# Unit - 4

## Introduction:

→ Soils are subjected to stresses due to loads from foundations of various shapes and self weight of soil.

→ The theory of elasticity is the basis of determination of stresses in soil mass due to loads from foundations of various shapes.

→ for elastic theory to be applicable there must be a constant ratio between stresses induced and corresponding strain.

→ It is not necessary that the soil be truly elastic; only the ratio of stress to strain should be constant. So elastic theory can used in case of non-elastic soil masses subjected to small order to stresses.

→ fortunately, the actual magnitude transmitted to soil from structural loading falls within this range.

→ Hence the elastic theory gives valid results.

→ Stress at any point 'p' in the soil mass due to point load 'Q' over the surface at point 'o' can be computed using
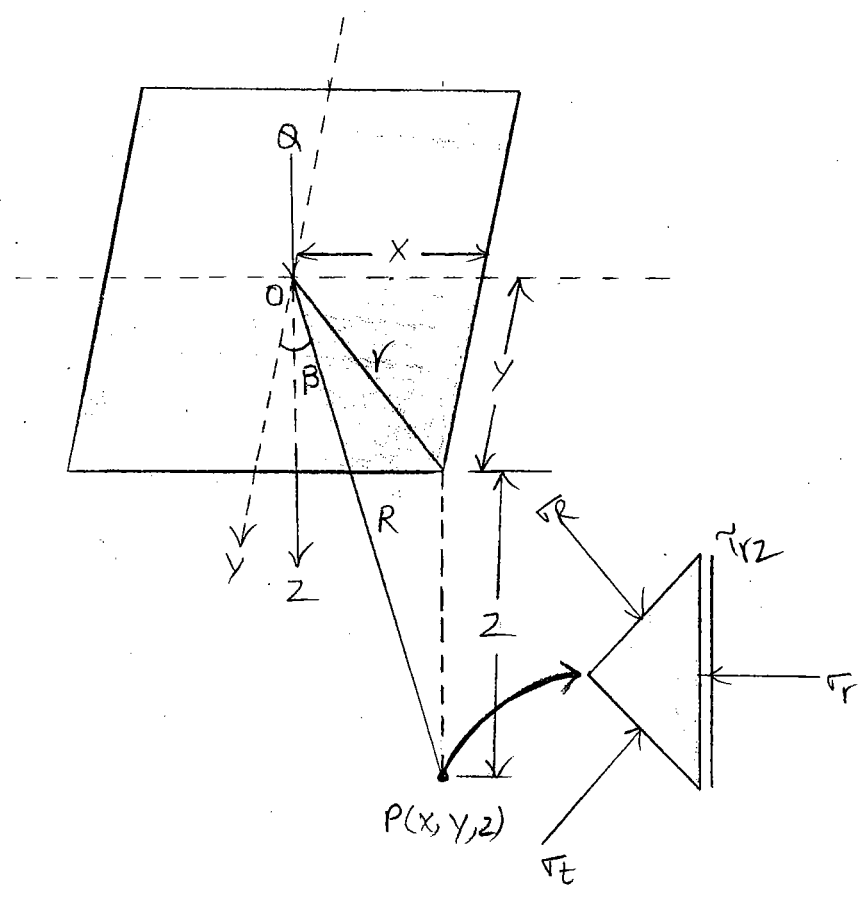
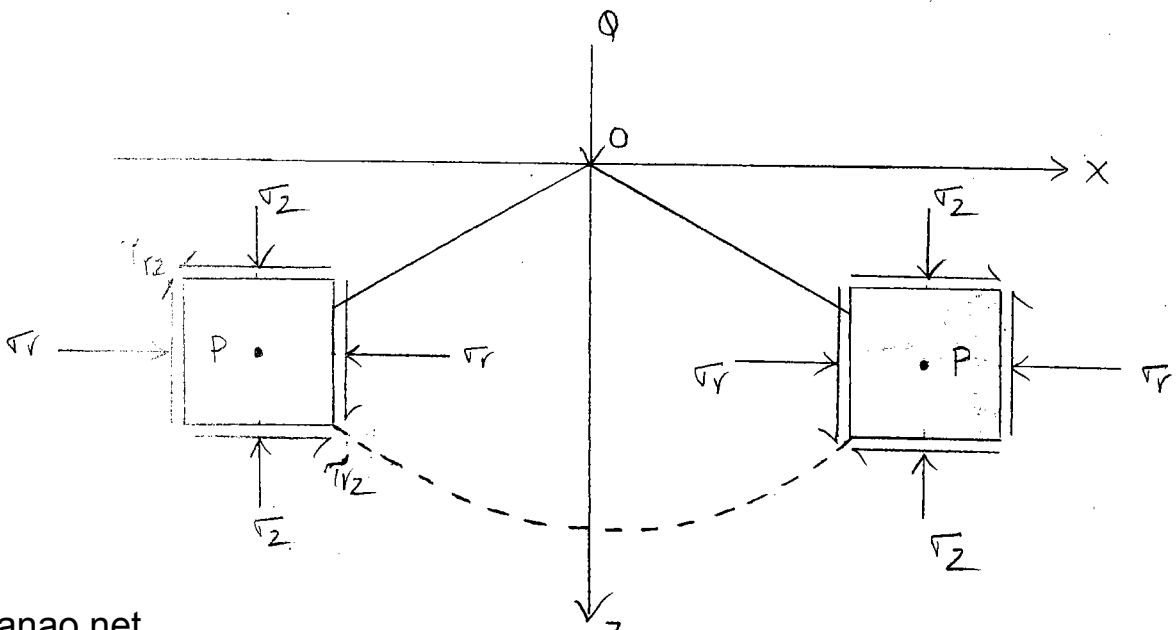   (a) Boussinesq's theory       (b) westergaard's theory

## (a) Boussinesq's theory:

### Assumptions:

→ Soil is assumed to be homogeneous, isotropic, semi-infinite and elastic.

→ Hooke's law is valid.

→ self weight of soil is neglected.

→ Soil is initially unstressed.

→ Any change in the volume of the soil due to the application of the load is neglected i.e., soil is incompressible.

$\rightarrow$ The top surface of the soil is free from any shear stress and is subjected to only point load.

$\rightarrow$ Distribution of the stresses is symmetric along the vertical axis.

$\rightarrow$ Continuity of stress exists in the medium.



Where $r = \sqrt{x^2 + y^2}$

$R = \sqrt{r^2 + z^2} = \sqrt{x^2 + y^2 + z^2}$

The boussinesq's equations are as follows:

1) Vertical stress, $\sigma_Z = K_B \dfrac{Q}{Z^2}$

Where $K_B$ = Boussinesq influence factor

$$= \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$K_B$ is a number and is a function of $r/z$ ratio.

2) Shear stress, $\tau_{rz} = \dfrac{3Q}{2\pi}\dfrac{rz^2}{[r^2+z^2]^{5/2}} = \sigma_Z \cdot \dfrac{r}{z}$

3) Radial stress, $\sigma_r = \dfrac{Q}{2\pi}\left[\dfrac{3zr^2}{R^5} - \dfrac{(1-2\mu)}{R(R+z)}\right]$
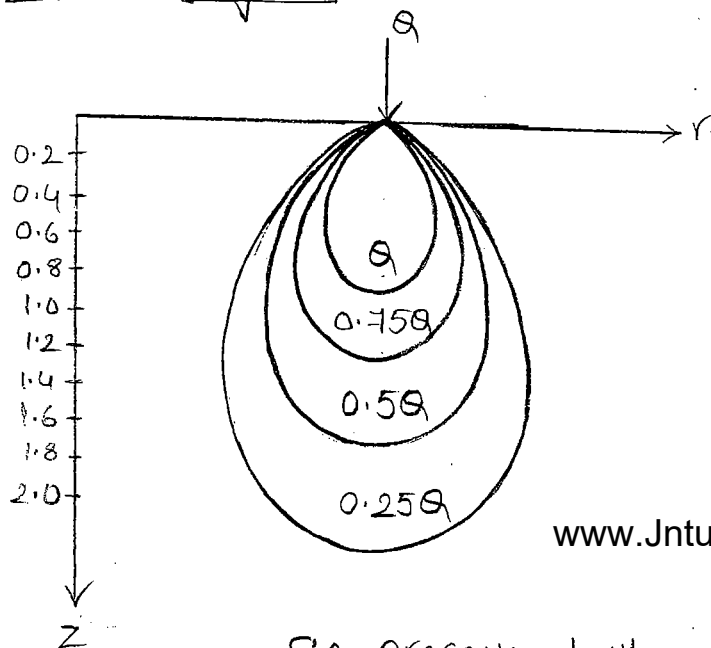
Where $\mu$ = Poisson's ratio

Note: The intensity of vertical stress directly below the point load on its axis of loading is given by where $r=0$

i.e, $\sigma_Z = \dfrac{3Q}{2\pi z^2}\left[\dfrac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}_{r=0}$

$$= 0.4775 \frac{Q}{z^2}$$

Vertical stress distribution diagrams:

1) vertical stress isobar diagram :



fig. Pressure bulb

→ Stress isobar is a curve (or contour joining the points of equal vertical pressure in the soil mass.

→ Since the vertical stress on a given horizontal plane is the same in all directions at points located at equal radial distances from the axis of loading.
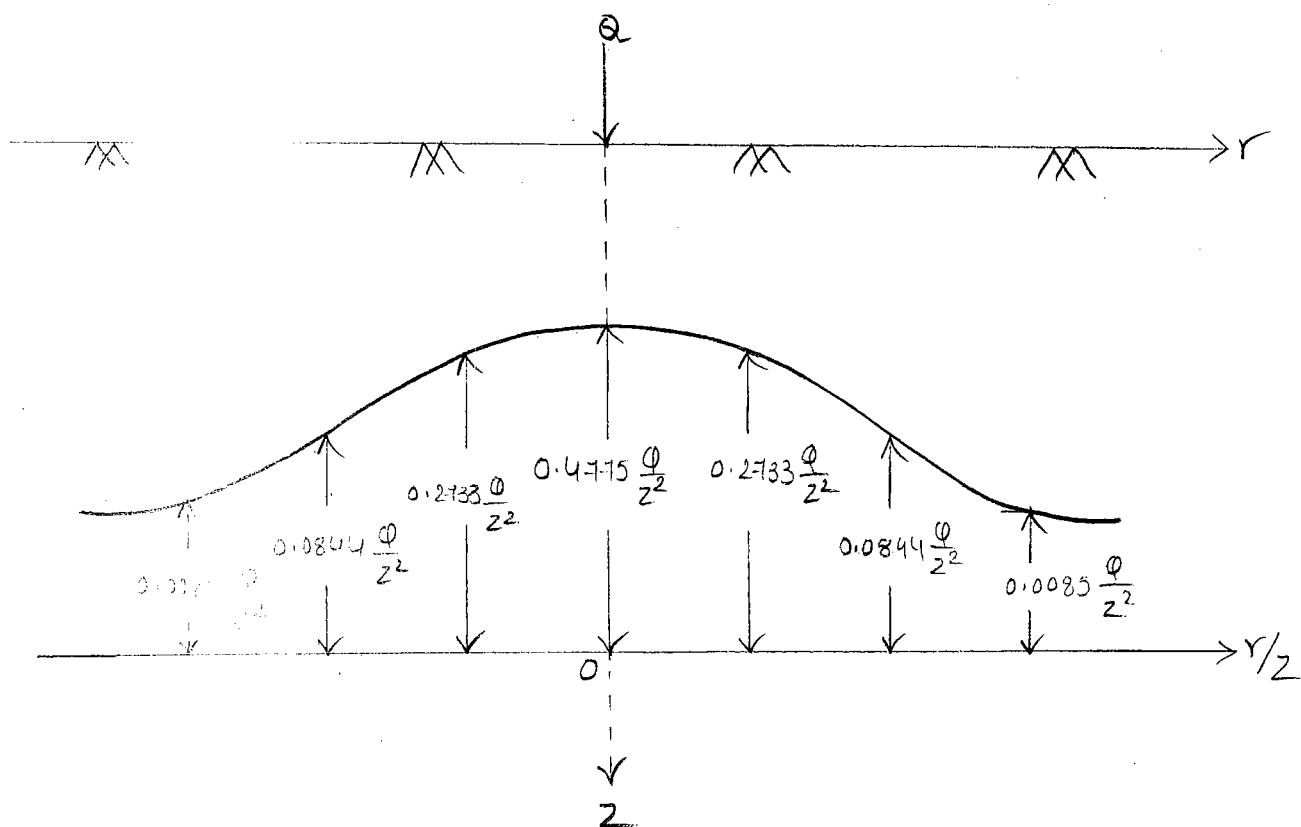
→ This isobar is spatial curved surface and resembles a onion bulb in shape, hence it is also called pressure bulb.

→ With the increasing of depth, vertical stress reduces. Hence, generally effect of vertical stress beyond the zone of 0.20 Q isobar is neglected.

Note: The depth at which vertical stress becomes 20% of applied load, means 0.2 Q, that depth is called seat of settlement. This is also a criteria of determining depth of exploration.

2) vertical stress distribution diagram on horizontal plane:



$$0.4775 \frac{Q}{z^2}$$

$$0.2733 \frac{Q}{z^2}$$

$$0.2733 \frac{Q}{z^2}$$

$$0.0844 \frac{Q}{z^2}$$

$$0.0844 \frac{Q}{z^2}$$

$$0.0085 \frac{Q}{z^2}$$

→ Vertical pressure distribution diagram on the horizontal plane at a constant depth of 'z' can be analysed by using the vertical pressure given by boussinesq theory.

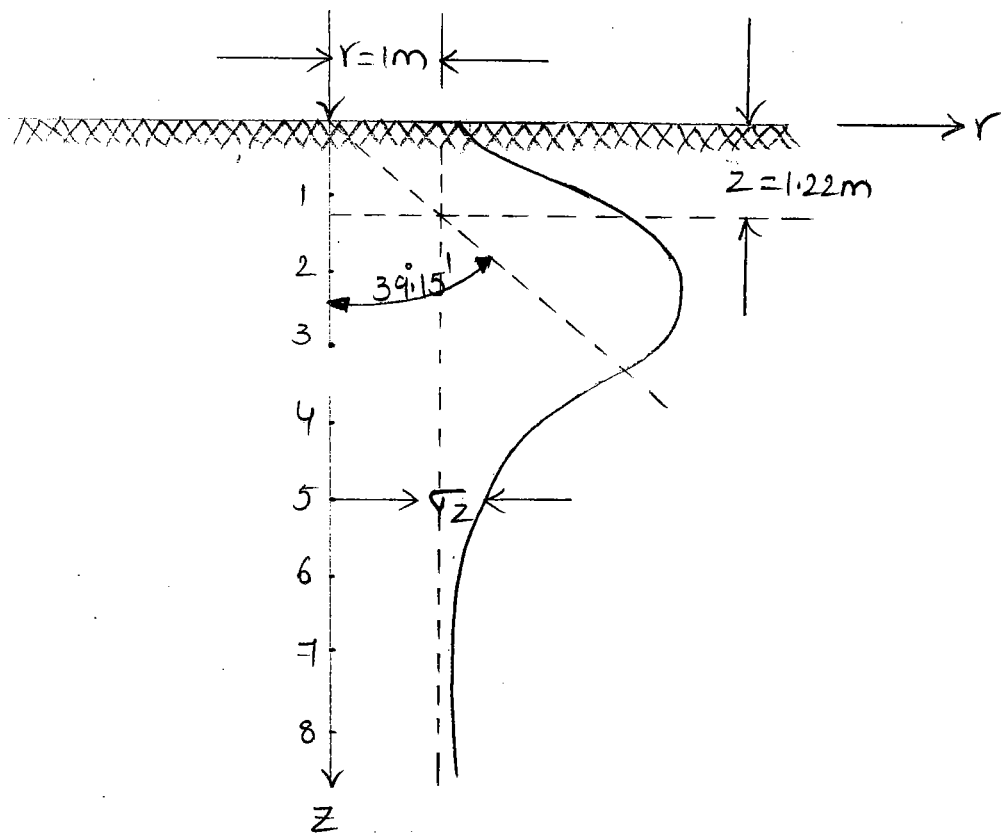$$\text{i.e., } \sigma_Z = K_B \frac{Q}{Z^2}$$

→ $\sigma_Z$ is maximum occurs just below the load i.e., r=0, which is equals to

$$\sigma_Z = 0.477 \frac{Q}{Z^2}$$

→ Vertical stress distribution diagram on a horizontal plane is bell-shaped. The maximum stress is just below the line of action of the concentrated load and stress decrease is asymptotic.

| $r/z$ | $K_B$ | $\sigma_Z$ | Description |
|-------|-------|-----------|-------------|
| 0 | 0.4775 | $0.4775 \frac{Q}{Z^2}$ | $\sigma_{max}$ |
| 0.5 | 0.2733 | $0.2733 \frac{Q}{Z^2}$ | 57.24% of $\sigma_{max}$ |
| 1 | 0.0844 | $0.0844 \frac{Q}{Z^2}$ | 17.67% of $\sigma_{max}$ |
| 2 | 0.0085 | $0.0085 \frac{Q}{Z^2}$ | 1.78% of $\sigma_{max}$ |
| 1.52 | 0.0239 | $0.0239 \frac{Q}{Z^2}$ | 5% of $\sigma_{max}$ |

→ If Q=1 (unity), the diagram is known as influence diagram for vertical stress at any point.

Stress on a vertical plane

→ Vertical pressure on the vertical line first increases with increase in depth and reaches upto its maximum value and starts decreasing beyond it with further increase in depth.

→ maximum vertical pressure on vertical line occurs when the angle B made by the polar ray attains a value of $39°13'53.3''$ from the point load.

for $\sigma_{z,max}$,  $\quad \dfrac{\partial \sigma_z}{\partial z} = 0$

$\therefore \quad \dfrac{\partial}{\partial z}\left[\dfrac{3Q}{2\pi} \cdot \dfrac{z^3}{(z^2+r^2)^{5/2}}\right] = 0$

or, $\dfrac{(z^2+r^2)^{5/2} \cdot 3z^2 - (z^3)\,\frac{5}{2}\,(z^2+r^2)^{3/2} \cdot 2z}{(z^2+r^2)^5} = 0$

or, $6z^2(z^2+r^2)^{5/2} - 10z^4(z^2+r^2)^{3/2} = 0$

or, $2z^2 (z^2+r^2)^{1/2} [3(z^2+r^2) - 5z^2] = 0$

or, $2z^2 (z^2+r^2)^{3/2} (3r^2 - 2z^2) = 0$

∴ $3r^2 - 2z^2 = 0$

or, $\dfrac{r}{z} = \sqrt{\dfrac{2}{3}} = \tan\beta$

∴ $\beta = 39°13'53.5''$

NOW $\quad \sigma_{z,max} \left[ \dfrac{r}{z} = \sqrt{\dfrac{2}{3}} = \dfrac{3}{2\pi} \left[ \dfrac{1}{1+\left(\frac{r}{z}\right)^2} \right]^{5/2} \dfrac{Q}{z^2} \right.$

$= \dfrac{3}{2\pi} \left[ \dfrac{1}{1+(2/3)} \right]^{5/2} \cdot \dfrac{Q}{\frac{3}{2}r^2}$

$= 0.0888 \dfrac{Q}{r^2}$

Shear stress, where $\sigma_{z,max}$ occurs, $\tau_{rz} = \sigma_z \cdot \dfrac{r}{z}$

$= 0.0888 \times \dfrac{Q}{r^2} \times \sqrt{\dfrac{2}{3}}$

$= \dfrac{Q}{r^2}$

Problem-1:

Laboratory results on a soil have shown that its unconfined compressive strength is $1.2 \, Kg/cm^2$. In a tri-axial compression test a specimen of the soil when subjected to a chamber pressure of $0.4 \, Kg/cm^2$ failed at an additional stress of $1.6 Kg/cm^2$. Estimate the shearing strength of the same soil along a horizontal plane at a depth of 4m in a deposit. The ground water table is at a depth of 2.5m from the ground level. Take dry unit weight of soil as $1.7 \, gm/cc$ and specific gravity as 2.7.

Sol: The unconfined compression test is also a sort of triaxial test, in which minor principal stress $(\sigma_3)$ is zero. The unconfined

We Know that

$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c \tan\left(45° + \frac{\phi}{2}\right)$$

Where $\phi$ is angle of internal friction and $c$ is cohesion.

Given $\sigma_1 = 1.2 \, kg/cm^2$ and $\sigma_3 = 0$

$$\therefore \quad 1.2 = 0 \times \tan^2\left(45° + \frac{\phi}{2}\right) + 2c \tan\left(45° + \frac{\phi}{2}\right)$$

$$c = \frac{0.6}{\tan\left(45° + \frac{\phi}{2}\right)} \quad\quad —\; ①$$

Triaxial test:

$$\sigma_3 = 0.4 \, kg/cm^2 \quad \text{and} \quad \sigma_d = 1.6 \, kg/cm^2$$

$$\therefore \quad \sigma_1 = \sigma_3 + \sigma_d = 0.4 + 1.6 = 2 \, kg/cm^2$$

then $\quad \sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c\tan\left(45° + \frac{\phi}{2}\right)$

$$\Rightarrow \quad 2 = 0.4 \tan^2\left(45° + \frac{\phi}{2}\right) + 2 \times \frac{0.6}{\tan\left(45° + \frac{\phi}{2}\right)} \times \tan\left(45° + \frac{\phi}{2}\right)$$

$$\tan^2\left(45° + \frac{\phi}{2}\right) = \frac{2 - 1.2}{0.4}$$

$$\tan\left(45° + \frac{\phi}{2}\right) = \sqrt{2}$$

$$\phi = 19.47°$$

Putting $\phi = 19.47°$ in eqⁿ ①

we get $\quad c = \dfrac{0.6}{\tan\left(45° + \frac{19.47°}{2}\right)}$

$$= 0.4243 \, kg/cm^2$$

But $\gamma = c + \bar{\sigma} \tan\phi$

Where $\gamma$ is the shear strength of soil and $\bar{\sigma}$ is the effective pressure.

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Effective pressure, $\sigma = \gamma_{dry} \times 2.5 + \gamma_{sub} \times 1.5$

Given, $\gamma_{dry} = 1.7 \, gm/cc$, $G = 2.7$

We know that $\gamma_{dry} = \dfrac{G\gamma_w}{1+e}$

$$1.7 = \dfrac{2.7 \times 1}{1+e}$$

$$e = 1.588 - 1$$

$$e = 0.588$$

and $\gamma_{sub} = \left(\dfrac{G-1}{1+e}\right)\gamma_w$

$$= \left(\dfrac{2.7-1}{1+0.588}\right) \times 1$$

$$= 1.07 \, gm/cc$$

$\therefore \ \bar{\sigma} = (1.7 \times 250) + (1.07 \times 150) \, g/cm^2$

$$= 585.5 \, g/cm^2$$

$$= 0.5855 \, Kg/cm^2$$

$$= 0.586 \, Kg/cm^2$$

Shear strength of soil, $\tau = c + \bar{\sigma} \tan\phi$

$$= 0.4243 + 0.586 \tan 19.47°$$

$$= 0.63 \, Kg/cm^2$$

## Assumptions:

→ medium is assumed to be homogeneous, semi-infinite, elastic & non-isotropic.

→ Here, medium is assumed to be laterally reinforced with fibres of negligible thickness i.e., medium is considered to be rigid horizontally and elastic vertically. It mean no lateral deformations but only vertical deformations.

Note: Since there is no lateral strain. Hence poisson's ratio in horizontal direction,

$$\mu = \frac{\varepsilon_{lat}}{\varepsilon_{log}} = 0$$

Young's modulus in horizontal direction will be

$$E_H = \frac{\sigma}{\varepsilon_{lat}} = \infty$$

Here the ratio of young's modulus in horizontal to vertical direction tends to infinity

$$E = \frac{E_H}{E_V} = \infty$$

→ westergaard equation for vertical stress for a point load, for a poisson's ratio value equal to zero is given by

$$\sigma_z = K_w \frac{Q}{z^2}$$

Where $K_w$ = westergaard's influence factor

$$= \frac{1}{\pi} \left[ \frac{1}{1 + 2\left(\frac{r}{z}\right)^2} \right]^{3/2}$$

→ $v_{z,max}$ occurs when $z$ is constant and $r$ is zero i.e.,

$\frac{r}{z} = 0$, then

$$K_W = -\frac{1}{\pi}\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}$$

$$= \frac{1}{\pi}\left[\frac{1}{1+2\times 0}\right]^{3/2}$$

$$= 0.3183$$

$$\nabla_{z,max} = 0.3183\,\frac{Q}{z^2}$$

→ It is to be noted that at $\frac{r}{z} = 1.52$, both the theories gives equal values
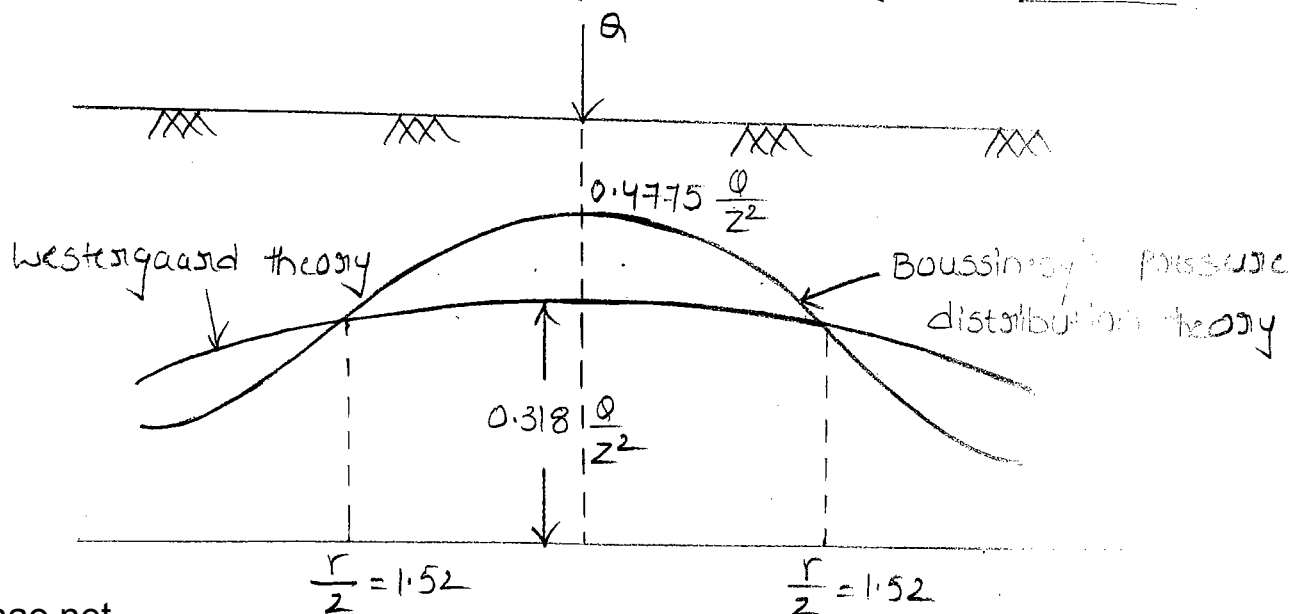
$$\sigma_{W,Bouss} = \sigma_{z,Wester}$$

$$K_B\,\frac{Q}{z^2} = K_W\,\frac{Q}{z^2}$$

$$\frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = \frac{1}{\pi}\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}$$

$$\frac{9}{4}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^5 = \left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^3$$

$$\frac{r}{z} = 1.52$$

<u>Comparision between boussinesq and westergaard theories:</u>

$\rightarrow$ If $\dfrac{r}{z} < 1.52$    $K_B > K_W$

$\dfrac{r}{z} = 1.52$    $K_B = K_W$

$\dfrac{r}{z} > 1.52$    $K_B < K_W$

Note: Results obtained by Westergaard's theory are more realistic than boussinesq theory but in the range where stresses are significant. Results obtained by Bossinesq's theory are more conservative than that of westergaard's theory. Hence, Boussinesq's theory is practically used for the computation of vertical stresses in the soil mass.

## Problem-1:

A concentrated load of 2500KN is applied at the ground surface. Determine the vertical stress at Point P which is at a depth of 5m but at a horizontal distance of 4m from the axis of load. Also calculate the percentage increase in stress when the point P is directly below the load at same depth of 5m. use Boussinesq's theory.

Sol: Case-1:

Given $r = 4m$, $z = 5m$

using    $\sigma_z = \dfrac{3Q}{2\pi z^2}\left[\dfrac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$

$\sigma_P = \dfrac{3 \times 2500}{2\pi \times (5)^2}\left[\dfrac{1}{1+\left(\frac{4}{5}\right)^2}\right]^{5/2}$

$= 13.86 \ KN/m^2$

Case - II : Here r becomes zero.

$$\sigma_P = \frac{3 \times 2500}{2\pi \times (5)^2} \left[ \frac{1}{1 + \left(\frac{0}{5}\right)^2} \right]^{5/2}$$
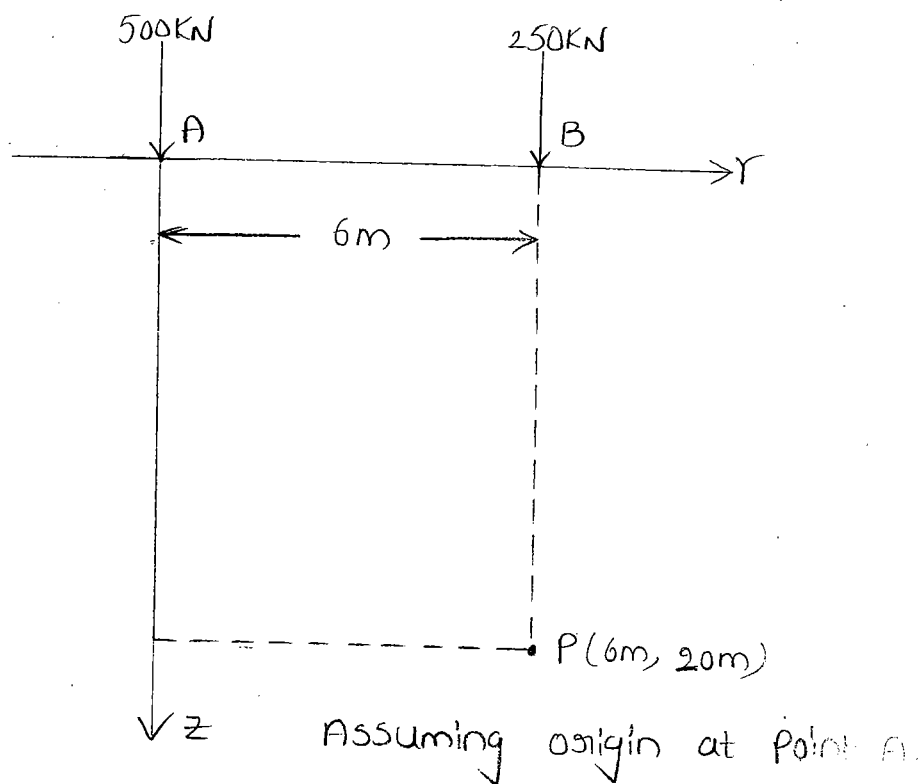
$$= 47.75 \ KN/m^2$$

% increase in stress , $\Delta\sigma = \dfrac{47.75 - 13.86}{13.86} \times 100$

$$= 244.5\%$$

## Problem-2:

Two columns A and B are situated 6m apart. Column A transfers a load of 500KN and column B, a load of 250KN. Determine the resultant vertical stress on a horizontal plane, 20m below the ground surface at point vertically below the points B.



Assuming origin at point A.

Sol: Effect of 500 KN load at point 'P':

$$Q = 500 \ KN, \ r = 6m, \ z = 20m$$

$$\therefore \ \sigma_1 = \frac{3Q}{2\pi z^2} \left[ \frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

$$= \frac{?}{2\pi (20)^2} \left[ \frac{}{1+\left(\frac{6}{20}\right)^2} \right]^{-\tfrac{5}{2}}$$

$$= 0.481 \, KN/m^2$$

Effect of 250 KN load at point 'p':

$$Q = 250 \, , \, r=0, \, z = 20m$$

$$\sigma_2 = 0.4775 \, \frac{Q}{z^2}$$

$$= 0.4775 \times \frac{250}{(20)^2}$$

$$= 0.2984 \, KN/m^2$$

Resultant stress, $\sigma = \sigma_1 + \sigma_2$
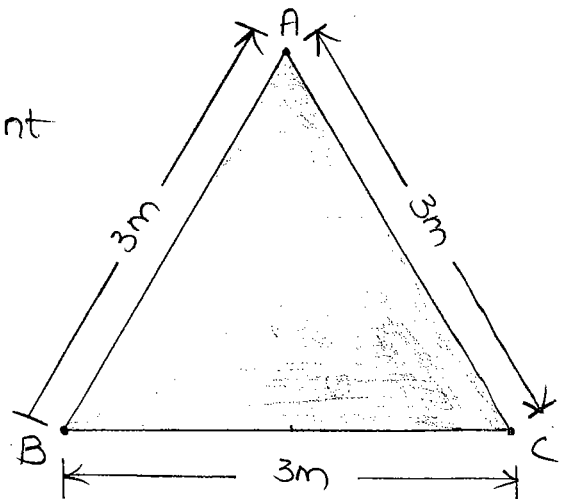
$$= 0.481 + 0.2984$$

$$= 0.7794 \, KN/m^2$$

Problem—3:

A three legged tower forms an equilateral triangle in plane of side 3m, as shown in fig. If the total weight of the tower is 450KN and is equally carried by all it's legs, compute the increase in the vertical stress caused in the soil by the installation of the tower at a depth of 4.5m directly below one of the leg of the tower.

sol: The total weight of tower = 450KN
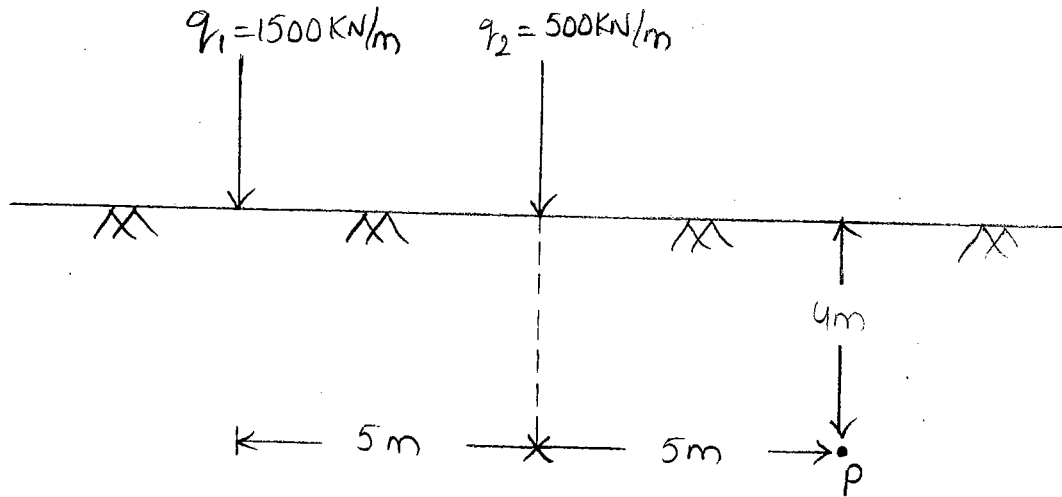The load can be approximated to a point load acting at all the legs



$$\therefore \text{Load on each kg} = \frac{450}{3}$$
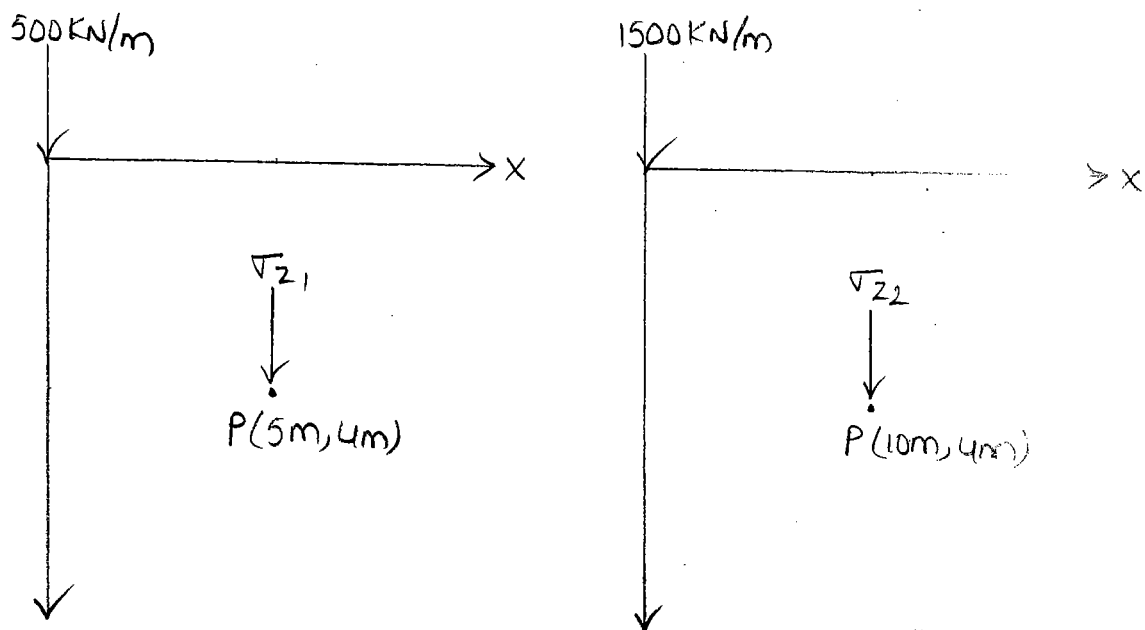
$$= 150 KN$$

Now vertical stress at 4.5m below on of the leg (say below (A)) is caused by three point loads coming through the legs A,B and C.

Problem-2:

figure shows two parallel line loads on the ground surface.
Determine the vertical stress at a point P as shown in figure.



$q_1 = 1500 \, KN/m$   $q_2 = 500 \, KN/m$

5 m ── ✳ ── 5 m

4m

•P

sol: The vertical stress at P can be obtained by super position.
Due to each line load the stresses at 'p' are computed seperately
and then they are added to give the total stress due to the two line
loads that acts simultaneously on the ground surface. This is shown
in figure below:



500 KN/m                     1500 KN/m

────────────→X              ────────────→X

$\sigma_{z_1}$                $\sigma_{z_2}$

P(5m, 4m)                    P(10m, 4m)

i) Effect of 500KN/m on point 'p'   ii) Effect of 1500 KN/m on point 'p'.

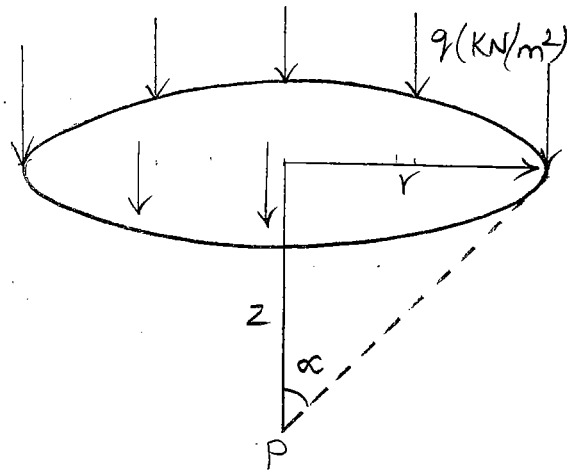Net stress on point 'p' will be $\sigma_z = \sigma_{z_1} + \sigma_{z_2}$

where   $\sigma_{z_1} = \dfrac{2q_1 z^3}{\pi (x_1^2 + z^2)^2}$ ⟹ $\dfrac{2 \times 500 \times 4^3}{\pi (5^2 + 4^2)^2}$ ⟹ $12.12 \, KN/m^2$

and $\quad \sigma_{z_2} = \dfrac{-P^2}{\pi(r_2^2+z^2)^2} \Rightarrow \dfrac{2 \times 10 \times \dots \times 1}{\pi(10^2+4^2)^2} \Rightarrow 4.54 \text{ KN/m}^2$

$$\sigma_z = 12.12 + 4.54$$

$$\sigma_z = 16.66 \text{ KN/m}^2$$

2) <u>vertical stress distribution below uniformly loaded circular area:</u>

for a circular area of radius 'r' loaded uniformly with 'q'/unit area the vertical stress at a depth 'z' is given as,



i) <u>using boussinesq's theory:</u>

$$\sigma_z = K \cdot q$$

where $\quad K = 1 - \cos^3 \alpha$

here $\quad \cos \alpha = \dfrac{z}{\sqrt{r^2+z^2}}$

Hence, $\quad \sigma_z = \left[ 1 - \left( \dfrac{1}{1+\dfrac{r^2}{z^2}} \right)^{3/2} \right] q$

i) <u>using westergaard theory:</u>

$$\sigma_z = \left[ 1 - \dfrac{1}{\sqrt{1+\left(\dfrac{r}{nz}\right)^2}} \right] q$$

where $\quad n = \sqrt{\dfrac{1-2\mu}{2-2\mu}}$ , $\mu$ is being poisson's ratio

i) vertical stress caused by load on leg A, below which the stress is computed.

Here, $Q = 150\,KN$, $r=0$, $z=4.5m$

$$\sigma_{z1} = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

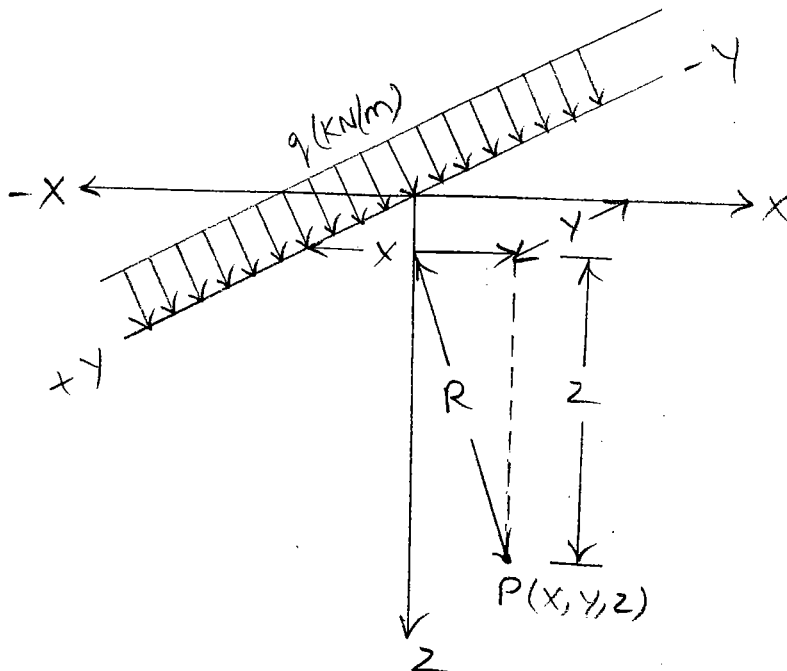$$= \frac{3\times150}{2\pi(4.5)^2}\left[\frac{1}{1+\left(\frac{0}{4.5}\right)^2}\right]^{5/2}$$

$$= 3.54\,KN/m^2$$

ii) vertical stress caused by load on either by the other two legs B and C.

Here $Q=150\,KN$, $r=3m$, $z=4.5m$

$$\therefore \sigma_{z2} = \sigma_{z3} = \frac{3\times150}{2\pi(4.5)^2}\left[\frac{1}{1+\left(\frac{3}{4.5}\right)^2}\right]^{5/2}$$

$$= 1.41\,KN/m^2$$

$$\therefore \sigma_z = \sigma_{z1}+\sigma_{z2}+\sigma_{z3}$$

$$= 3.54 + 1.41 + 1.41$$

$$= 6.36\,KN/m^2$$

1) <u>vertical stress due to a uniform line load:</u>

$\rightarrow$ For this two-dimensional problem, using the principles of elastic theory, the vertical stress $\sigma_z$ at a point 'p' in the soil mass with coordinate $x,z$ is given as
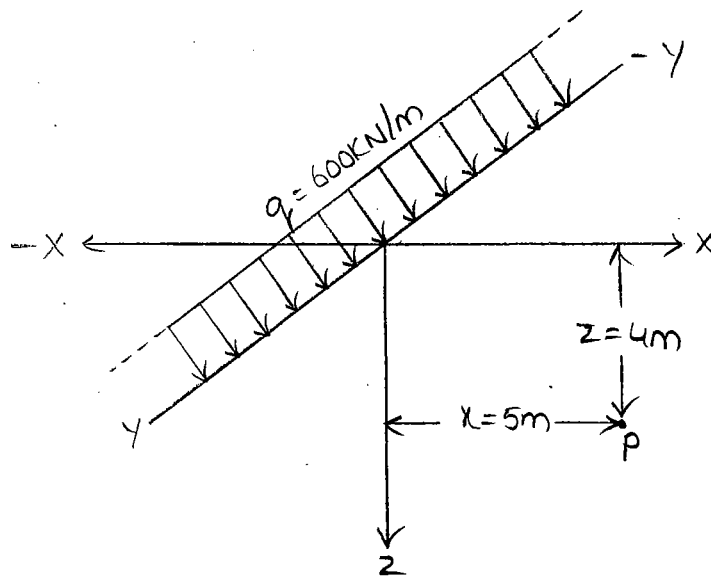
$$\sigma_z = \frac{2q}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]$$

$\rightarrow$ If point 'p' is just below the line load at a depth 'z' i.e., $x=0$ then

$$\sigma_z = \frac{2q}{\pi z}.$$

## Problem-1:

The infinite line load as shown in figure has an intensity of 600 KN/m. Determine the vertical stress at a point A with coordinates $x=5m$ and $z=4m$.

sol: using equation,

$$\sigma_z = \frac{2q}{\pi z}\left[\frac{1}{1+\frac{x^2}{z^2}}\right]^2$$

Substituting $q=600 KN/m$, $x=5m$ and $z=4m$, we get

$$\sigma_z = \frac{2\times 600\times 4^3}{\pi(5^2+4^2)}$$

$$\sigma_z = 14.54 KN/m^2$$

Problem-11

A circular foundation rests on the horizontal upper surface of a semi-infinite soil mass, whose properties comply with the usual elasticity requirement and carries a load of 1000 KN. The contact pressure is uniform and the foundation is flexible. The base of foundation is frictionless. The diameter of the foundation is 3m. Determine the vertical stress distribution on horizontal planes along the central axis of the foundation to a depth of 10m below the surface.

Sol: Total load = 1000KN

Area of circular foundation = $\dfrac{\pi D^2}{4} = \dfrac{\pi \times 3^2}{4} = 7.069 m^2$

∴ uniform load intensity, $q = \dfrac{1000}{7.069} = 141.47 KN/m^2$

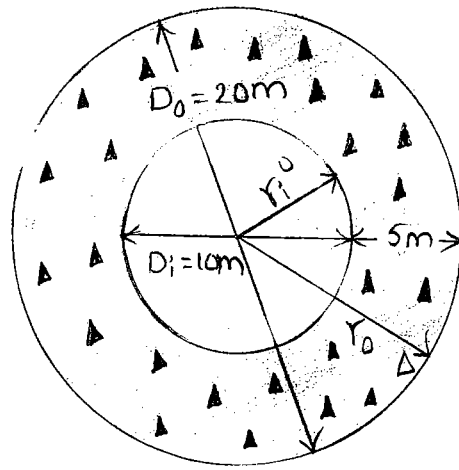The vertical stress caused by uniform load on circular area is given by

$$\sigma_z = \left[ 1 - \left( \dfrac{1}{1+\dfrac{r^2}{z^2}} \right)^{3/2} \right] q$$

Here, $q = 141.47 KN/m^2$, $r = 1.5m$, $z = 10m$

$$\sigma_z = \left[ 1 - \left( \dfrac{1}{1+\left(\dfrac{1.5}{10}\right)^2} \right)^{3/2} \right] \times 141.47$$

$$\sigma_z = 4.64 KN/m^2$$

Problem 2:

A loading of 50 KN/m² is acting on an annular foundation of width 5m and inside diameter of 10m. Find the vertical stress intensity at a depth of 10m below the centre of the foundation.
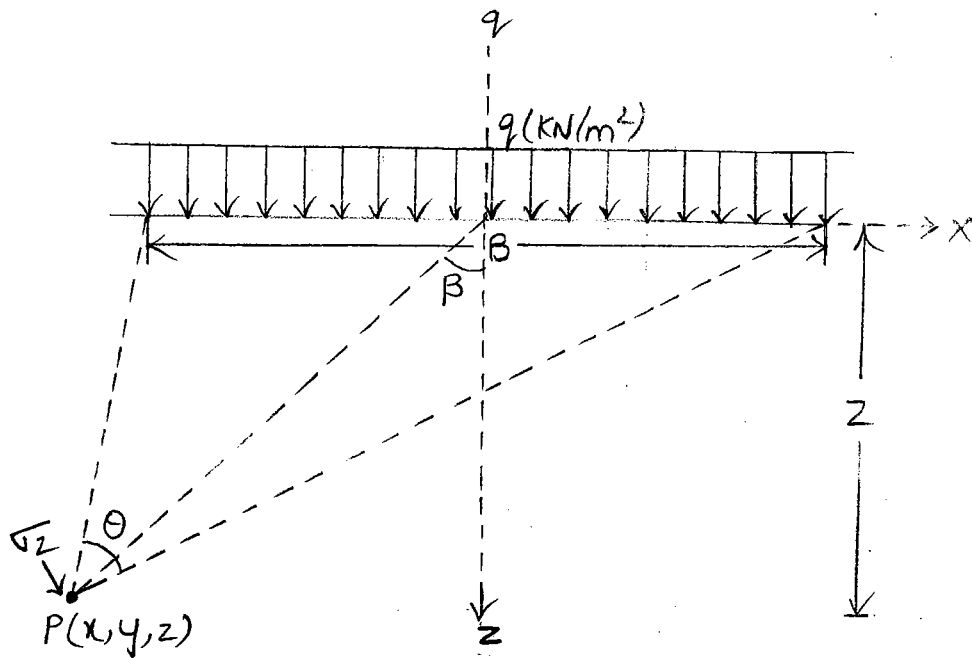


Sol: Here, $D_i = 10m$ ; $r_i = 5m$

$D_0 = 20m$ ; $r_0 = 10m$

$q = 50 KN/m^2$ ; $z = 10m$

In this case entire area is not loaded to find vertical stress, assume whole area of radius $r_0$ is loaded by 50KN/m², whereas the circular area of radius 5m is also loaded with same intensity but opposite in nature. Hence,

$$\sigma_z = q\left[1 - \left(\frac{1}{1+\left(\frac{r_0}{z}\right)^2}\right)^{3/2}\right] - q\left[1 - \left(\frac{1}{1+\left(\frac{r_i}{z}\right)^2}\right)^{3/2}\right]$$

$$= 50\left[1 - \left(\frac{1}{1+\left(\frac{10}{10}\right)^2}\right)^{3/2}\right] - 50\left[1 - \left(\frac{1}{1+\left(\frac{5}{10}\right)^2}\right)^{3/2}\right]$$

$$= 50\left[1 - \left(\frac{1}{2}\right)^{3/2}\right] - 50\left[1 - \left(\frac{1}{1.25}\right)^{3/2}\right]$$

$$= 50\left[1 - 0.353\right] - 50\left[1 - 0.716\right]$$

$$= 32.32 - 14.22$$

$$= 18.10 \ KN/m^2$$

## 3) Vertical stress due to strip load:



→ The vertical stress at a point 'p' due to a uniform load of intensity 'q' on a strip of width 'B' and semi-infinite length is given by
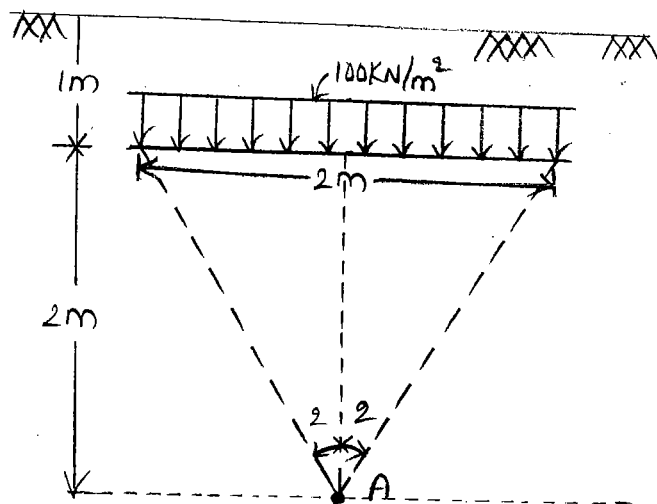
$$\sigma_Z = \frac{q}{\pi}\left[\theta + \sin\theta \cos2\beta\right]$$

→ If the point 'p' is directly below the centre of the strip i.e, $\beta = 0$

then $\sigma_Z = \frac{q}{\pi}\left[\theta + \sin\theta\right]$

## Problem -1:

A 2m wide strip footing is to be placed as shown in figure below. Determine the vertical stress at a point 'A' shown in figure.

sol: The vertical stress at a point, which is directly below the centre of the strip is given by

$$\sigma_z = \frac{q}{\pi}(\theta + \sin\theta)$$
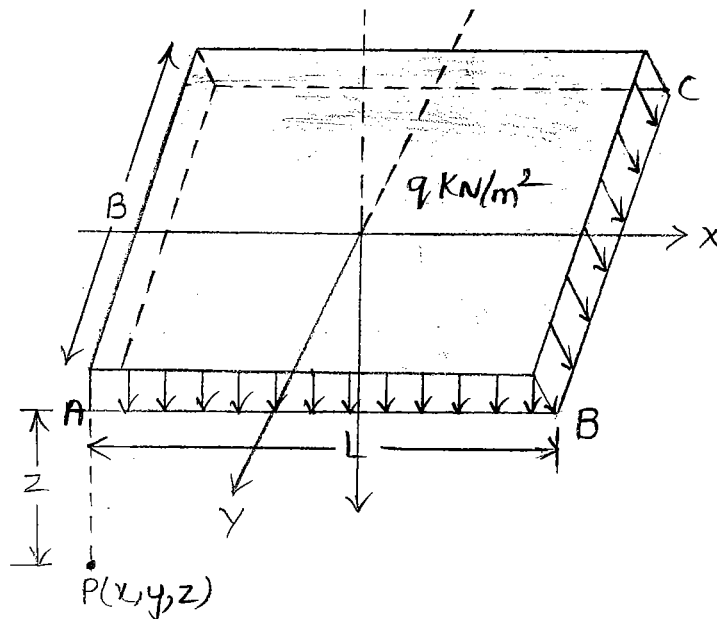
$$\tan\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = \tan^{-1}\left(\frac{1}{2}\right) = 26.56°$$

$$\theta = 53.13° \text{ (or) } 0.927 \text{ radian}$$

$$\sigma_z = \frac{100}{\pi}(0.927 + \sin 53.13)$$

$$\sigma_z = 54.98 \text{ KN/m}^2$$

**1) Vertical pressure below the corner of uniformly loaded rectangular area:**



→ Vertical stress at a point below the corner of a rectangular area loaded uniformly is given by Newmark's equation.

$$\sigma_z = K \cdot q.$$

$$K = \frac{1}{4\pi}\left[\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+m^2n^2+1} \cdot \frac{(m^2+n^2+2)}{(m^2+n^2+1)} + \tan^{-1}\left(\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1}\right)\right]$$
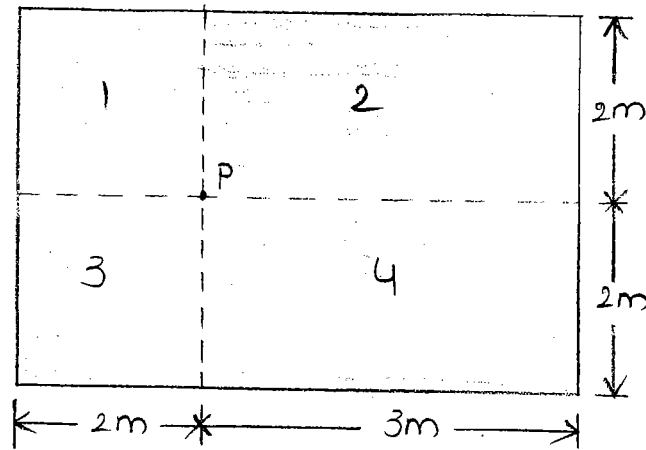
Where $m = \frac{L}{z}$ and $n = \frac{B}{z}$

→ As per Westergaard theory - assuming Poisson's ratio zero

$$\sigma_z = K \cdot q$$

Where, $K = \frac{1}{2\pi}\left[\cot^{-1}\left(\frac{1}{2m^2} + \frac{1}{2n^2} + \frac{1}{4m^2n^2}\right)^{1/2}\right]$

calculate the intensity of stress at a point 'p' located as shown in fig at a depth of 2.5m due to a load of $2KN/m^2$ over the footing area of 4m×5m.



**Sol:** for area $A_1$ and $A_3$:

$$m = \frac{L}{Z} = \frac{2}{2.5} = 0.80, \quad n = \frac{B}{Z} = \frac{2}{2.5} = 0.8$$

for area $A_2$ and $A_4$:

$$m = \frac{L}{Z} = \frac{3}{2.5} = 1.20, \quad n = \frac{B}{Z} = \frac{2}{2.5} = 0.8$$

using boussinesq equation

$$K = \frac{1}{4\pi}\left[\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+m^2n^2+1} \cdot \frac{(m^2+n^2+2)}{(m^2+n^2+1)} + \tan^{-1}\left(\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1}\right)\right]$$

$$K_1 = K_3 = \frac{1}{4\pi}\left[\frac{2\times0.8\times0.8(0.8^2+0.8^2+1)^{1/2}}{0.8^2+0.8^2+0.8^2\times0.8^2+1} \cdot \frac{(0.8^2+0.8^2+2)}{(0.8^2+0.8^2+1)} + \right.$$

$$\left. \tan^{-1}\left(\frac{2\times0.8\times0.8(0.8^2+0.8^2+1)^{1/2}}{0.8^2+0.8^2-0.8^2\times0.8^2+1}\right)\right]$$

$$= \frac{1}{4\pi}(1.0329+45.94°) \Rightarrow \frac{1}{4\pi}(1.0329+0.8021 \text{ rad.}) \Rightarrow 0.1461$$

$$K_2 = K_4 = \frac{1}{4\pi}\left[\frac{2\times1.20\times0.8(1.20^2+0.8^2+1)^{1/2}}{1.20^2+0.8^2+1.20^2\times0.8^2+1} \cdot \frac{(1.20^2+0.8^2+2)}{(1.20^2+0.8^2+1)} + \right.$$

$$\left. \tan^{-1}\left(\frac{2\times1.20\times0.8(1.20^2+0.8^2+1)^{1/2}}{1.20^2+0.8^2-1.20^2\times0.8^2+1}\right)\right]$$

$$= \frac{1}{4\pi}\left[1.1154+57.35°\right] \Rightarrow \frac{1}{4\pi}\left[1.1154+1.00 \text{ rad}\right] \Rightarrow 0.1684$$

$$= (0.1461 + 0.1684 + 0.1461 + 0.1684) \times 250$$

$$= 0.629 \times 250$$

$$\sigma_z = 157.24 \ KN/m^2$$

using westergaard's equation

$$K = \frac{1}{2\pi} \left[ \cot^{-1} \left( \frac{1}{2m^2} + \frac{1}{2n^2} + \frac{1}{4m^2n^2} \right)^{1/2} \right]$$

$$K_1 = K_3 = \frac{1}{2\pi} \left[ \cot^{-1} \left( \frac{1}{2 \times 0.8^2} + \frac{1}{2 \times 0.8^2} + \frac{1}{4 \times 0.8^2 \times 0.8^2} \right)^{1/2} \right]$$

$$= 0.0948$$

$$K_2 = K_4 = \frac{1}{2\pi} \left[ \cot^{-1} \left( \frac{1}{2 \times 1.20^2} + \frac{1}{2 \times 0.8^2} + \frac{1}{4 \times 1.20^2 \times 0.8^2} \right)^{1/2} \right]$$
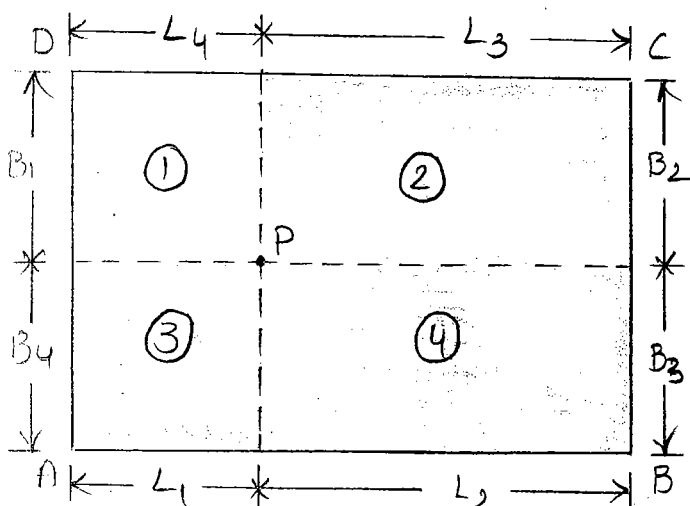
$$= 0.1168$$

$$\sigma_z = (K_1 + K_2 + K_3 + K_4) \cdot q$$

$$= (2 \times 0.0948 + 2 \times 0.1168) \times 250$$

$$\sigma_z = 105.8 \ KN/m^2$$

special case:
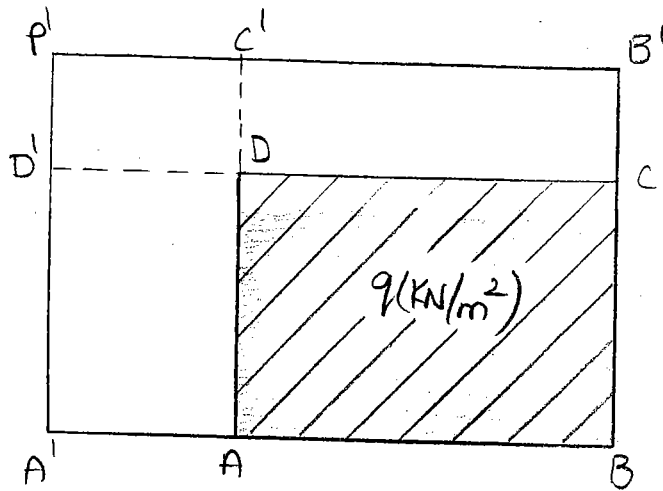
1) when point 'P' is located inside the loaded area:



$q (KN/m^2)$

→ If point 'P' at which $\sigma_z$ is required is located within the rectangular loaded area but not below any corner. Then the whole rectangular area is divided into four rectangles so that the point under consideration i.e, 'P' become the common corner for each rectangle.

→ The sum of the four stresses thus found will be the vertical stress $\sigma_z$ at a depth 'z' below the point 'p'.

$$\therefore \sigma_z = (K_1 + K_2 + K_3 + K_4)\, q$$

2) When point 'p' is located outside the loaded area:



→ Let ABCD is loaded area for which the stress is required point 'p' below the depth 'z' as shown in figure.

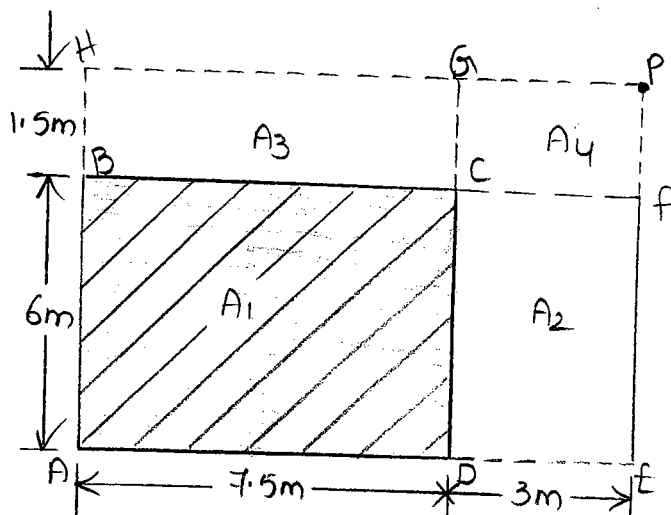→ Construct two imaginary rectangle PC'AA' and PB'CD' having common corner 'p' as shown in fig.

→ The required value is obtained by adding (or) subtracting their stress contribution

Here, $\sigma_z = \left[ K_{PA'BB'} - K_{PA'AC'} - K_{PD'CB'} + K_{PD'DC'} \right] q$

Problem—1:

A rectangular loaded area 6m×7.5m carries a load of 200KN/m². Determine the vertical stress at point p located outside at a depth of 7.5m.

Sol:

$$\sigma_Z = [K_{AEPH} - K_{BFPH} - K_{DEPG} - K_{CFPG}] \cdot P$$

for rectangular area AEPH,

$$K = \frac{1}{4\pi}\left[\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+m^2n^2+1} \cdot \frac{(m^2+n^2+2)}{(m^2+n^2+1)} + \tan^{-1}\left(\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2-m^2n^2+1}\right)\right] — ①$$

$$m = \frac{L}{Z} = \frac{10.5}{7.5} = 1.40 , \quad n = \frac{B}{Z} = \frac{7.5}{7.5} = 1$$

on substituting values of m, n we get

$$K_{AEPH} = 0.1914$$

for area BFPH,

$$m = \frac{L}{Z} = \frac{10.5}{7.5} = 1.40 , \quad n = \frac{B}{Z} = \frac{1.5}{7.5} = 0.20$$

$$\therefore K_{BFPH} = 0.0589$$

for area DEPG,

$$m = \frac{L}{Z} = \frac{7.5}{7.5} = 1 , \quad n = \frac{B}{Z} = \frac{3}{7.5} = 0.40$$

$$\therefore K_{DEPG} = 0.1013$$

for area CFPG

$$m = \frac{L}{Z} = \frac{1.5}{7.5} = 0.2 , \quad n = \frac{3}{7.5} = 0.40$$

$$\therefore K_{CFPG} = 0.0328$$

Therefore, $\sigma_Z = [0.1914 - 0.059 - 0.1013 + 0.0328] \times 200$

$$= 12.8 \ KN/m^2$$

## Approximate methods for vertical stress computation:

### 1) Newmark's influence chart method:

→ In this method, soil is assumed to be homogeneous, semi-infinite, elastic and isotropic.

→ These charts are based on Boussinesq's equation so these charts are not applicable for stratified soils.

→ The greatest advantage of this method is that it can be

used for any shape of the loaded area

→ The influence chart consists of a no. of concentric circles and radial lines which divides it into different area units where influence of each area unit at the centre of the chart is same and is referred as __INFLUENCE FACTOR__

→ Generally, 10 concentric circles and 20 radial lines are considered which divides it into 200 area units.

$$i.e., \sigma_Z = K_B \frac{Q}{Z^2}$$

$$I = \frac{1}{m \times n}$$

Where m = No. of concentric circles,

n = No. of radial lines

Here, $I = \frac{1}{10 \times 20}$

$$= \frac{1}{200}$$

→ In order to use the influence chart for the determination of vertical stresses at any point below a uniformly loaded area of any shape, the following steps are followed.

) Draw the plan of the loaded area with a scale such that the depth Z at which stress is being computed equals the length AB shown on the chart.
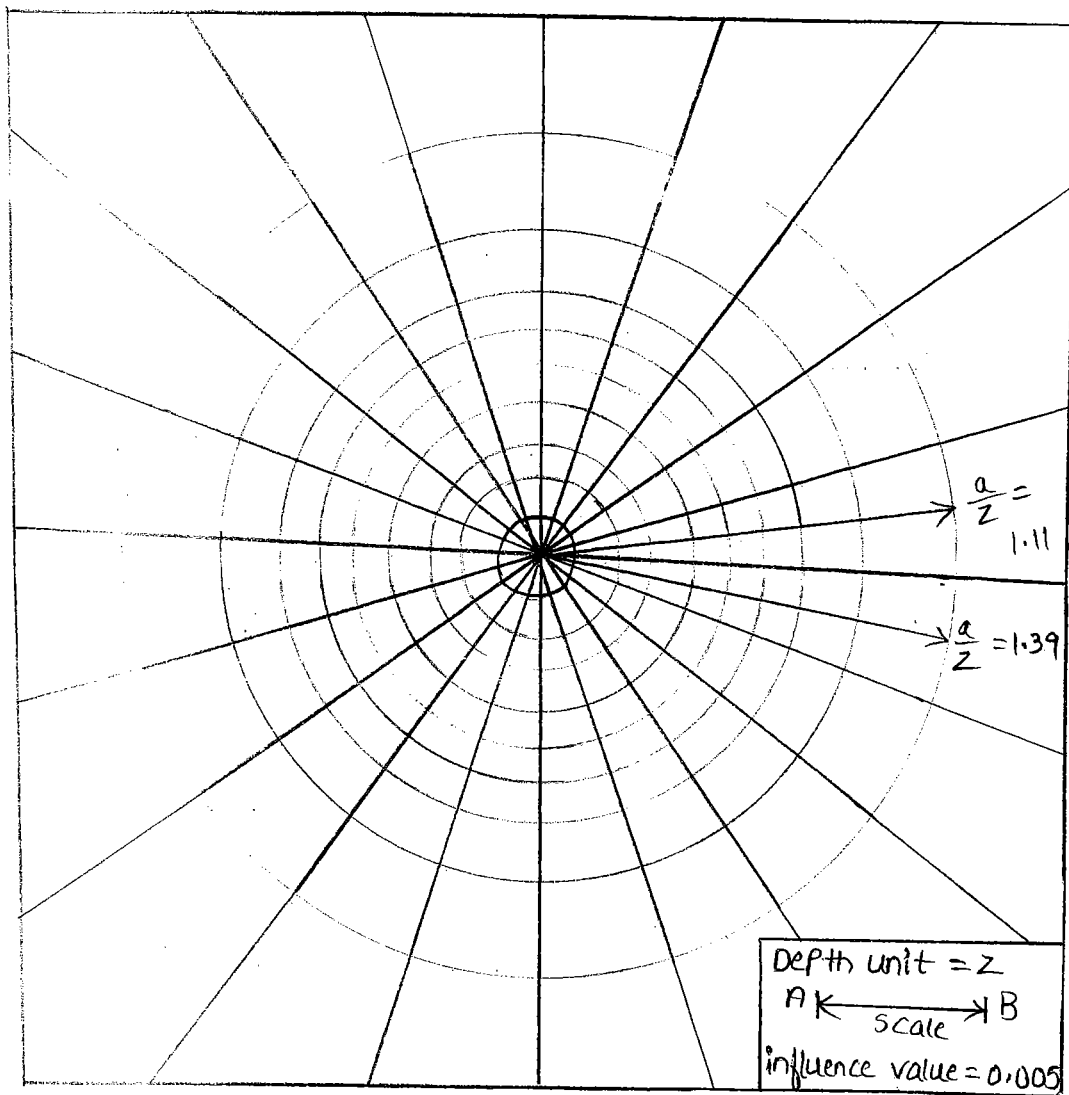


fig. Neimark influence chart for vertical stress at any depth z=AB

This plan of the loaded area is then kept on the influence chart such that point at which stress are required, coincides with the centre of the chart.
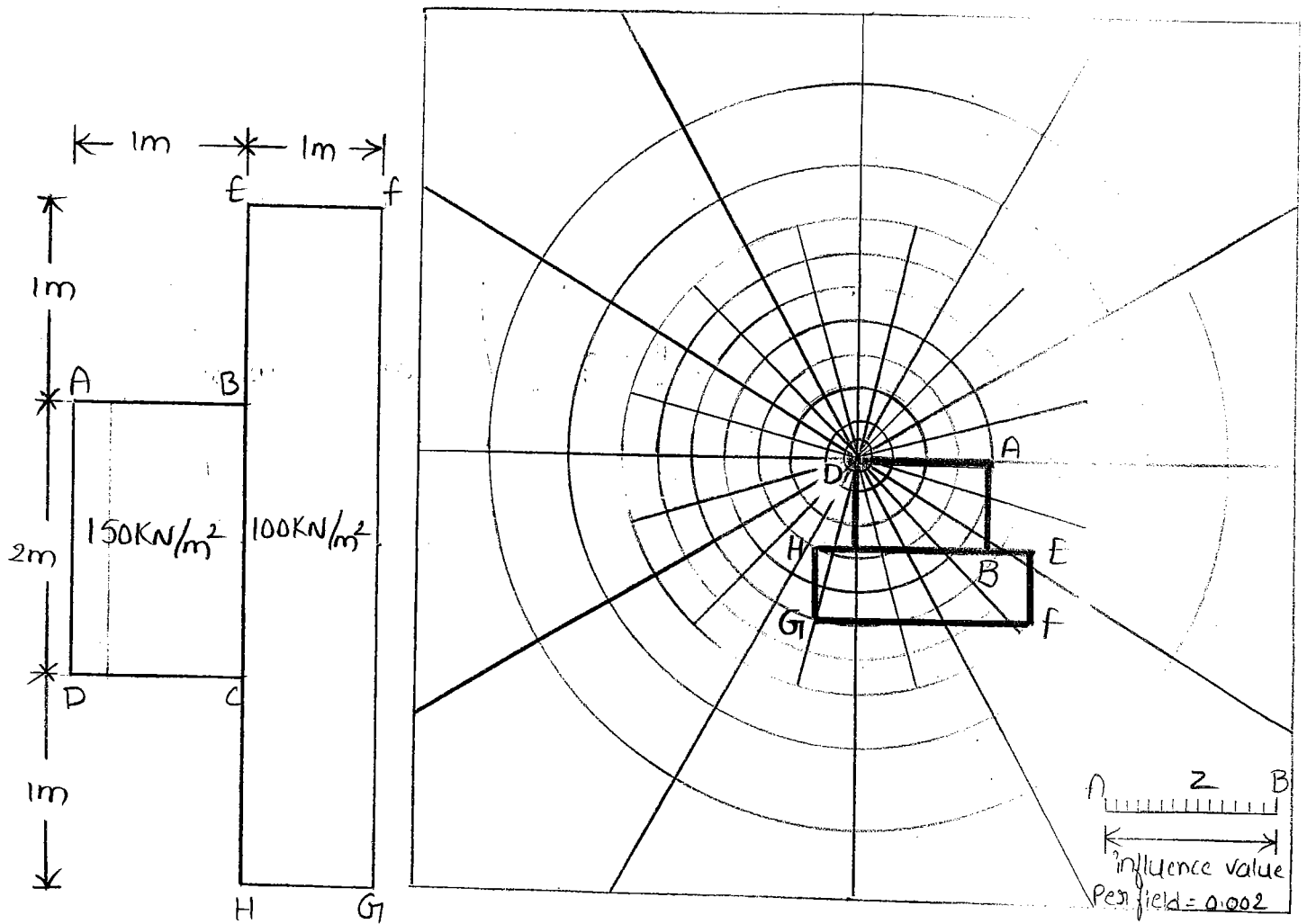
Count the number of area units covered by the plan area on the influence chart including the fractional ones

if $I_f$ is influence factor, $N_A$ = Total No. of area units occupied
the vertical stress is given by $\sigma_Z = I_f \times N_A \times q$

$$= \frac{1}{m \times n} \times N_A \times q$$

Problem-11

A foundation is constructed to take a stress of $150 \, KN/m^2$ and is flush with another existing foundation shown in figure taking a load of $100 \, KN/m^2$. find the vertical stress at a depth of 2m below the point D. use the Newmark's chart given in below figure with an influence value of 0.002



Considering the depth scale AB=2m, draw the loaded area of this scale with D at the centre of the chart. This is drawn and shown in above figure.

The number of stress blocks occupied by areas ABCD and EfGH are seperately counted and given as
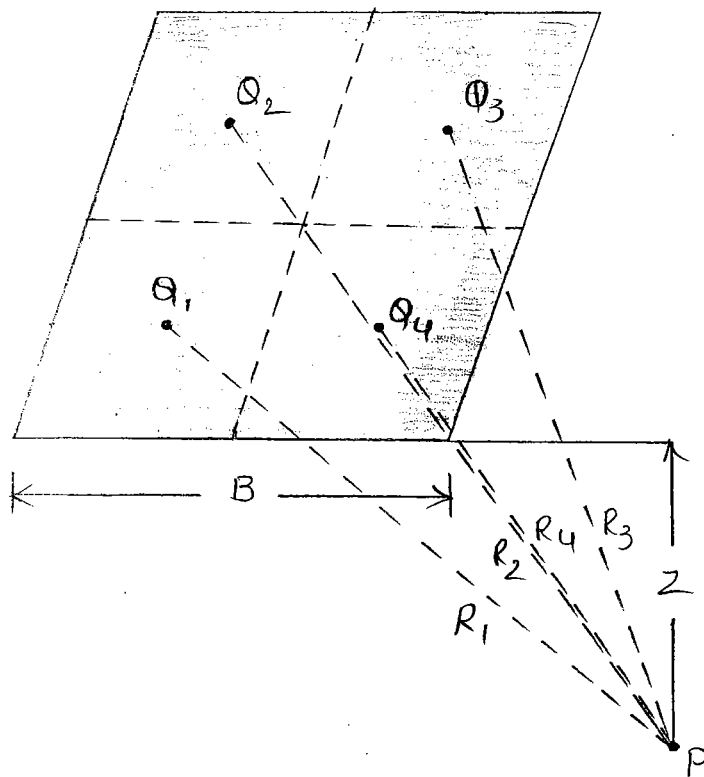
No. of blocks in $ABCD = N_1 = 72$

No. of blocks in $EfGH = N_2 = 50$

stress at that point $\sigma = q_1 \triangle N \times n_1 + q_2 \triangle N \times n_2$

$$= \triangle N (q_1 n_1 + q_2 n_2)$$

$$= 0.002 (150 \times 72 + 100 \times 50)$$

$$= 31.6 \, KN/m^2$$

## ii) equivalent load method:

→ The given loaded area is divided into number of smaller units and total load from each unit is assumed to act the centroid as a point load.

→ Principal of superposition is then applied and the required stress at a specified point is obtained by summing up the contributions of the individual points loads from each of the units.



→ If the influence values of $K_{B_1}, K_{B_2}, K_{B_3}, K_{B_4}$ for the point loads $Q_1, Q_2, Q_3$ and $Q_4$. then
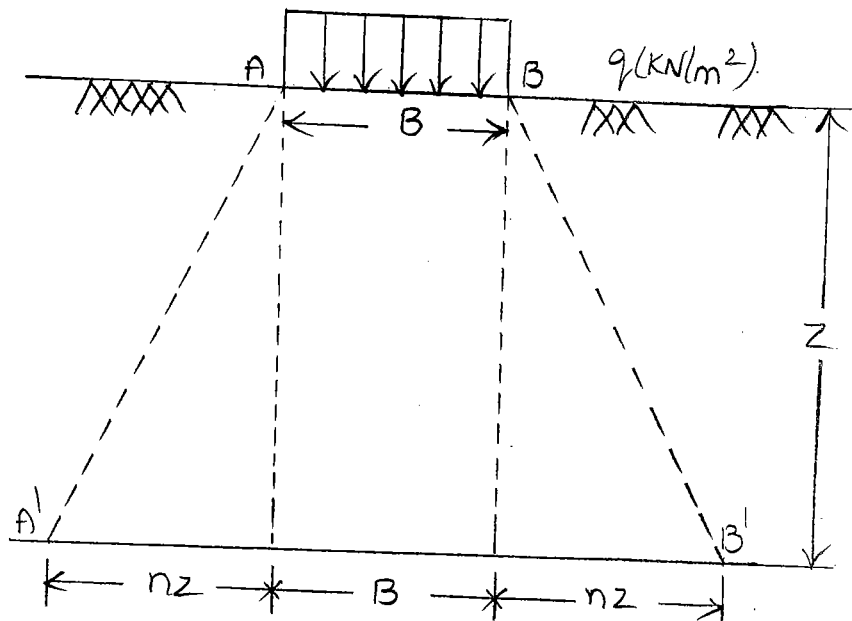
$$\sigma_Z = K_{B_1} Q_1 + K_{B_2} Q_2 + K_{B_3} Q_3 + K_{B_4} Q_4$$

NOTE: The accuracy of this method depends upon the size of area units considered. If the length of the side of the area unit is less than the 1/3rd the depth at which the stress is required, the error involved in this analysis is less than 3% only.

3) Trapezoidal method/Load strip method:

→ It involves assumption that the stresses get distributed uniformly on to areas the edges of which are obtained by taking the angle of distribution at 1 vertical to n horizontal.

→ Generally, 2V:1H spread is considered below the loaded area.



1) for rectangular area:

$$\sigma_z = \frac{q(BL)}{(B+2nz)(L+2nz)}$$

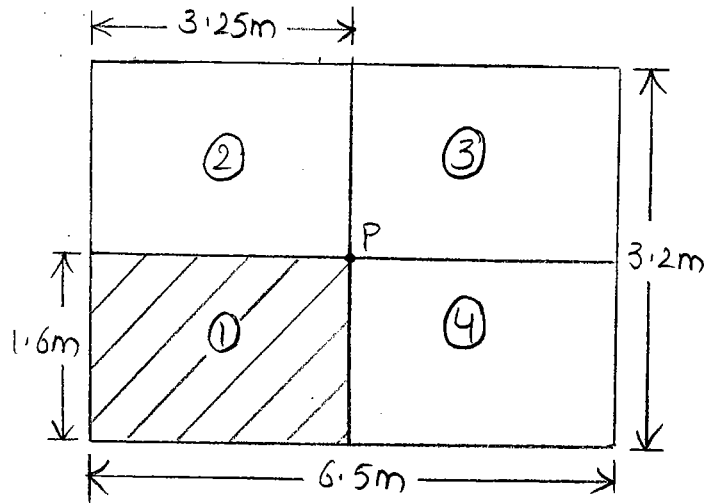2) for square area:

$$\sigma_z = \frac{q \cdot B^2}{(B+2nz)^2}$$

3) for strip load:

$$\sigma_z = \frac{q \cdot (BL)}{(B+2nz)(L+2nz)}$$

Since L>>B, L+2nz ≃ L.

$$\frac{qB}{B+2nz}$$

culate the intensity of stress below the centre of footing at a depth
13m due to load of 1600KN/m² over the footing area of 3.2m×6.5m
by using equivalent load method.



$\frac{3}{2\pi}$:  The radial distance of 'P' from the C.G of divided
rectangular areas.

$$r_1 = r_2 = r_3 = r_4 = \sqrt{(1.625)^2 + (0.8)^2} = 1.81$$

$$\text{and} \quad z = 13m$$

Equivalent load on the C.G of area 1,

$$Q_1 = \text{Area} \times q$$

$$= (3.25 \times 1.6) \times 1600$$

$$= 8320 KN$$

using boussinesq's equation

$$\sigma_{z_1} = K_1 \cdot \frac{Q_1}{z^2}$$

$$\text{where} \quad K_1 = \frac{Q}{2\pi} \left[ \frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

$$= \frac{3}{2\pi} \left[ \frac{1}{1 + \left(\frac{1.81}{13}\right)^2} \right]^{5/2}$$

$$= 0.455$$

$$\therefore \sigma_{z_1} = 0.455 \times \frac{8320}{(13)^2}$$

$$= 22.4 \, KN/m^2$$

Total stress due to all four smaller rectangles

$$\sigma_{\bar{z}} = 4 \times \sigma_{z_1}$$

$$= 4 \times 22.4$$

$$= 89.6 \, KN/m^2$$

using westergaard's equation

$$\sigma_{z_1} = K_i \, \frac{q}{z^2}$$

where $K_1 = \frac{1}{\pi} \left[ \frac{1}{1 + 2\left(\frac{r}{z}\right)^2} \right]^{3/2}$

$$= \frac{1}{\pi} \left[ \frac{1}{1 + 2\left(\frac{1.81}{13}\right)^2} \right]^{3/2}$$

$$= 0.300$$

$$\sigma_{z_1} = 0.300 \times \frac{8320}{(13)^2} = 14.77 \, KN/m^2$$

$$\sigma_{\bar{z}} = 4 \, \sigma_{z_1}$$

$$= 4 \times 14.77$$

$$= 59.077 \, KN/m^2$$