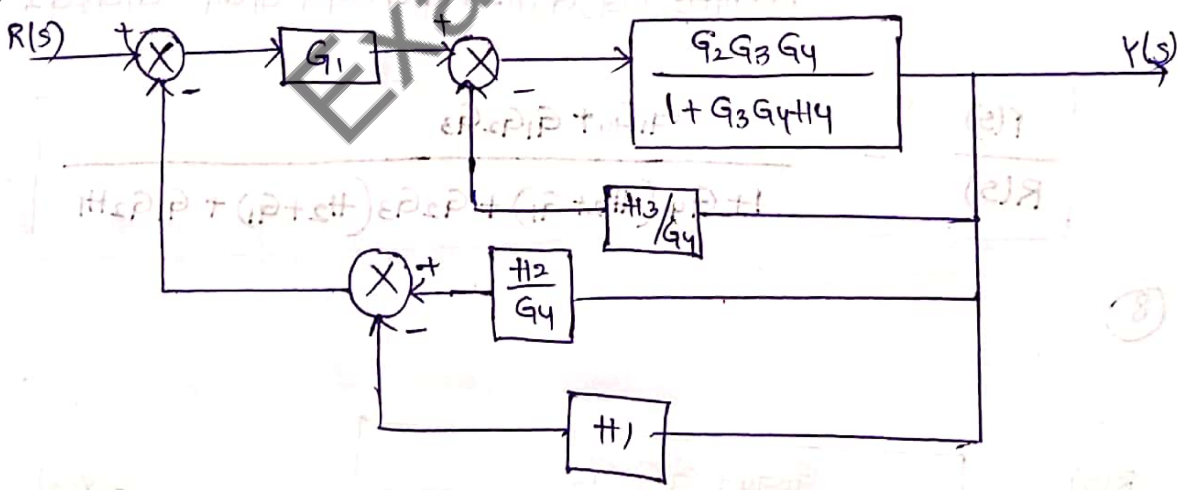


① Moving pickoff point A behind block.



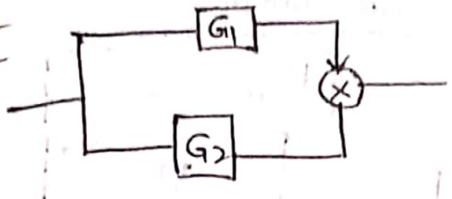
(last 4 four lines join at same point).

②

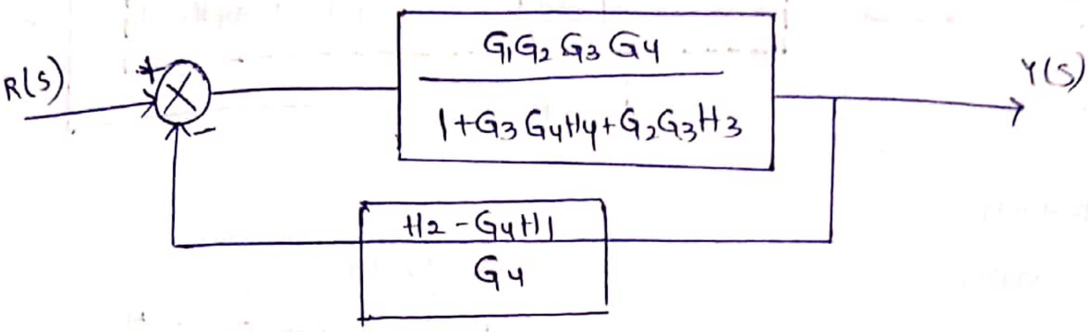


$$\frac{\dfrac{G_2G_3G_4}{1+G_3G_4H_4}}{1+\dfrac{G_2G_3G_4}{1+G_3G_4H_4}\dfrac{H_3}{G_4}} = \frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}$$

III



$G_1 \pm G_2$ $\qquad$ $\dfrac{H_2 - G_4H_1}{G_4}$

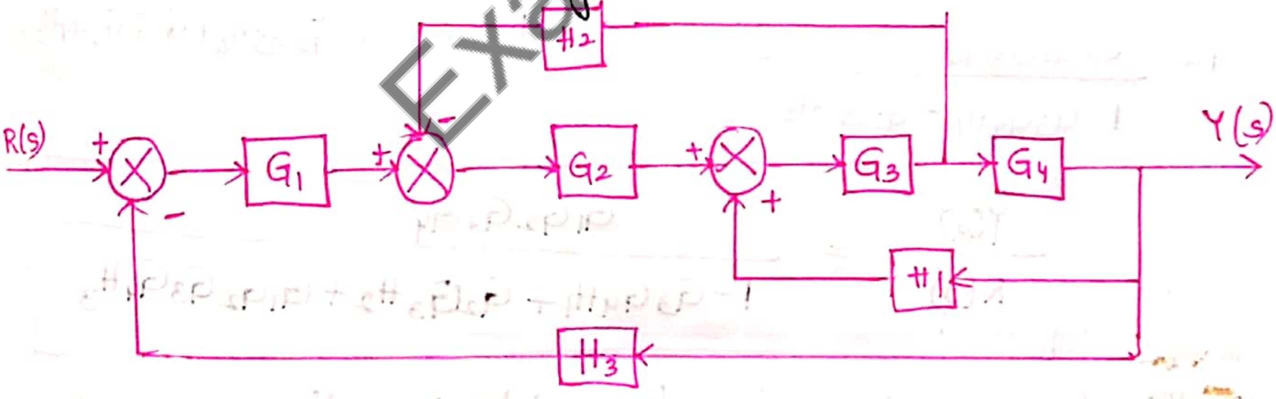


$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}}{1+\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}\left(\dfrac{H_2-G_4H_1}{G_4}\right)}$$

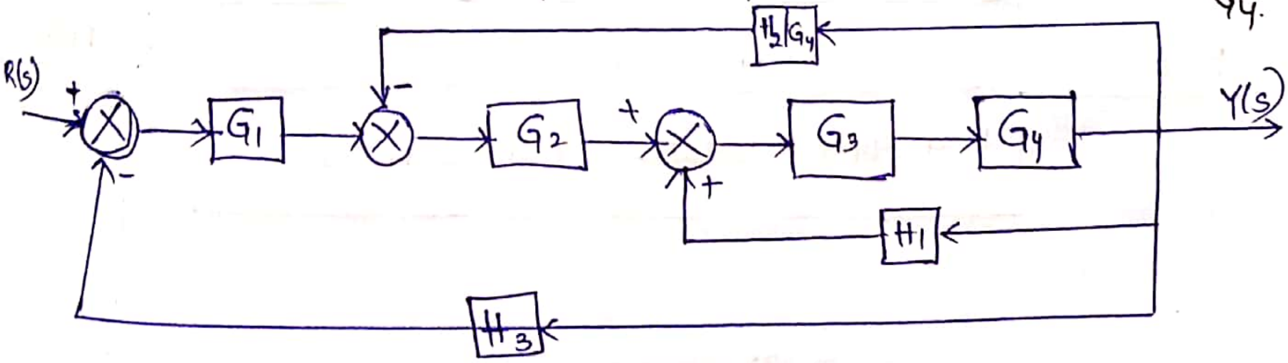
$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}}$$

Example-10:

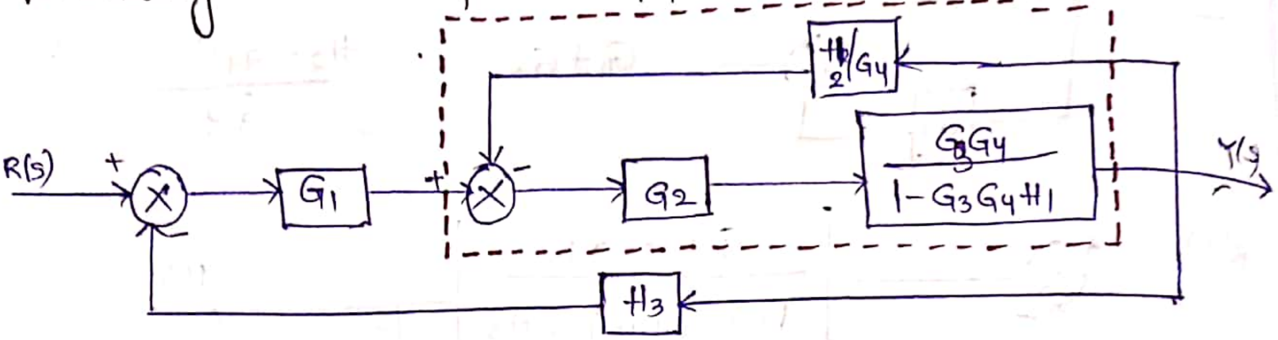
Reduce the block diagram.



First to eliminate the loop $G_3G_4H_4$ we move H_2 behind block G_4.

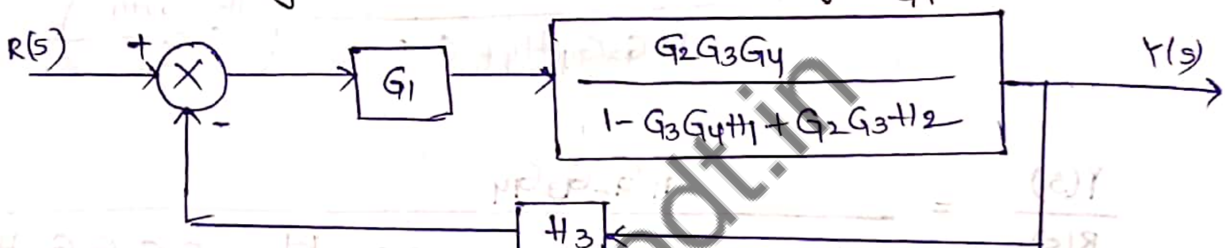


Eliminating the loop $G_3G_4H_1$ we obtain



$$\frac{\dfrac{G_2G_3G_4}{1-G_3G_4H_1}}{1+\dfrac{G_2G_3G_4}{1-G_3G_4H_1}\dfrac{H_2}{G_4}} = \frac{G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}$$

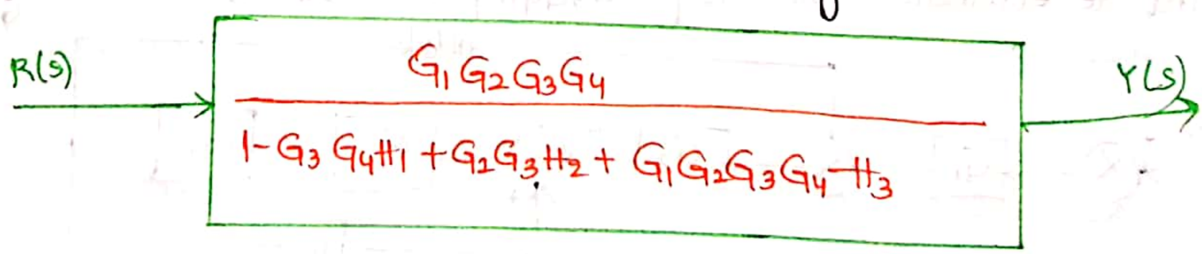
Then, eliminating the inner loop containing $\frac{H_2}{G_4}$ we obtain



$$\frac{\dfrac{G_1 \cdot G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}}{1+\dfrac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}H_3} = \frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}.$$

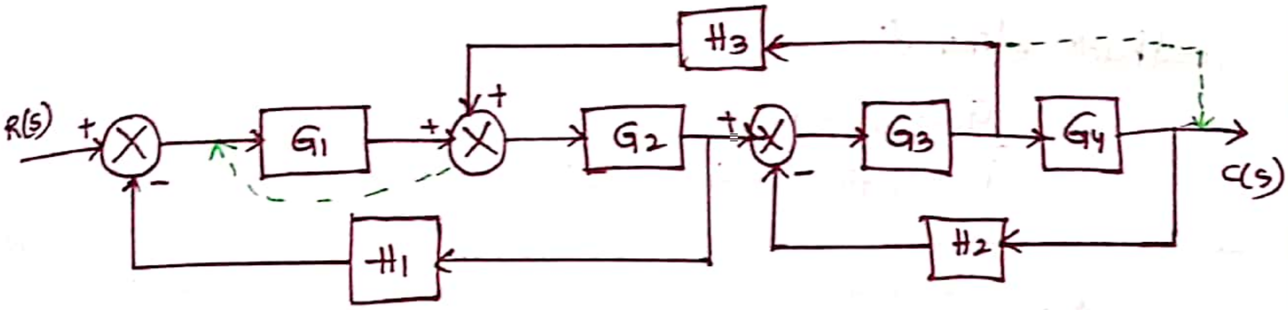
$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}}$$

Finally by reducing the loop containing H_3, we obtain.



Example-11:

Simplify the block diagram then obtain the close loop transfer function $\frac{C(s)}{R(s)}$.



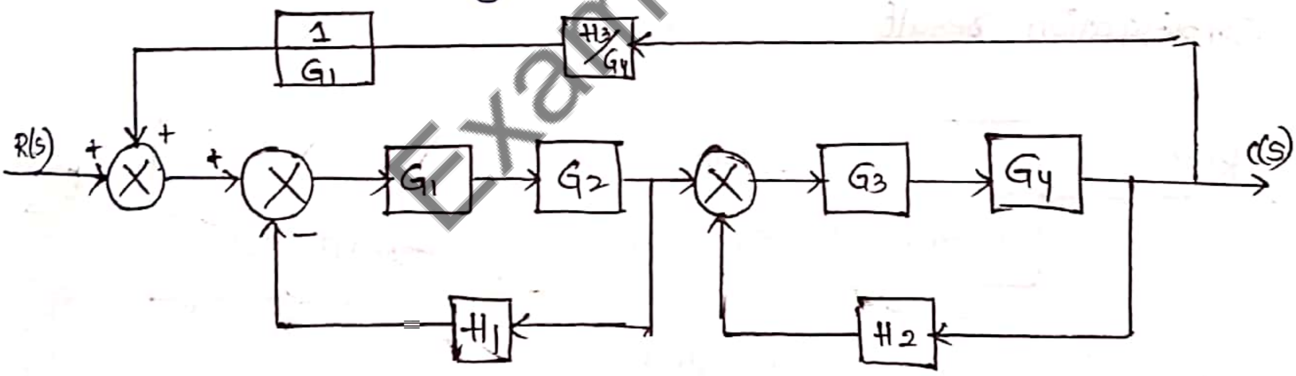
Step-1: Shift take off point after G_4.

Step-2: Shift Summing point ahead

Step-3: Swap Summation point before G_1

(or)

First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4 and H_2. Then move the Summining point between G_1 & G_2 to the left-hand side of the first Summing point.

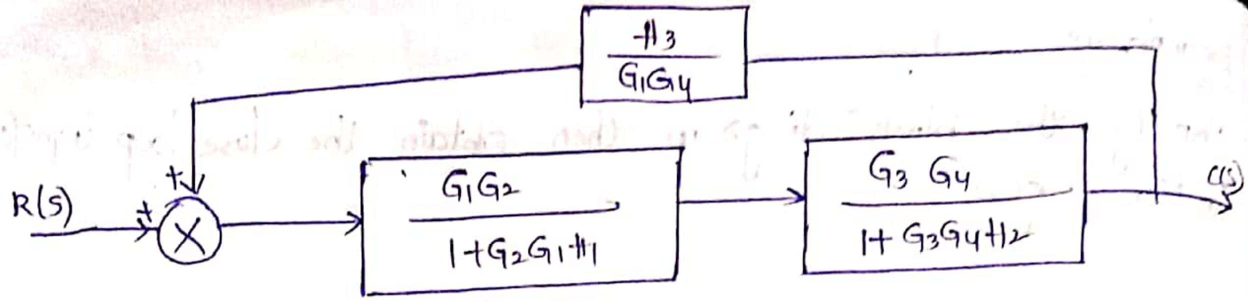


$$\frac{G_1G_2}{1+G_1G_2H_1}$$

$$\frac{G_3G_4}{1+G_3G_4H_2}$$

$$\frac{H_3}{G_1G_4}$$

Simplifying each loop, the block diagram can be modified as:



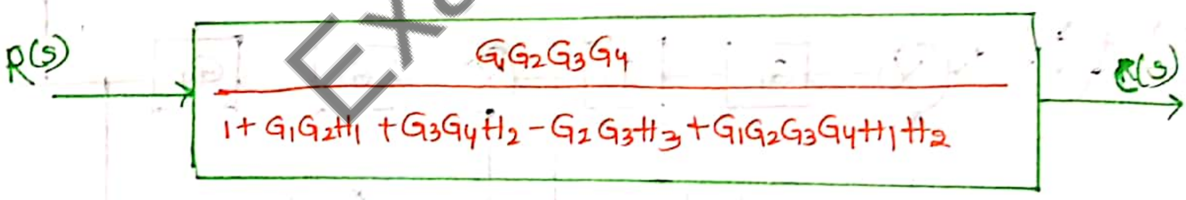
Feedback elimination,

$$\frac{\dfrac{G_1G_2G_3\ G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}{1 - \dfrac{G_1G_2G_3\ G_4}{(1+G_1G_2H_1)\ (1+G_3G_4H_2)} \cdot \dfrac{H_3}{G_1G_4}}$$

$$\frac{C(S)}{R(S)} = \frac{G_1G_2\ G_3\ G_4}{(1+G_1G_2H_1)\ (1+G_3G_4H_2) - G_2G_3H_3}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4}{1+G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}}$$

Simplification result:



Superposition of Multiple Inputs:

Step-1: Set all inputs except one equal to zero

Step-2: Transform the block diagram to canonical form using the transformations.

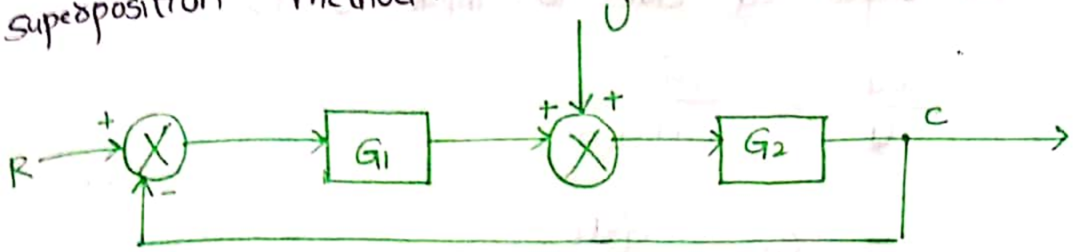
Step-3: Calculate the response due to the chosen input acting alone.

Step-4: Repeat steps 1 to 3 for each of the remaining inputs.

Step-5: Algebraically add all the responses (outputs) determined in steps 1 to 4. this sum is the total output of the system with all inputs acting simultaneously.

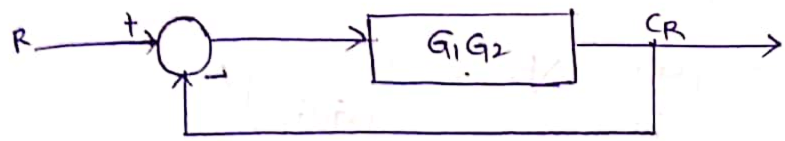
Example – 12: Multiple Input System.

Determine the output C due to inputs R and U using the superposition method.



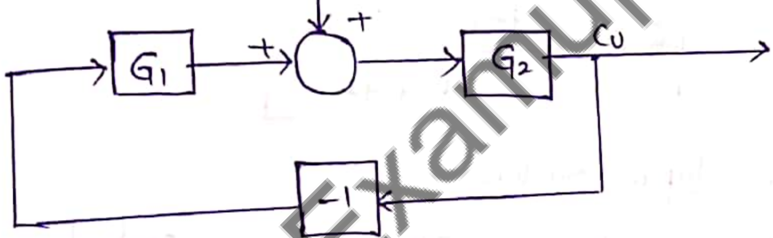
Step-1 : Put $U = 0$

Step-2 : The system reduces to :



Step-3 : The output C_R due to input R is

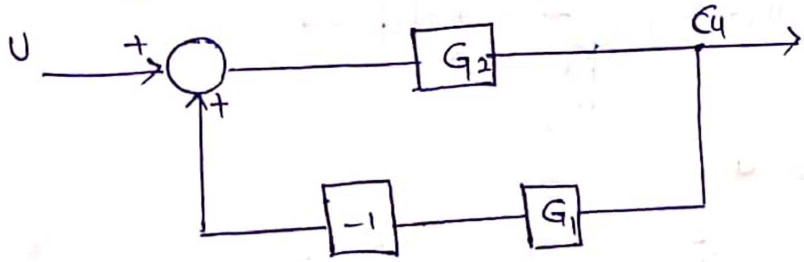
$$\boxed{C_R = \frac{G_1 G_2 R}{1 + G_1 G_2}} \quad \left[\because \frac{C_R}{R} = \frac{G_1 G_2}{1 + G_1 G_2}\right]$$



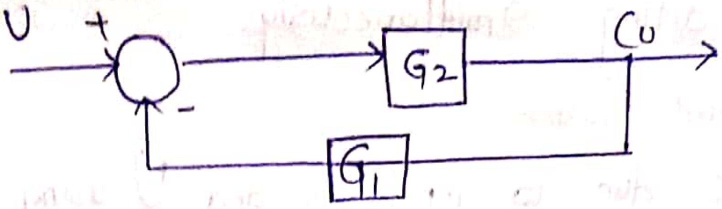
Step-4(a): Put $R = 0$

Step-4(a) : put −1 into a block, representing the −ve feedback effect.

Rearrange the block diagram:



Let the −1 block be absorbed into the summing point.



Step-4c: The output C_U due to input U is

$$\frac{C_U}{U} = \frac{G_2}{1+G_1G_2}$$

$$\therefore \boxed{C_U = \frac{G_2U}{1+G_1G_2}}$$

Step-5: The total output is

$$C = C_R + C_U$$

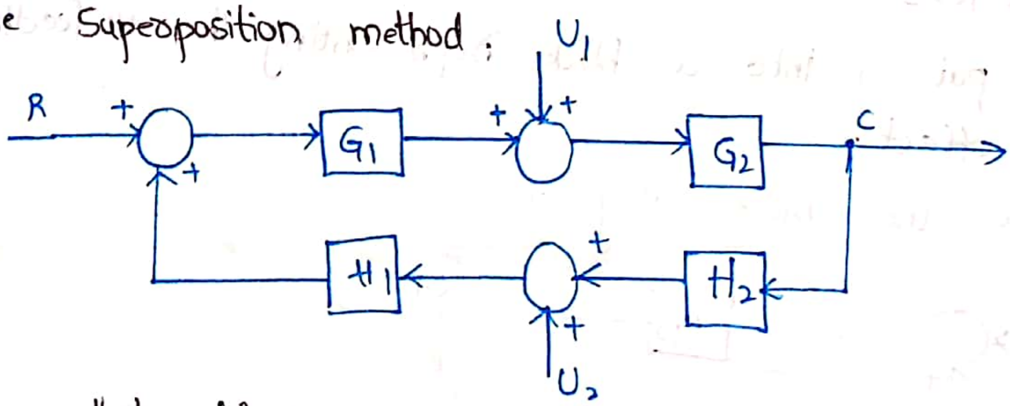
$$= \left(\frac{G_1G_2}{1+G_1G_2}\right)R + \left(\frac{G_2}{1+G_1G_2}\right)U$$

$$C = \frac{G_1G_2R + G_2U}{1+G_1G_2}$$

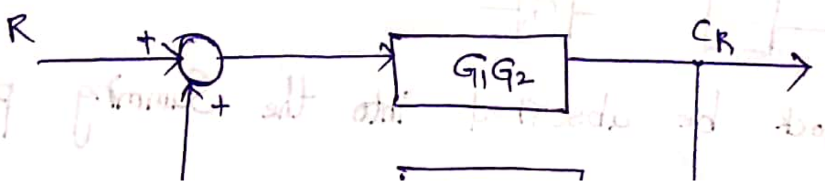
$$\therefore \boxed{C = \frac{G_2(G_1R+U)}{1+G_1G_2}}$$

Example-13: Multiple Input system.

Determine the Output C due to inputs R, U_1 and U_2 u the Superposition method.



Let $U_1 = U_2 = 0$

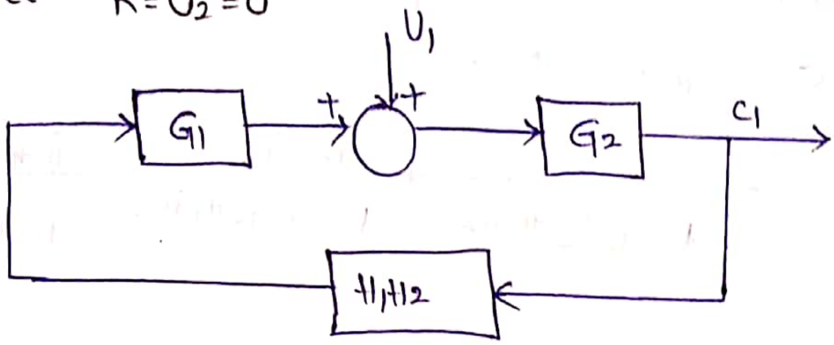


$$\frac{C_R}{R} = \frac{G_1 G_2}{1 - G_1 G_2 H_1 H_2}$$

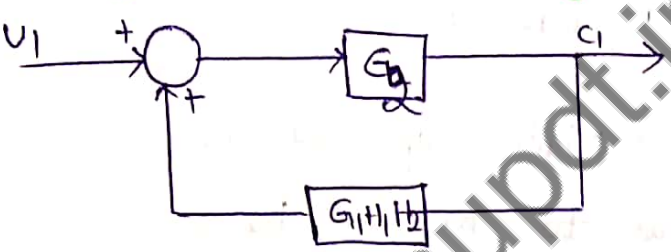
$$\boxed{C_R = \frac{G_1 G_2 R}{1 - G_1 G_2 H_1 H_2}}$$

Where C_R is the output due to R input alone.

Now let $R = U_2 = 0$



Rearranging the blocks, we get.

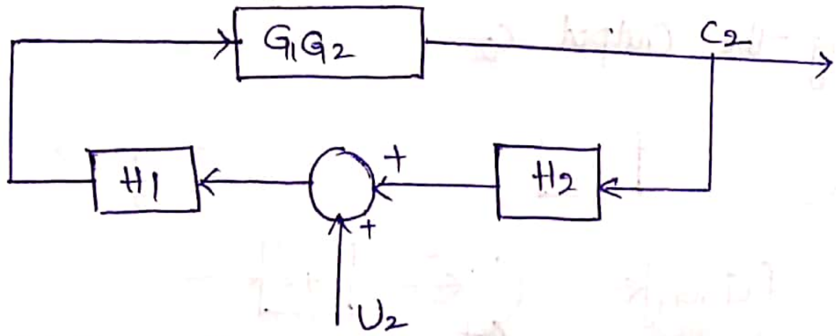


$$\frac{C_1}{U_1} = \frac{G_2}{1 - G_1 G_2 H_1 H_2}$$

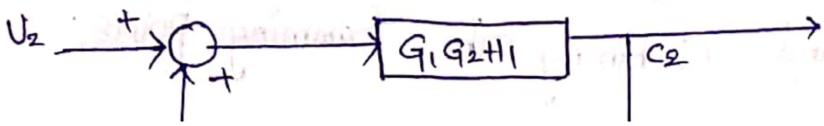
$$\therefore \boxed{C_1 = \frac{G_2 U_1}{1 - G_1 G_2 H_1 H_2}}$$

Where C_1 is the response due to U_1 acting alone.

Let $R = U_1 = 0$



Rearranging the blocks, we get



$$\frac{C_2}{U_2} = \frac{G_1G_2H_1}{1-G_1G_2H_1H_2}$$

$$\therefore \boxed{C_2 = \frac{G_1G_2H_1U_2}{1-G_1G_2H_1H_2}}$$

Where C_2 is output due to U_2 input alone.

By Superposition, the total output is

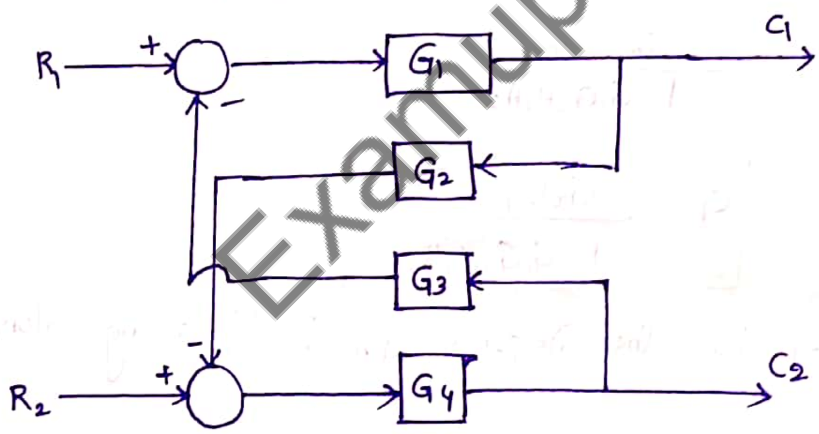
$$C = C_R + C_1 + C_2$$

$$= \frac{G_1G_2R}{1-G_1G_2H_1H_2} + \frac{G_2U_1}{1-G_1G_2H_1H_2} + \frac{G_1G_2H_1U_2}{1-G_1G_2H_1H_2}$$

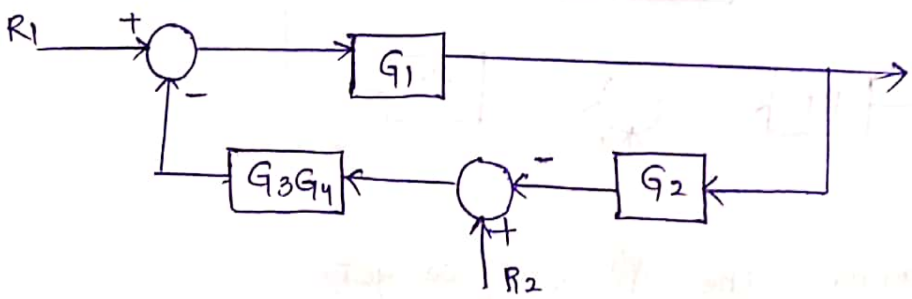
$$\boxed{C = \frac{G_1G_2R + G_2U_1 + G_1G_2H_1U_2}{1-G_1G_2H_1H_2}}$$

Example - 14: Multi-Input Multi-Output system.

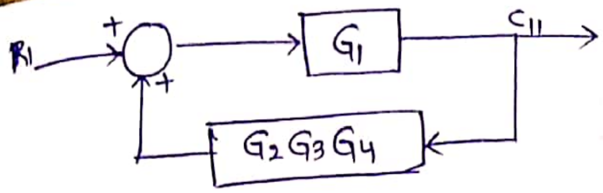
Determine C_1 & C_2 due to R_1 and R_2.



First ignoring the Output C_2.



Let $R_2 = 0$ and Combining the Summing points,

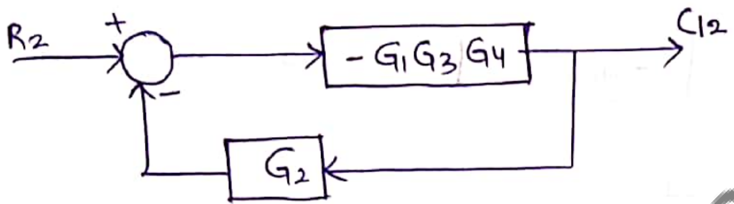


$$\frac{C_{11}}{R_1} = \frac{G_1}{1 - G_1G_2G_3G_4}$$

$$\boxed{C_{11} = \frac{G_1R_1}{1 - G_1G_2G_3G_4}}$$

Where C_{11} is the output of R_1 alone.

For $R_1 = 0$



$$\frac{C_{12}}{R_2} = \frac{-G_1G_3G_4}{1 - G_1G_2G_3G_4}$$

$$\boxed{C_{12} = \frac{-G_1G_3G_4R_2}{1 - G_1G_2G_3G_4}}$$

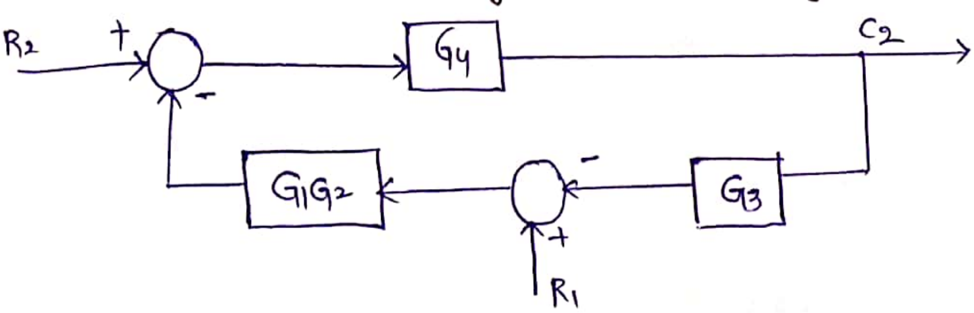
Where C_{12} is the output of R_2 alone at C_1

Thus, $C_1 = C_{11} + C_{12}$

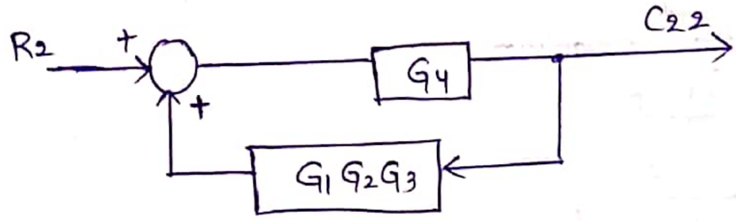
$$= \frac{G_1R_1}{1 - G_1G_2G_3G_4} - \frac{G_1G_3G_4 \; R_2}{1 - G_1G_2G_3G_4}$$

$$\therefore \boxed{C_1 = \frac{G_1(R_1 - G_3G_4R_2)}{1 - G_1G_2G_3G_4}}$$

Now, we reduce the original block diagram, ignoring output C_1.



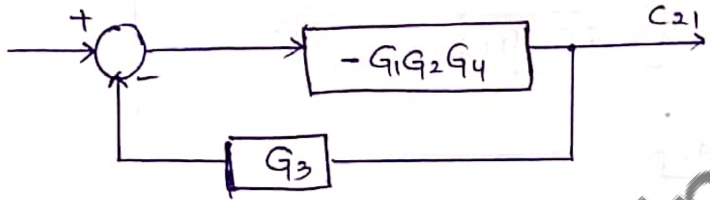
When $R_1 = 0$



$$\frac{C_{22}}{R_2} = \frac{G_4}{1 - G_1 G_2 G_3 G_4}$$

$$\boxed{C_{22} = \frac{G_4 R_2}{1 - G_1 G_2 G_3 G_4}}$$

When $R_2 = 0$



$$\frac{C_{21}}{R_1} = \frac{-G_1 G_2 G_4}{1 - G_1 G_2 G_3 G_4}$$

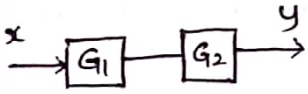
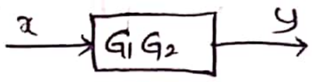
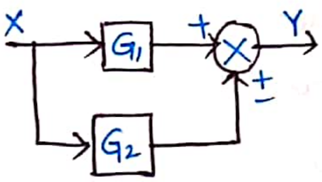
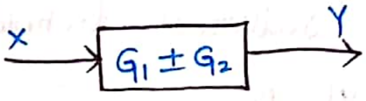
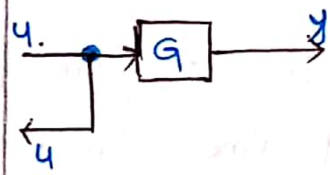
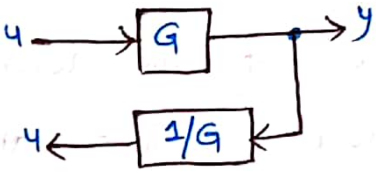
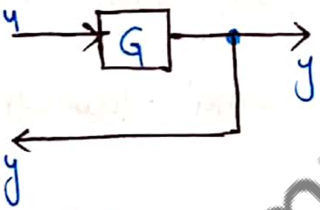
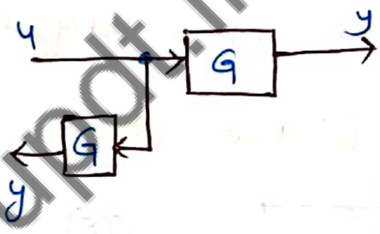
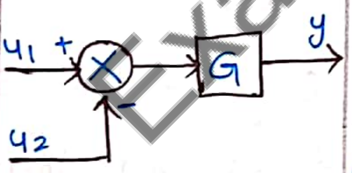
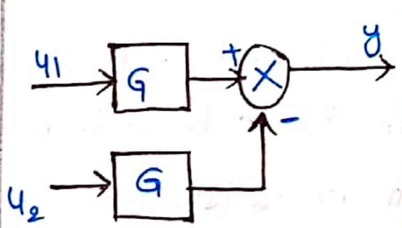
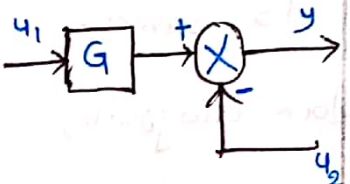
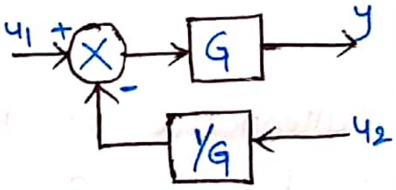
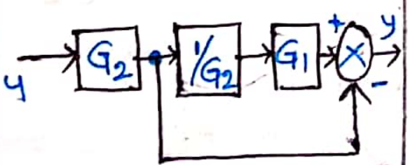
$$\boxed{C_{21} = \frac{-G_1 G_2 G_4 R_1}{1 - G_1 G_2 G_3 G_4}}$$

Thus $C_2 = C_{21} + C_{22}$

$$= \frac{-G_1 G_2 G_4 R_1}{1 - G_1 G_2 G_3 G_4} + \frac{G_4 R_2}{1 - G_1 G_2 G_3 G_4}$$

$$\boxed{C_2 = \frac{G_4 (R_2 - G_1 G_2 R_1)}{1 - G_1 G_2 G_3 G_4}}$$

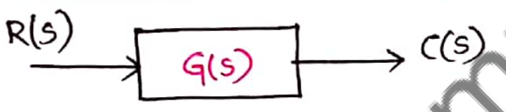
Finally, $C_2 = C_{22} + C_{21}$

Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation.
Combining blocks in cascade			$Y = (G_1 G_2) X$
Combining blocks parallel (or) Eliminating a Forward loop			$Y = (G_1 \pm G_2) X$
Moving a pickoff point behind a block			$y = Gu$ $u = \frac{1}{G} y$
Moving a pickoff point ahead of a block			$y = Gu$
Moving a Summing point behind a block			$e_2 = G(u_1 - u_2)$
Moving a Summing point ahead of a block			$y = Gu_1 - u_2$
			$y = (G_1 - G_2) u$

Signal Flow Graph:

- SFG is a diagram which represents a set of simultaneous equations.
- This method was developed by S.J Mason. This method doesn't require any reduction technique.
- It consists of nodes and three nodes are connected by a directed line called branches.
- Every branch has an arrow which represents the flow of signal.
- For complicated systems, when Block diagram (BD) reduction method becomes tedious and time consuming then SFG is a good choice.

Comparision of Block diagram (BD) and SFG:

Block Diagram (BD)	Signal Flow Graph (SFG).
	
In this case at each step block diagram is to be redrawn. That's why it is tedious method. So wastage of time and space.	Only one time SFG is to be drawn and then Mason's gain formula is to be evaluated. so time and space is saved.

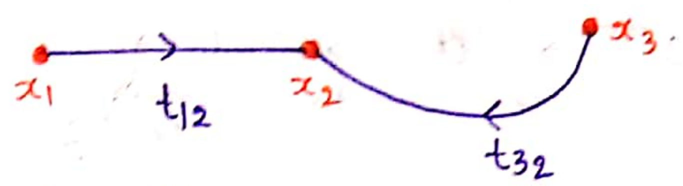
- For SFG, alternative to block diagram; Consists only branches (systems) and nodes (signals)

Branches	Nodes
 G(s)	 V(s)

...inition of terms required in SFG

...de: It is a point representing a Variable.

$$x_2 = t_{12}x_1 + t_{32}x_3$$



In this, SFG there are 3 nodes.

Branch: A line joining two nodes.

Input node: Node which has only outgoing branches.

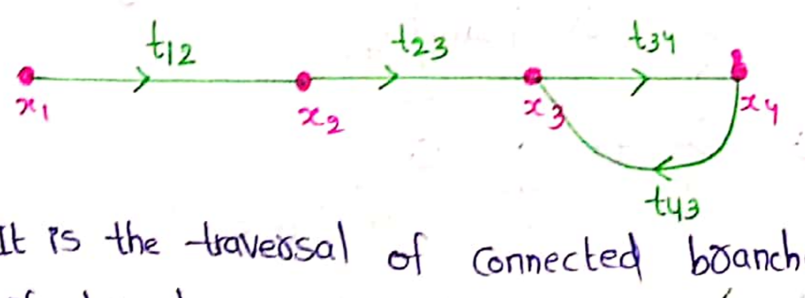
x_1 is input node (source node).

Output node: Only incoming branches.

Output node / Sink node.

Mixed nodes: Has both incoming and outgoing branches.

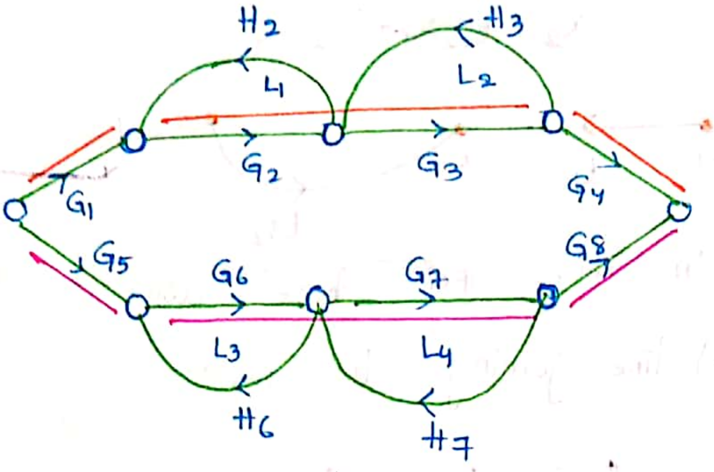
- Transmittance: It is the gain between two nodes. It is generally written on the branch near the arrow.



- Path: It is the traversal of connected branches in the direction of branch arrows, such no node is traversed more than once.

- Forward path: A path which originates from the input node and terminates at the output node and along which no node is traversed more than once.

• Forward path gain : It is the product of branch transmittances of a forward path :



Forward path - 1 : $P_1 = G_1 G_2 G_3 G_4$

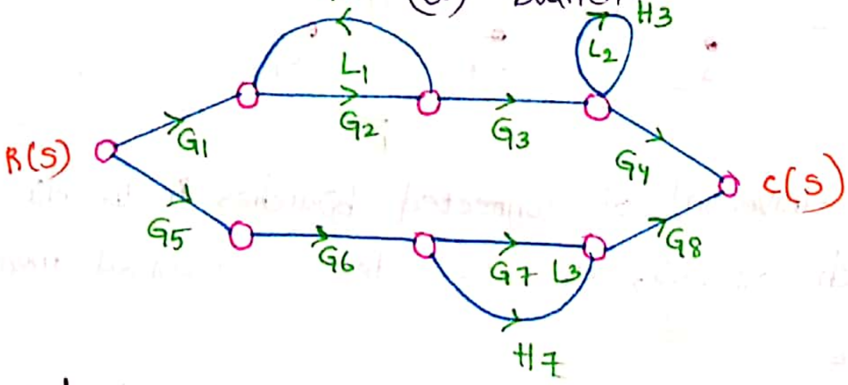
Forward path 2 : $P_2 = G_5 G_6 G_7 G_8$

• Loop : Path that originates and terminates at the same node and along which no other node is traversed more than once.

• Self loop : Path that originates and terminates at the same node.

• loop gain : It is the product of branch transmittances of a loop.

• Non-touching loops : Loops that don't have any common node (or) branch.



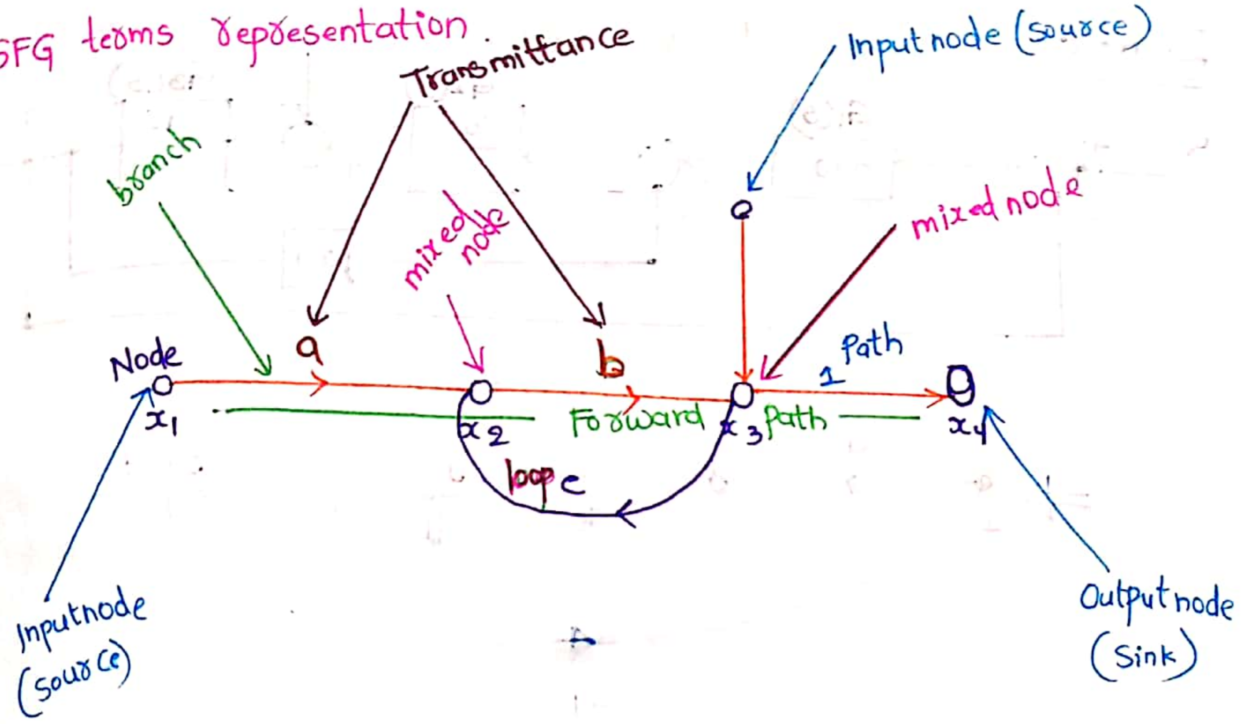
Loop - 1 : $L_1 = G_2 H_2$

Loop 2 : $L_2 = H_3$ (self loop)

Loop - 3 : $L_3 = G_7 H_7$

Non-touching loops : $L_1 \& L_2$, $L_1 \& L_3$ and $L_2 \& L_3$.

SFG terms representation



Mason's Gain Formula :

- A technique to reduce a signal-flow graph to a single transfer function requires the application of one formula.
- The transfer function, $\frac{C(s)}{R(s)}$ of a system represented by a signal flow graph is

$$\boxed{G(s) = \frac{C(s)}{R(s)} = \frac{\sum_K P_K \Delta_K}{\Delta}}$$

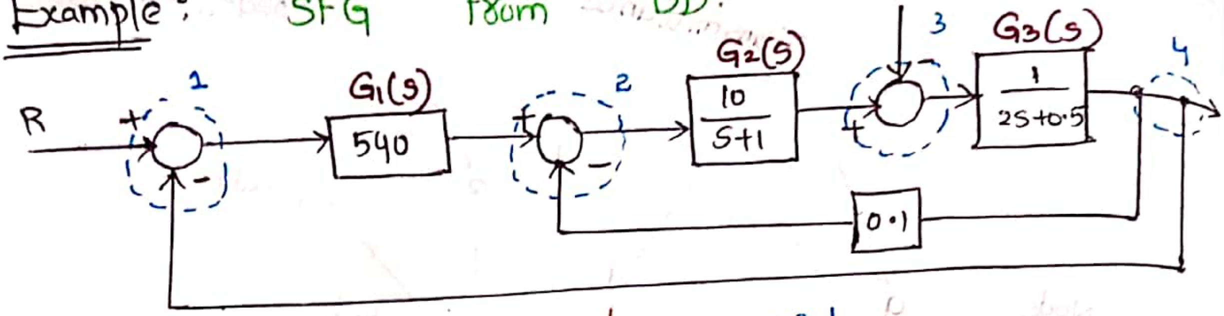
Where K = Number of forward path

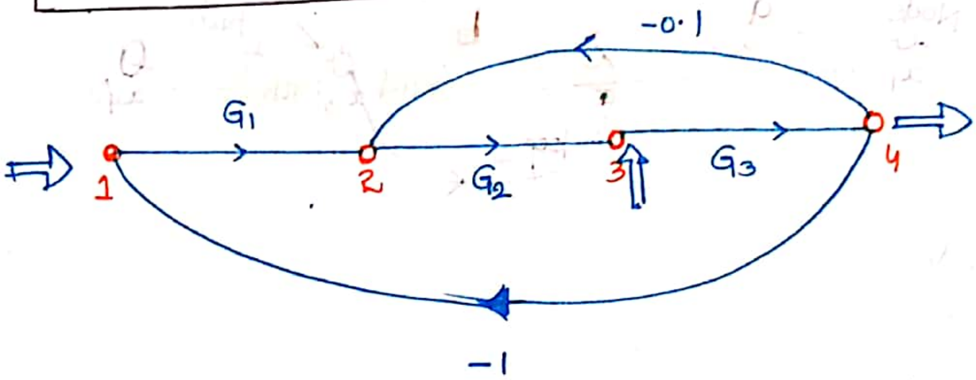
P_K = The K^{th} forward path gain

Δ = 1 − (Σ loop gains) + (Σ non-touching loop gains taken two at a time) − (Σ non-touching loop gains taken three at a time) + so on.

Δ_K = 1 − (loop-gain which does not touch the forward path).

Example: SFG From BD.

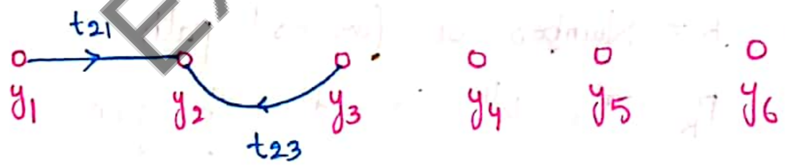




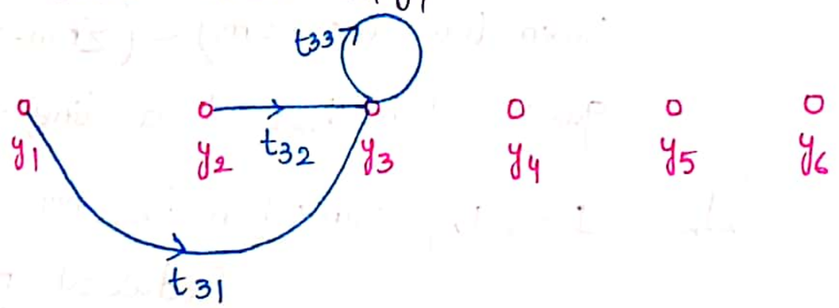
Construction of SFG from simultaneous equations.

$$y_2 = t_{21}y_1 + t_{23}y_3$$
$$y_3 = t_{32}y_2 + t_{33}y_3 + t_{31}y_1$$
$$y_4 = t_{43}y_3 + t_{42}y_2$$
$$y_5 = t_{54}y_4$$
$$y_6 = t_{65}y_5 + t_{64}y_4$$

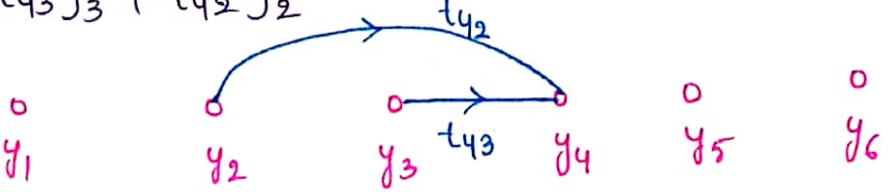
- $y_2 = t_{21}y_1 + t_{23}y_3$



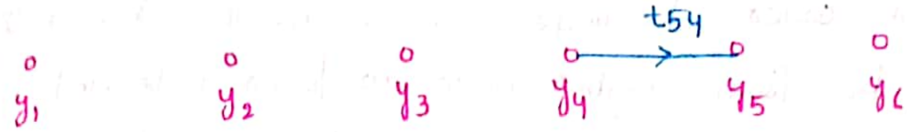
- $y_3 = t_{32}y_2 + t_{33}y_3 + t_{31}y_1$



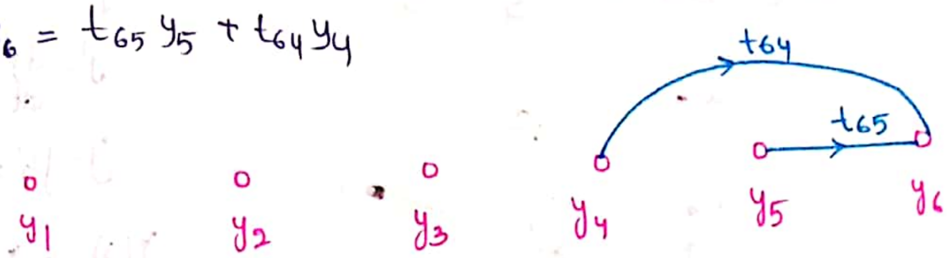
- $y_4 = t_{43}y_3 + t_{42}y_2$



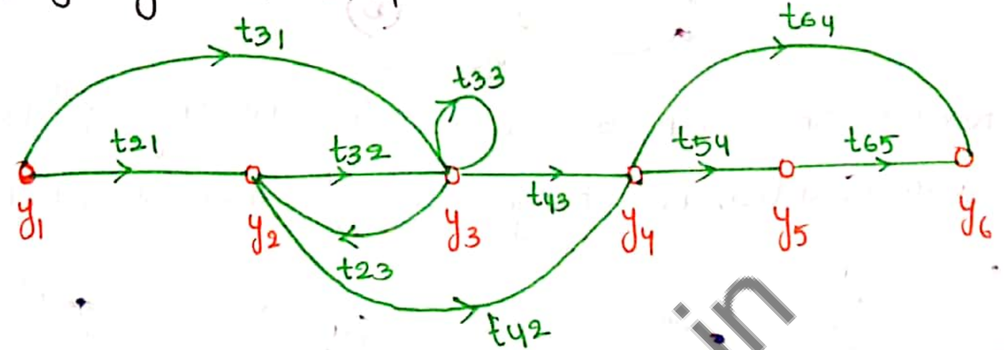
- $y_5 = t_{54} y_4$



- $y_6 = t_{65} y_5 + t_{64} y_4$



After joining all SFG



SFG from Differential equations:

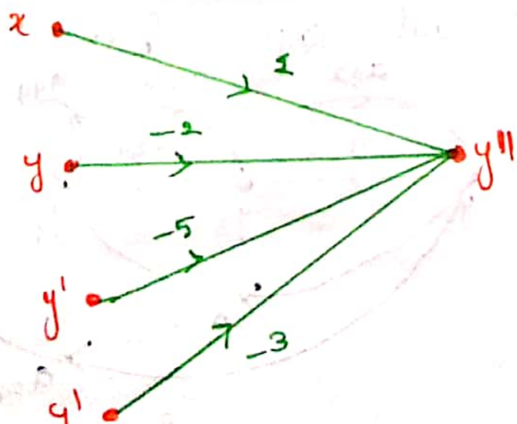
- Consider the differential equation $y'' + 3y' + 5y' + 2y = x$

Step-1: Solve the above equation for highest order:

$$y'' = x - 3y' - 5y' - 2y$$

Step-2: Consider the left hand terms (highest derivative) as dependant variable and all other terms on right hand side as independent variables.

Construct the branches of signal flow graph as shown below:



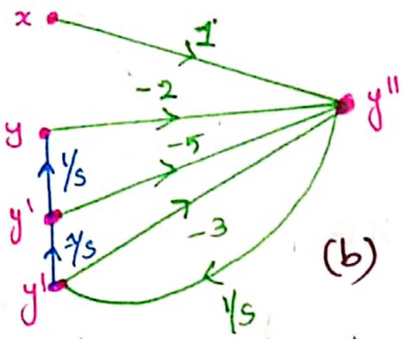
(a)

Step-3: Connect the nodes of highest order derivatives to the lowest order der. node and so on. The flow of signal will be from higher node to lower node and transmittance will be $\frac{1}{s}$ as shown in figure (b)

$$y' = \frac{dy}{dt}$$

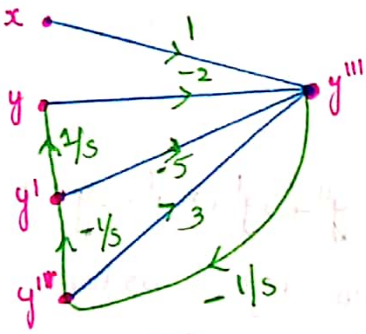
$$y = \int y' dt$$

$$Y(s) = \frac{1}{s} Y'(s)$$

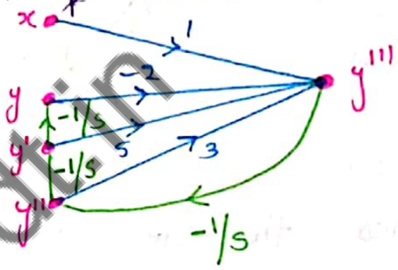


(b)

Step-4: Reverse the sign of a branch Connecting Y''' to Y'', with Condition no change in ~~T~~ Transfer function.

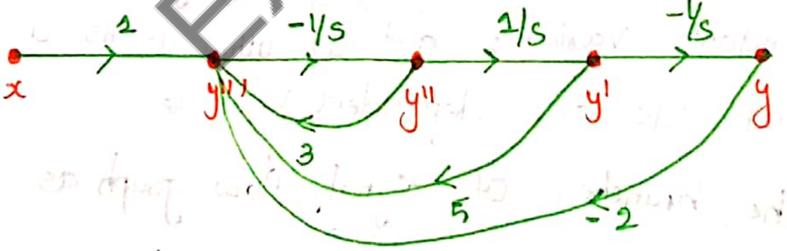


(c)

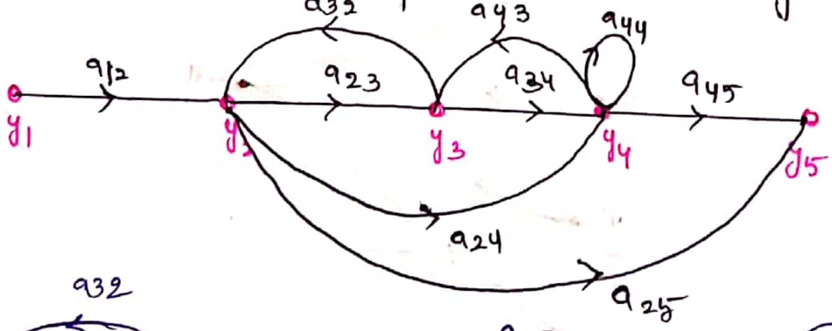


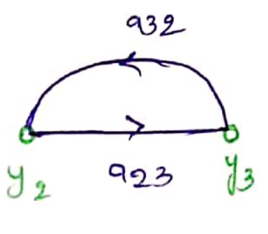
(d)

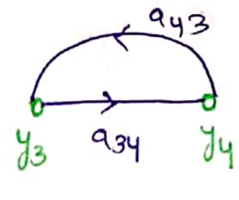
Step-5: Redraw the SFG as shown.

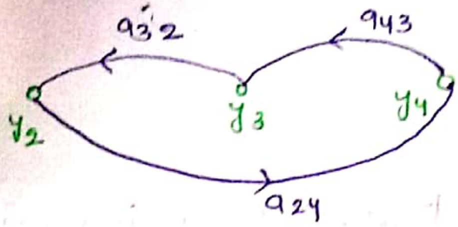


Problem: To find out loops from the given SFG

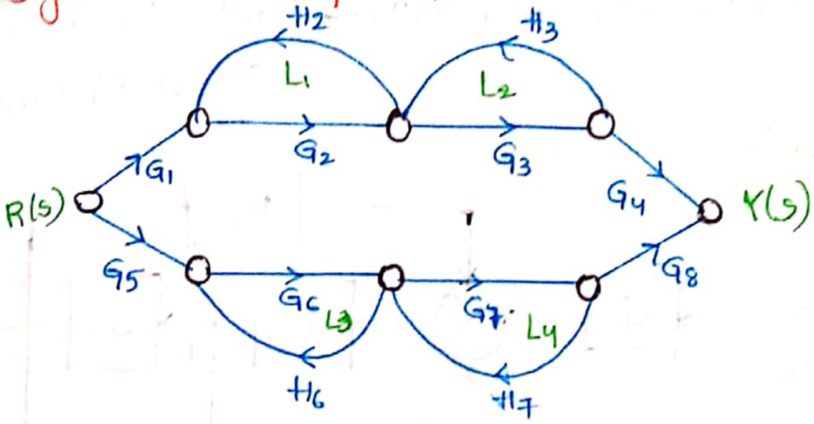








Signal-Flow Graph models



Forward path $P_1 = G_1G_2G_3G_4$

$P_2 = G_5G_6G_7G_8$

Loops $L_1 = G_2H_2$

$L_2 = G_3H_3$

$L_3 = G_6H_6$

$L_4 = G_7H_7$

pair of Non-touching loops : $L_1L_3,\ L_1L_4,\ L_2L_3,\ L_2L_4$

$\Delta = 1 - \Sigma$ loop gains + Σ product of non touching loops

$$\Delta = 1-(G_2H_2+G_3H_3+G_6H_6+G_7H_7)+(G_2H_2G_6H_6+G_2H_2G_7H_7+G_3H_3G_6H_6+G_3H_3G_7H_7)$$

$\Delta_1 = 1-$ (non touching loop gain of 1).

$$\Delta_1 = 1-(G_6H_6+G_7H_7)$$

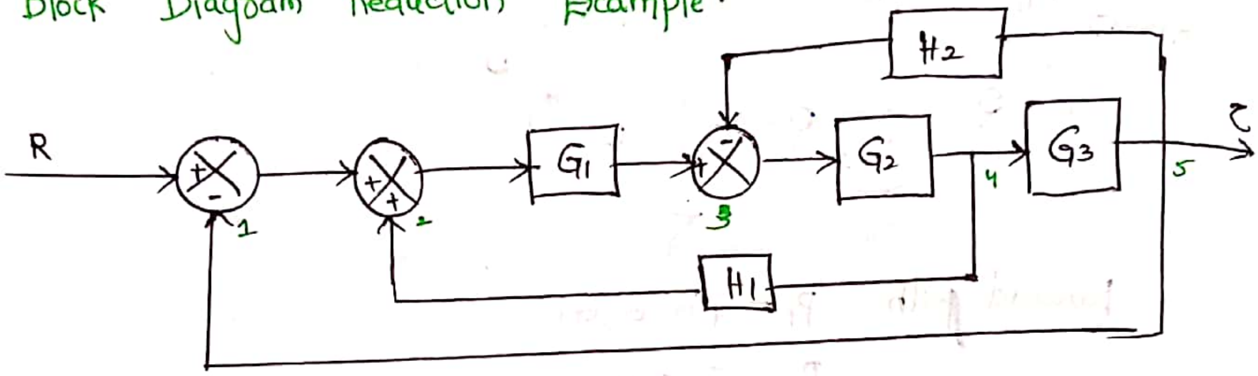
$\Delta_2 = 1-$ (non-touching loop gain of 2)

$$\Delta_2 = 1-(G_2H_2+G_3H_3)$$

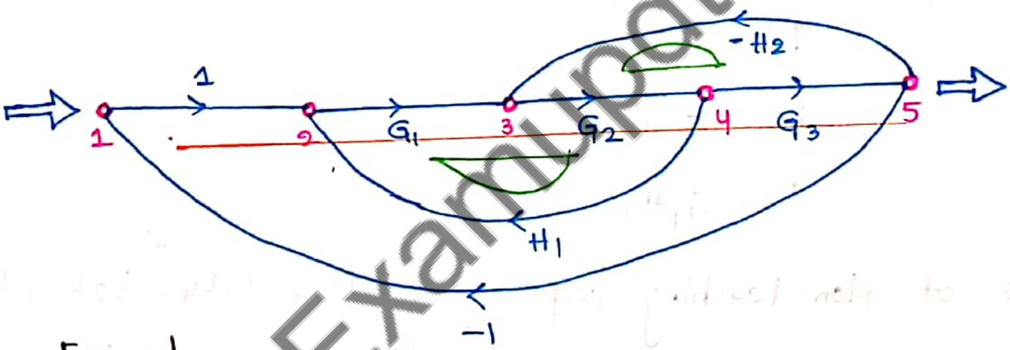
$$\frac{Y}{R} = \frac{\sum_K P_K \Delta_K}{\Delta}$$

$$\boxed{\frac{Y}{R} = \frac{G_1G_2G_3G_4\left(1-G_6H_6-G_7H_7\right) + G_5G_6G_7G_8\left(1-G_2H_2-G_3H_3\right)}{1-\left(G_2H_2+G_3H_3+G_6H_6+G_7H_7\right)+\left(G_2H_2G_6H_6+G_2H_2G_7H_7+G_3H_3G_6H_6+G_3H_3G_7H_7\right)}}$$

Block Diagram Reduction Example:



$$\frac{C}{R} = \frac{G_1G_2G_3}{1-G_1G_2H_1+G_1G_2G_3+G_2G_3H_2}$$



Forward path $P_1 = G_1G_2G_3$ $\qquad \Delta_1 = 1$

Loops : $L_1 = G_1G_2H_1$ $\qquad \cancel{\Delta_2}\ \Delta = 1-(L_1+L_2+L_3)$

$L_2 = -G_2G_3H_2$

$L_3 = -G_1G_2G_3$

Non-touching loops : Nil

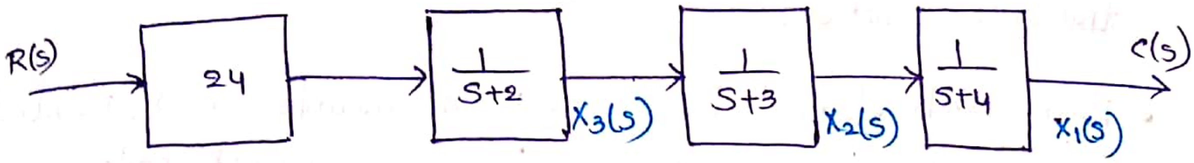
$$T = \frac{\sum_K P_K \Delta_K}{\Delta}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3 \cdot 1}{1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3}$$

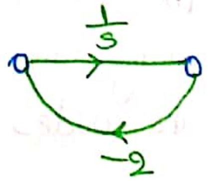
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1-G_1G_2H_1+G_1G_2G_3+G_2H_2G_3}}$$

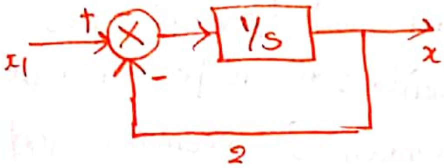
SFG from given Transfer function

$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)}$$

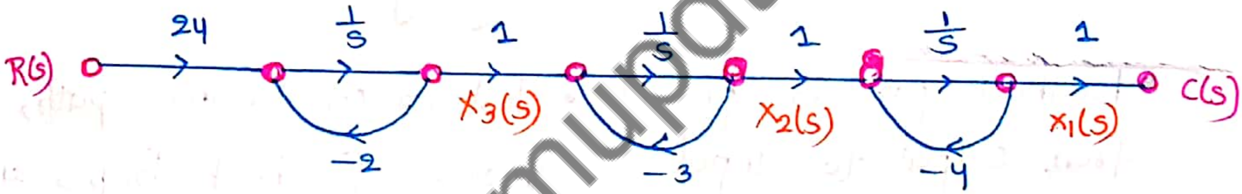


We can write $\frac{1}{s+2} = \frac{s^{-1}}{(1+2s^{-1})}$.





$$\frac{G}{1 \pm Gs} = \frac{\frac{1}{s}}{1+\frac{1}{s}(2)} = \frac{s^{-1}}{1+2s^{-1}}$$



Block Diagram	Signal Flow Graph.
• Basic importance is given to the elements and their transfer functions.	• Basic importance is given to the variables of the systems.
• Each block element is represented by a block.	• Each variable is represented by a seperated node.
• Transfer function of the element is shown inside the corresponding block.	• The transfer function is shown along the branches connecting the nodes.
• Summing points and takeoff points	• Summing and takeoff points are absent. Any node can have any number of incoming and outgoing branches.
• Feedback path is present from output to input	• Instead of feedback path, various feedback loops are considered for the analysis
• For a minor feedback loop present, the formula $\frac{G}{1 \pm GH}$ can be used	• Gains of various forward paths and feedback loops are just the product of associative branch gains. No such formula $\frac{G}{1 \pm GH}$ is necessary.
• Block diagram reduction rules can be used to obtain the resultant transfer function	• The Mason's gain formula is available which can be used directly to get resultant transfer function without reduction of SFG

Hardware Components of Control Systems:

- Servo position Control mechanism.
- Dc Servo mechanism.
- Armature Controlled Dc servo
- Field Controlled Dc Servo.
- Ac servo mechanism.
- Synchro Transmitter Receiver mechanism.

Servomotor.

- A servo is a small device which has an output shaft which positions on coded signal. It is a rotary (or) linear actuator that allows for precise control of angular (or) linear position, velocity and acceleration.
- The servo motor is which respond to signal abruptly and accelerate the load quickly are called servo motor.

Dc servo motor:

- Dc servo motors are seperately excited dc motor (or) permanent dc motor.
- They are controlled by armature voltage.
- The armature is designed to have large resistance so that the torque-speed characteristics are linear and have a large negative slope.
- Dc servo motor provides very accurate and also fast respond to start (or) stop command signals due to the low armature inductive reactance. Dc servo motors are used in similar equipments and computerized numerically controlled machines.
- Dc motor (servo motor) are of 2 types. They are:
  1) Armature Controlled Dc servo motor.
  2) Field Controlled Dc servo motor.

Ac Servo motor

- Ac servo motor is an Ac motor that includes encoder is used with controllers for giving closed loop control and feedback.
- This motor can be placed to high accuracy and also controlled precisely as compulsory for the applications.
- Frequently these motors have higher designs of tolerance (or) better bearings and some simple designs also use higher voltages in order to accomplish greater torque.

Construction:

Stator: From the position of the rotor, a rotating magnetic field is created to efficiently generate torque.

Winding: Current flows in the winding to create a rotating magnetic field.

Bearing: Ball-bearing

Shaft: The part transmitts the output power of motor. The load is driven through the transfer mechanism (such as the coupling).

Rotor: A high-function rare earth (or) other permanent magnet is positioned externally to the shaft.

Encoder: The optical encoder always watches the number of rotations and the position of the shaft.

Advantages of Servomotor.

- High output power relative to motor size and power.
- Resonance and vibration free operation.
- High efficiency.
- High speed operation is possible.

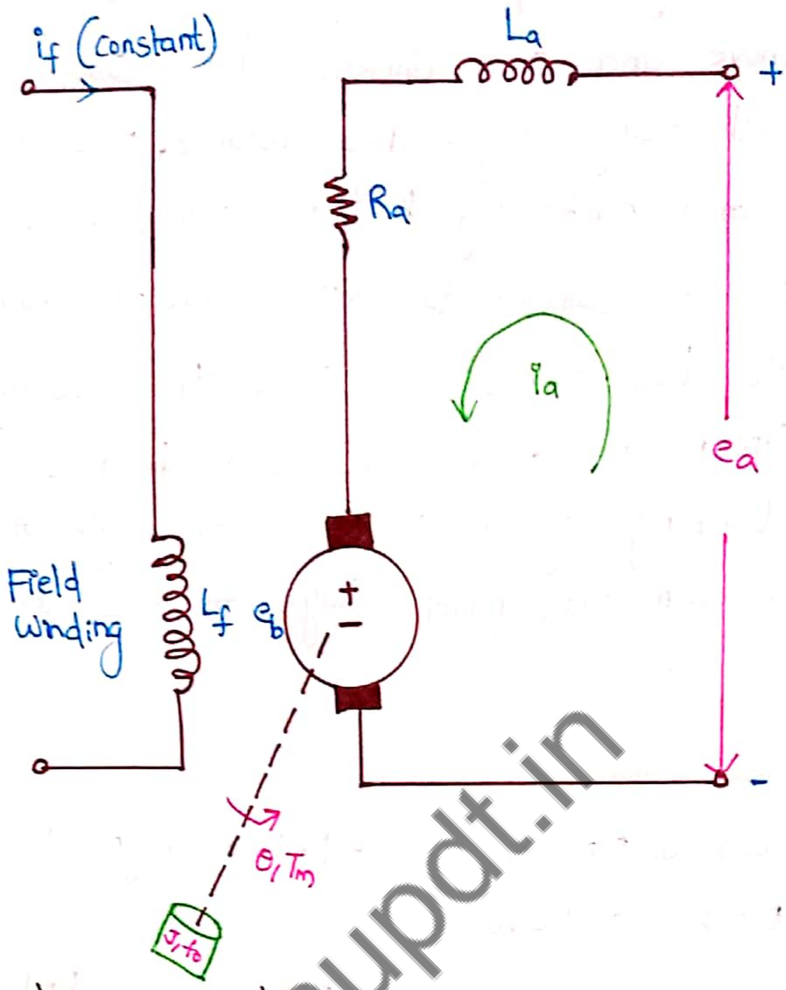
Applications of Servomotor:

- Computers, toys, CD/DVD players, Robotics.
- Start, move and stop conveyor belts carrying the product along with many stages. For instance, product labeling, bottling and packaging built into the camera to correct a lens of the camera to improve out of focus images.
- Automatic door openers to control the door in public places like supermarkets, hospitals and theatres.
- Solar tracking system to correct the angle of the panel so that each solar panel stays to face the sun.

Comparision of Servo motor:

Ac Servo motor	Dc Servo motor.
• Low power output of about 0.5 W to 100 W	• Deliver high power output
• Efficiency is less	• Efficiency is high
• Maintenance is less	• Frequent maintenance required
• Stability problems are less.	• More problems of stability
• No radio frequency noise	• Brushes produce RF noise
• Compare to Dc Servo motor it is relatively stable and smooth operation	• It's a noisy operation

Transfer Function of Armature Controlled DC servo motor



The parameters are taken as

R_a = Armature winding resistance

L_a = Armature Winding inductance

i_a = Armature Current

i_f = Field Current

e_a = applied armature (control) Voltage

e_b = Motor back emf

T_m = torque developed by motor.

θ = Angular displacement of motor shaft

J = Equivalent moment of inertia of load and motor reffered to motor shaft.

f_0 = Equivalent viscous friction coefficient of motor and load referred to motor shaft.

- In armature controlled DC motors, the armature input voltage e_a controls the motor shaft output while the field current i_f (i_f) remains constant.
- For servo applications, the DC motor operates in the linear region, so the air gap flux is proportional to the field current

$$\phi \propto i_f$$

$$\boxed{\phi = K_f i_f}$$

- The torque developed T_m by the motor is proportional to the armature current and the air gap flux.

$$T_m \propto \phi i_a$$

$$T_m \propto K_f i_f i_a$$

$$T_m = K_1 K_f i_f i_a$$

$$\boxed{T_m = K_T i_a}$$

- In armature controlled motors, field current is constant and K_T is the torque constant.
- The motor back emf being proportional to the speed is given by

$$\boxed{e_b = K_b\left(\frac{d\theta}{dt}\right)} \quad \text{—— (1)}$$

Where K_b is the back emf constant

- Using KVL on the armature circuit, the differential equation of the armature circuit is

$$\boxed{L_a\left(\frac{di_a}{dt}\right) + R_a i_a + e_b = e_a} \quad \text{—— (2)}$$

- The differential equation governing the mechanical rotational systems in terms of shaft torque is

$$\boxed{J\left(\frac{d^2\theta}{dt^2}\right) + f_0\left(\frac{d\theta}{dt}\right) = T_m = K_T i_a} \longrightarrow (3)$$

Taking Laplace transform of the equations (1), (2) & (3),

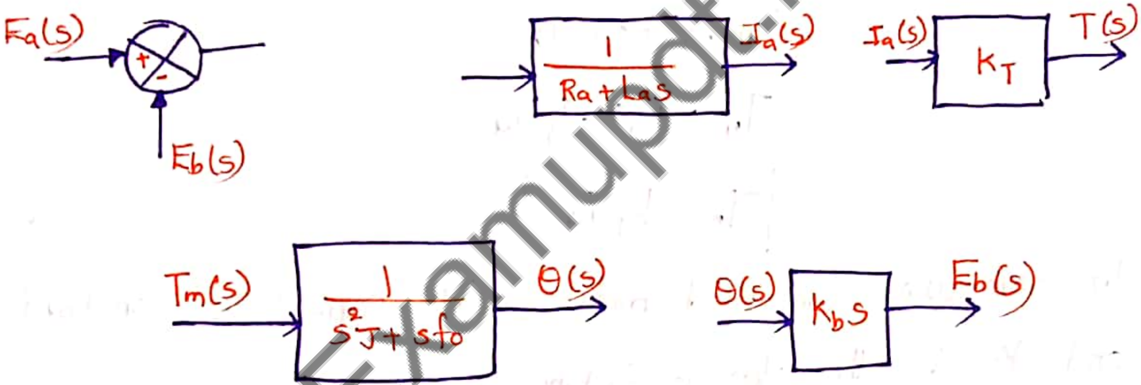
$$\boxed{E_b(s) = sK_b\theta(s)} \longrightarrow (4)$$

$$(sL_a + R_a) I_a(s) + E_b(s) = E_a(s)$$

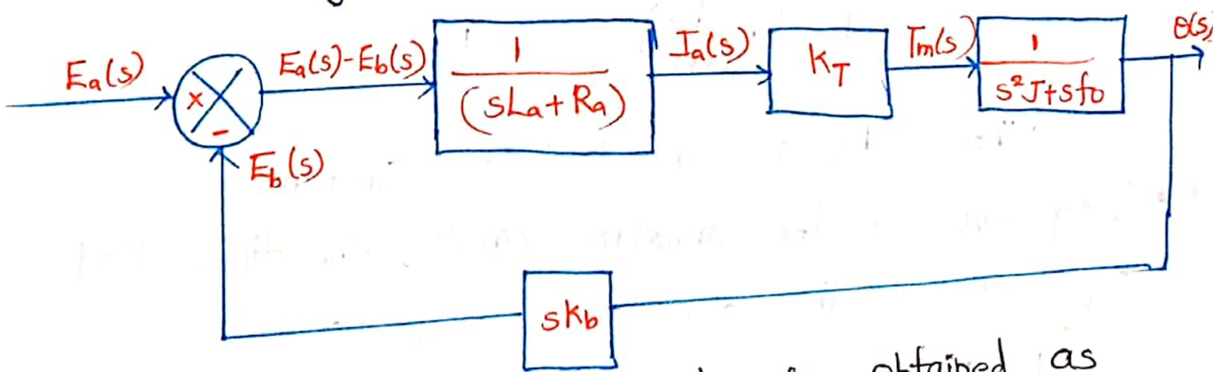
$$\boxed{I_a(s) = \frac{E_a(s) - E_b(s)}{sL_a + R_a}} \longrightarrow (5)$$

$$(Js^2 + sf_0)\,\theta(s) = K_T I_a(s)$$

$$\boxed{\theta(s) = \frac{K_T}{s^2 J + sf_0} I_a(s)} \longrightarrow (6)$$



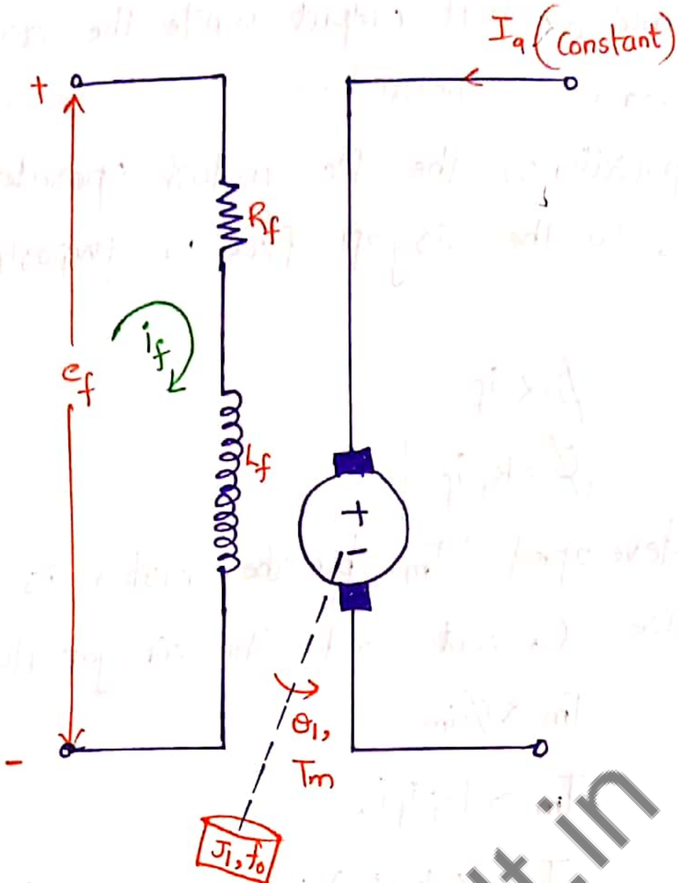
- The block diagram of the armature controlled motor is developed using the equations (4), (5) & (6)



- The transfer function of the system is obtained as

$$\therefore \boxed{G(s) = \frac{\theta(s)}{E_a(s)} = \frac{K_T}{s\left[(sL_a+R_a)(sJ+f_0) + K_T K_b\right]}}$$

Transfer Function of Field Controlled DC servo motor



- The parameters are taken as

R_f = Field Winding resistance

L_f = Field Winding inductance

i_a = Armature Current

i_f = Field Current

e_f = Supplied field (control) Voltage.

e_b = Motor back emf.

T_m = torque developed by motor.

θ = Angular displacement of motor shaft.

J = Equivalent moment of inertia (of load and motor) referred to motor shaft

f_0 = Equivalent Viscous friction Coefficient of motor and load referred to motor shaft.

- In field Controlled DC Motors, the field input voltage e_f Controls the motor shaft output while the armature current i_a remains Constant.
- For servo applications, the DC motor operates in the linear region, so the airgap flux is proportional to the field current

$$\phi \propto i_f$$

$$\boxed{\phi = K_f i_f}$$

- The torque developed T_m by the motor is proportional to the armature Current and the air gap flux.

$$T_m \propto \phi i_m$$

$$T_m \propto K_f i_f i_a$$

$$T_m = (K_1 K_f I_a) i_f$$

$$\boxed{T_m = K_T' i_f}$$

- In armature Controlled motors, field Current is Constant and K_T' is the torque Constant.
- Using KVL on the armature Circuit, the differential equation of the armature circuit is

$$\boxed{L_f \left(\frac{di_f}{dt}\right) + R_f i_f = e_f} \quad \text{--- (1)}$$

- The differential equation governing the mechanical rotational systems in terms of shaft torque is

$$\boxed{J\left(\frac{d^2\theta}{dt^2}\right) + f_0\left(\frac{d\theta}{dt}\right) = T_m = K_T' i_f} \quad \text{--- (2)}$$

Taking laplace transform of the equations (1),(2)

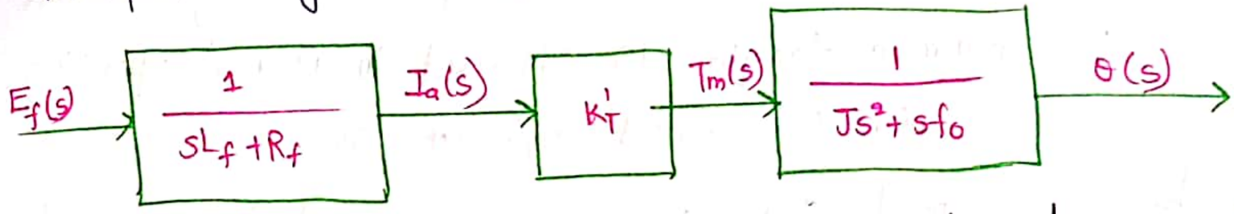
$$(sL_f + R_f) I_f(s) = E_f(s)$$

$$\boxed{I_f(s) = \frac{E_f(s)}{sL_f + R_f}} \quad \text{--- (3)}$$

$$(s^2J + sf_0)\,\theta(s) = K_T' \, I_f(s)$$

$$\boxed{\theta(s) = \frac{K_T' \, I_f(s)}{s^2J + sf_0}} \quad \text{--- (4)}$$

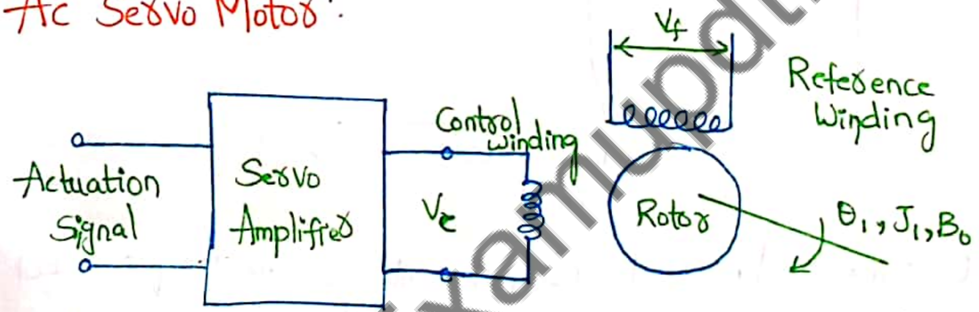
- The block diagram of the armature Controlled motor is developed using the equations (3) & (4)



- The transfer function of the system is obtained as

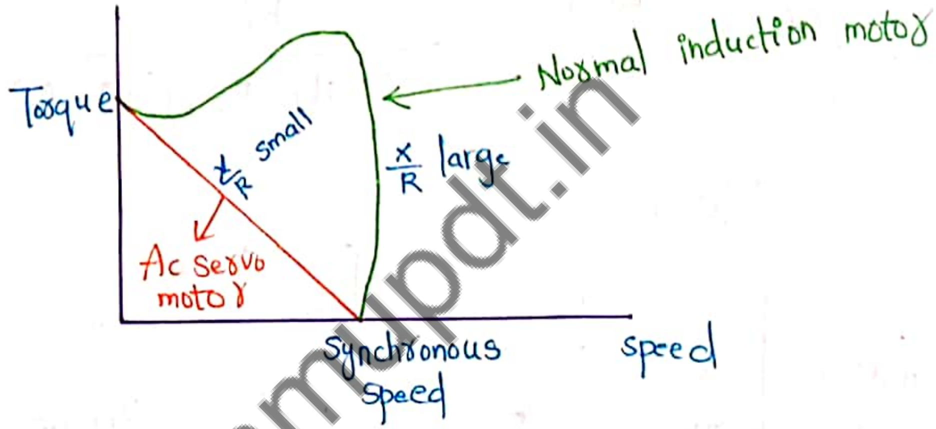
$$\therefore \boxed{G(s) = \frac{\theta(s)}{E_f(s)} = \frac{K_T'}{s(sL_f + R_f)(sJ + f_0)}}$$

Ac Servo Motor:

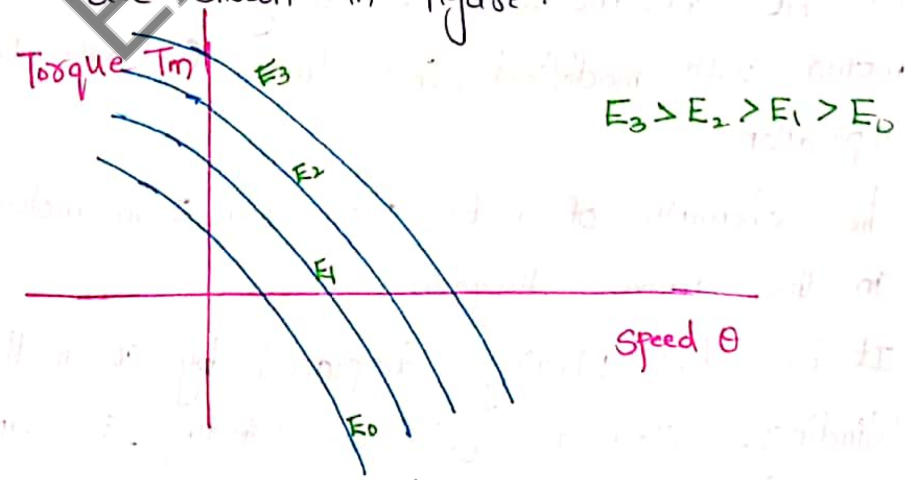


- An AC Servomotor is essentially a two phase induction motor with modified Constructional features to suit Servo applications.
- The schematic of a two phase (or) Servo motor is as shown in the above diagram.
- It has two Windings displaced by 90° on the stator One Winding, called as reference Winding, is Supplied with a Constant Sinusoidal Voltage
- The second Winding, Called Control winding, is supplied with a Variable Control Voltage which is displaced by −90° Out of phase from the reference Voltage.

- The motor major differences between the normal induction motor and an AC servo motor are:
  - * The rotor winding of an AC servo motor has high resistance (R) compared to its inductive reactance (X) so that its $\frac{X}{R}$ ratio is very low.
  - * For a normal induction motor, $\frac{X}{R}$ ratio is high so that the maximum torque is obtained in normal operating region which is around 5% of slip.
- The torque speed characteristics of a normal induction motor and an AC servo motor are shown



- For different rms values of control voltage the torque speed characteristics are shown in figure.



- The torque varies approximately linearly with respect to speed and also controls voltage.
- The torque speed characteristics can be linearized at the operating point and the transfer function of the motor

can be Obtained.

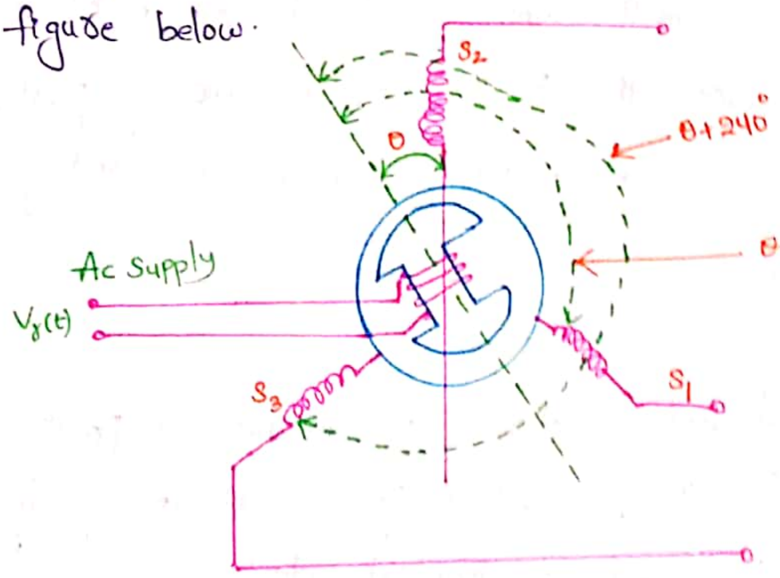
- The supplies to the two windings of AC servo motor are not balanced as in the case of a normal induction motor.
- The control voltage varies both in magnitude and phase with respect to the constant reference voltage applied to the reference winding.
- The direction of rotation of the motor depends on the phase ($\pm 90°$) of the control voltage with respect to the reference voltage.
- The torque speed characteristics of a normal induction motor is highly nonlinear and has a positive slope for some portion of the curve.
- This is not desirable for control applications as the positive slope makes the systems unstable
- The torque speed characteristic of an AC servo motor is fairly linear and has negative slope throughout.
- The rotor construction is usually squirrel cage (or) drag cup type for an AC servo motor.
- The diameter is small compared to the length of the rotor which reduces the inertia of the moving parts.
- Thus it has good accelerating characteristic and good dynamic response.

Synchros

- A commonly used error detector of mechanical positions of rotating shafts in AC control systems in the synchro.
- It consists of two electro mechanical devices.
  - * Synchro transmitter.

* Synchro receiver (or) Control transformer

- The principle of operation of these two devices is same but they differ slightly in their construction.
  - * The construction of a synchro transmitter is similar to a phase alternator.
  - * The stator consists of a balanced three phase winding and is star connected.
  - * The rotor is of dumbbell type construction and is wound with a coil to produce a magnetic field.
- When a no voltage is applied to the winding of the rotor, a magnetic field is produced.
- The coils in the stator link with this sinusoidal distributed magnetic flux and voltages are induced in the three coils due to transformer action.
- Than the three voltages are in time phase with each other and the rotor voltage.
- The magnitudes of the voltages are proportional to the cosine of the angle between the rotor position and the respective coil axis.
- The position of the rotor and the coils are shown in figure below.



$V_R(t) = V_r \sin\omega t$

$V_{S1n} = KV_r \sin\omega_r t \cos(\theta + 120)$

$V_{S2n} = KV_r \sin\omega_r t \cos\theta$

$V_{S3n} = KV_r \sin\omega_r t \cos(\theta + 240)$

$V_{S_1S_2} = V_{S1n} - V_{S2n} = \sqrt{3} KV_r \sin(\theta + 240) \sin\omega_r t$

$V_{S_2S_3} = V_{S2n} - V_{S3n} = \sqrt{3} KV_r \sin(\theta + 120) \sin\omega_r t$

$V_{S_3S_1} = V_{S3n} - V_{S_1n} = \sqrt{3} KV_r \sin\theta \sin\omega_r t$

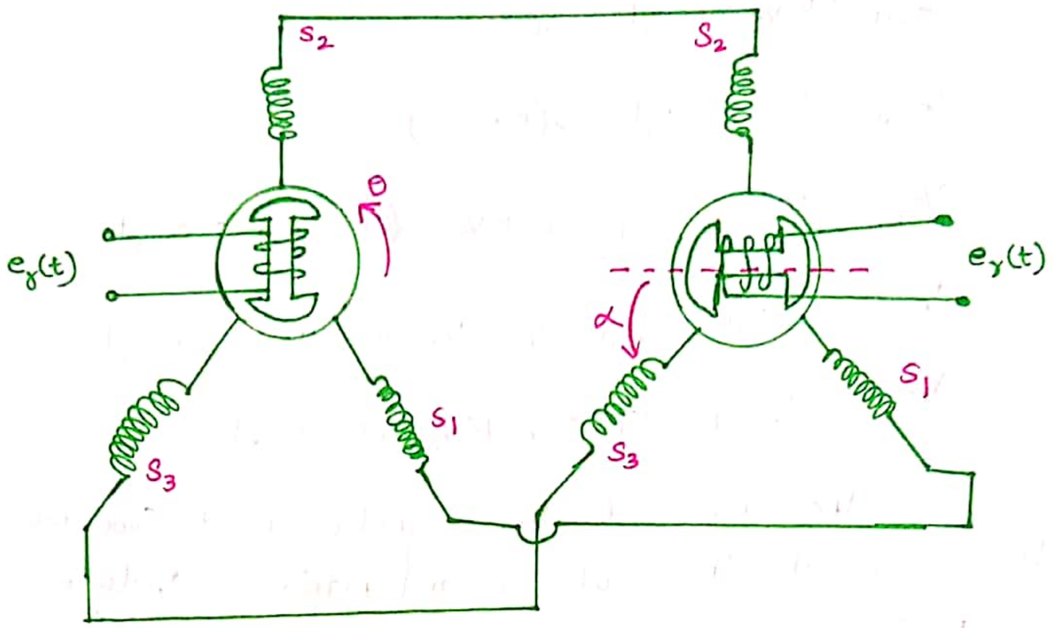
- When $\theta = 90°$ the axis of the magnetic field coincides with the axis of the coil S_2 and maximum voltage is induced in it as seen.
- For this position of the rotor, the voltage c is zero, this position of the rotor is known as the Electrical zero of the transmitter and is taken as reference for specifying the rotor position.
- In summary, it can be seen that the input to the transmitter is the angular position of the rotor and the set of three single phase voltages is the output.
- The magnitudes of these voltages depend on the angular position of the rotor as given.

Hence,

$$\boxed{e_r(t) = K_1 V_r \cos\phi \sin\omega_r t}$$

- Now consider these three voltages to the applied to the stator of a similar device. Called Control transformer (or) Synchro receiver.
- The construction of a control transformer is similar to that of the transmitter except that the rotor is made cylindrical in shape whereas the rotor of transmitter

is dumbell in shape.



- Since the rotor is Cylindrical, the air gap is uniform and the reluctance of the magnetic path is Constant.
- This makes the Output impedance of rotor to be a Constant.
- Usually, the rotor winding of Control transformer is Connected teas amplifier which requires Signal with Constant impedance for better performance.
- A synchro transmitter is usually required to Supply Several Control transformers and hence the stator winding of control transformer is wound with higher impedance per phase.
- Since, Some Currents flow through the stators of the synchro transmitter and receiver, the Same pattern of flux distribution will be Produced in the air gap of the Control transformer.
- The Control transformer flux axis is in the Same Position as that of the Synchro transmitter.
- Thus, the Voltage induced in the rotor coil of Control transformer is proportional to the cosine of the angle between the two rotors.