Compressibility and Consolidation Of Soil

* Compressibility

It is the process of decrease in volume of Soilmass due to increase in loading

The volume reduction in the soil may be due to:

(i) Compression and expulsion of pore air

(ii) Expulsion of pore water

(iii) Compression of water and solid molecules

(iv) Change in the Orientation of molecules.

Since water and soil molecules are considered non compressible. Hence on these account volume change is neglected.

Compressibility depends upon magnitude of the effective stress acting on the soil at that time and soiltype and its structure.

* Consolidation:

The decrease in soil volume by the Squeezing out of the porewater on account of gradual dissipation of excess hydrostatic pressure induced by an imposed total stress, is defined as "Consolidation"

Total Consolidation of a soil is divided into :

(a) Primary Consolidation :

- This process begins when soil is fully saturated. Due to increase in effective stress over the saturated soil mass, pore pressure increase. As a result, explusion of porewater occur if drainage facility is provided. Primary Consolidation is completed when expulsion of pore water stops.

- During the process of consolidation, soil remains saturated ($S=1$) and flow of water is under laminar condition, i.e $Re < 1$ [Darcy's Law applicable]

- The volume of expelled water is equal to decrease in volume of soil which can be measured. Since soil mass is considered semi-infinite, therefore, change in volume is equal to the change in depth.

[Semi-infinite means infinite area and finite depth in which area remain constant]

- Due to primary consolidation, settlement occur which is time dependent. The time required will depend upon:

1. Rate of application of load
2. Coefficent of permeability of soil
3. Drainage facility available (one-way (or) Two-way)
4. Length of drainage path which depends upon thickness of soil and type of drainage (one-way or Two way)

- If on a saturated soil mass, increase in effective stress is $\Delta\bar{\sigma}_1$, then initially it will be taken by pore water. Hence, increase in pore pressure [EXCESS PORE PRESSURE] $u_i = \Delta\bar{\sigma}_1$
- If expulsion of pore water takes place, then excess pore pressure reduces and when expulsion of water stops, then excess pore pressure becomes zero. At this stage, primary consolidation is complete for $\Delta\bar{\sigma}_1$.
- If effective stress is further increased by $\Delta\bar{\sigma}_2$, then consolidation will again occur and so on.

(b) Secondary Consolidation :

- After completion of primary consolidation When expulsion of porewater is stopped and load continues to act, then at very slow rate further volume change may be recorded which is due to plastic readjustment of solids. This is called Secondary Consolidation and it is time-dependent which is much slower than primary consolidation.
- Secondary Consolidation is more in plastic soils and in highly plastic clays, it may be 10-20% of total volume change. In coarse grained soils (Gravels and Sands), it is negligible.

Normal and Over Consolidated Soils.

Normal Consolidated Soils

Normally consolidated soils are those which are loaded for the first time to the present applied effective stress. It means past applied effective stress was lower than the present applied effective stress. Such soils are more compressible.

Over Consolidated Soils.

Over consolidated soils are those which have been subjected to effective stress in the past greaterthan present applied effective stress. The over consolidation / normal consolidation can be differentiated using over consolidation ratio.

Over consolidated soils are also called Pre-consolidated or pre-compressed soils. Such soils are less compressible and have greater shear strength and more stability.

Causes of over-consolidation or pre-consolidation.

1. In the past, over burben pressure or surcharge was placed, which is removed later.
2. Continous erosion of overburden soil
3. Melting of glacier which covered the soilmass in past.

4. Effect of capillary pressure which is later destroyed by rise of water table.

5. During the drying of soil, effective stress reduces and soil becomes over consolidated. This process is known as "Dessication of soil"

6. If initially soil was subjected to downward seepage pressure but later seepage stops then effective stress reduces.

7. If there was no seepage but later vertically upward seepage occurs, the effective stress also reduces.

8. Due to effects of tectonic forces.

Over Consolidation Ratios (O.C.R).

It is defined as the ratio of maximum applied effective stress in the past to the present applied effective stress.

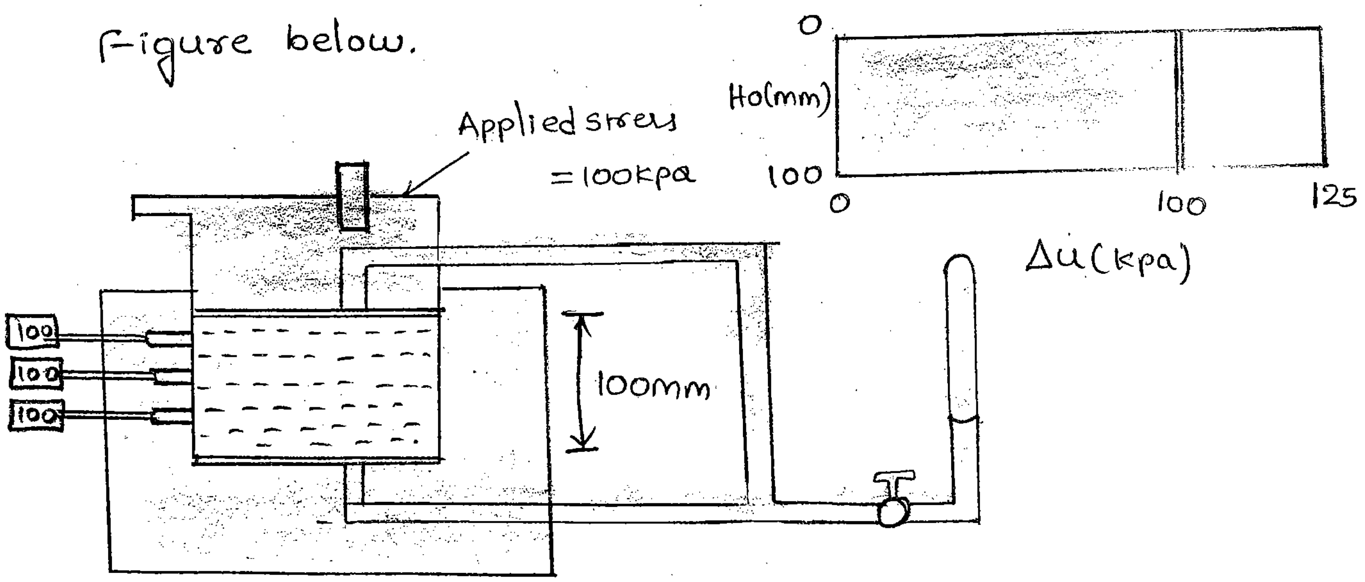
$$O.C.R = \frac{\text{Max. applied stress in past } (\bar{\sigma}_0)}{\text{Present applied effective stress } (\bar{\sigma})}$$

- For over consolidated soils, $OCR > 1$
- For normally consolidated soils $OCR \leq 1$.

- If $\bar{\sigma} < \bar{\sigma}_0$, then soil is called Over Consolidated Soil.
- If $\bar{\sigma} > \bar{\sigma}_0$, the soil is called Normally consolidated soil.

* Compression Test : Oedometer Test

The compressibility and consolidation characteristics of soil are determined in the labouratoly by the Oedometer or consolidometer test shown in figure below.



Instantaneous or initial excess porewater pressure when a vertical load is applied

- This test involves the measurement of one dimensional compression of saturated soil under several increments of vertical stress.
- Under each increment of loading, soil is allowed to consolidate till there is no or little further compression.

- Each increment is maintained for atleast 24 hours. The comparison of the specimen is noted at several intervals of time for each increment

- During consolidation under the maximum load, the excess porewater has dissipated completely. Now specimen is unloaded in two or three stages and the soil is allowed to swell during the unloading stage final swell readings are taken.

- After complete unloading, the wet and dry weight of sample are determined.

- The results of ~~od~~ Oedometer test are plotted on graph between the void ratio at the end of each increment period V/s corresponding effective stress.

* Computation of Void Ratio

The void ratio at the end of each increment period is known as increment void ratio. The increment void ratio may be found out the following two methods

(a) Height of solid method

(b) Change in void ratio method.

(a) Height Of Solid Method.

- During compression of a soil specimen, the volume of soil solids remain constant, when the volume of voids decreases.

- For a constant height of solids over the area of cross-section A of sample, knowing the dry weight of sample and specific gravity, the height of the soil solids hs, can be computed as

$$H_{s'} = \frac{W_s}{G\gamma_w \cdot A}$$

- Now the equilibrium void ratio can be calculated as

$$e = \frac{H - H_s}{H_s}$$

Where W_s = dry weight of soil

G_s = Specific gravity of soil

A = Cross sectional area of soil specimen.

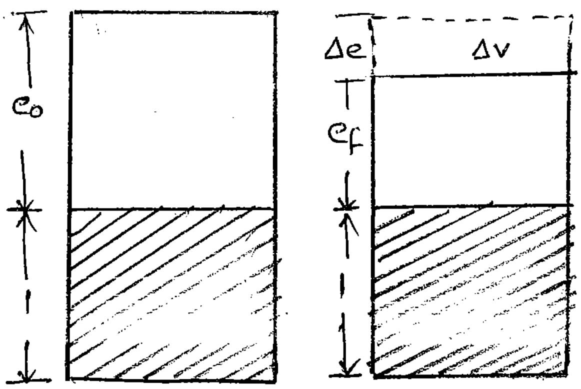
H = height of specimen at the end of particular stress increment ($H = H_1 \pm \Delta H$)

H_1 = Height of specimen at beginning of stress increment

ΔH = change in thickness under stress increment

H_s = Height of solid.

(b) Change in Void Ratio Method :



- In this method, it is assumed that the specimen is fully saturated at the end of the test. Now void ratio of the saturated soil at the end of the test is calculated from the equation

Hence,
$$S \cdot e = W \cdot G \qquad [\because S = 1]$$
$$e_f = W_f G_s$$

Where W_f = final water content at the end of test

- If the change in volume is a consequence of a decrease in void ratio of Δe, then

$$\frac{\Delta H}{H} = \frac{\text{change in volume}}{\text{Original volume}} = \frac{\Delta e}{1+e_0}$$

Where e_0 is initial void ratio of sample before any compression.

- Now, knowing ΔH, H and e_0 we can calculate change in void ratio Δe.

- Knowing the Δe, incremental void ratio at the end of every effective stress can be calculated as

$$e = e_0 - \Delta e.$$

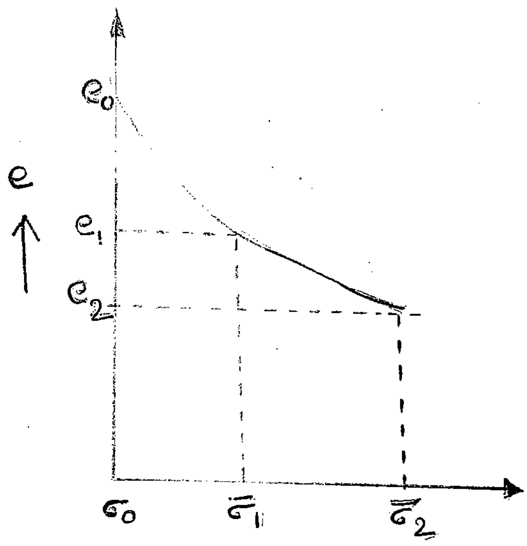
- The change in void ratio can be also found out directly by

$$\Delta e = \frac{\Delta H}{H_f}(1+e_f)$$

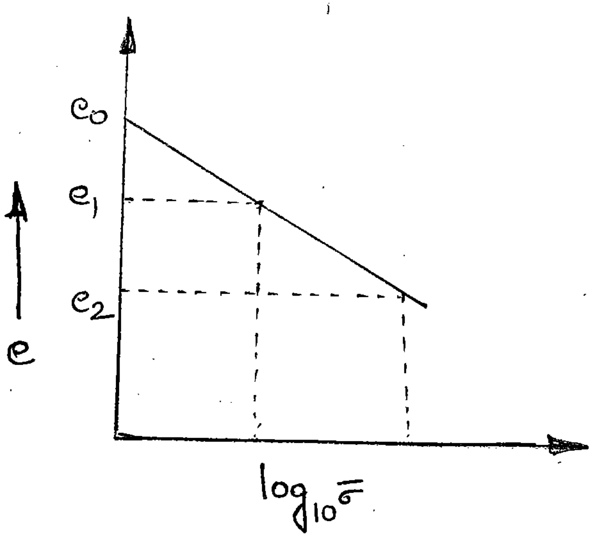
- Change in void ratio Δe under each stress increment is calculated from above formula working from backward from the known value of e_f, the equiliburium void ratio at the end of each stress increment can be deducted.

Graph between Void Ratio and Effective Stress.

1. Normally consolidated clay

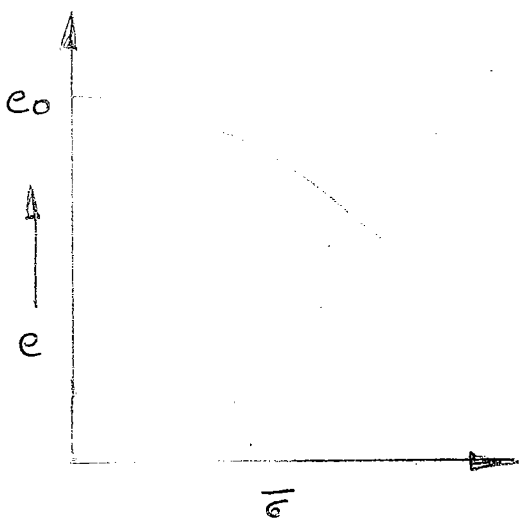


(a) On Arithmetic scale

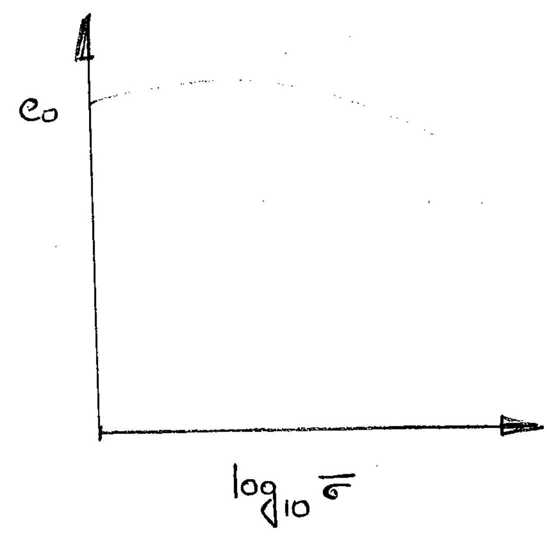


(b) On semi-log scale.

2. Over Consolidated clays

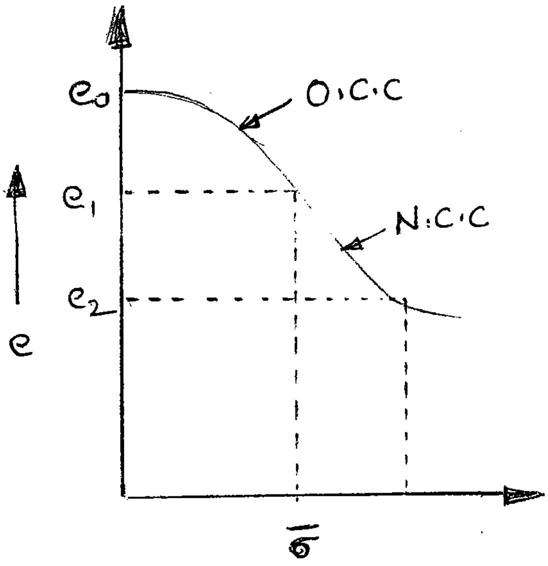


(a) On Arithmetic scale

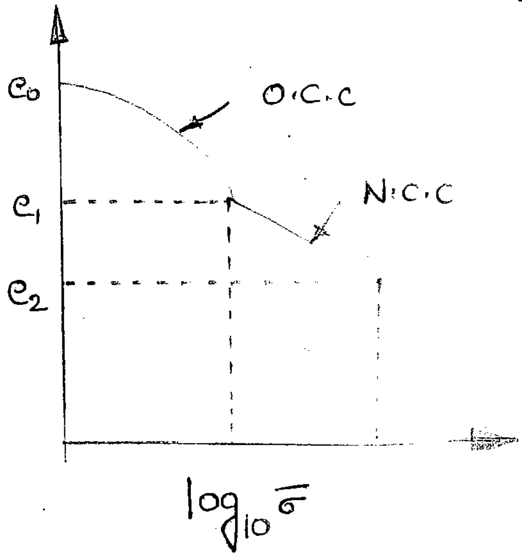


(b) Semi-log scale.

3. Initially Over Consolidated clay becomes Normally Consolidated clay

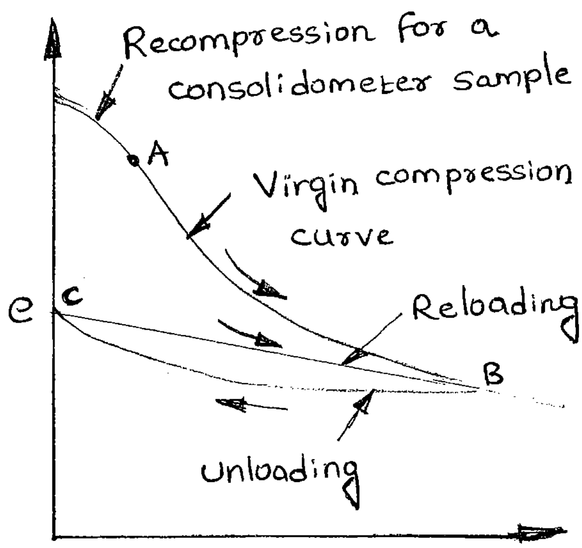


(a) On Arithmetic scale

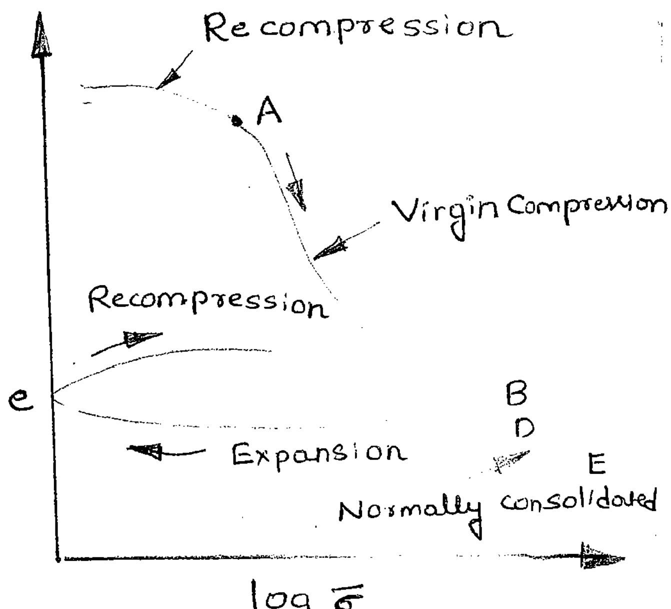


(b) Semi-log scale

4. When clay undergoes Recompression and Compression.



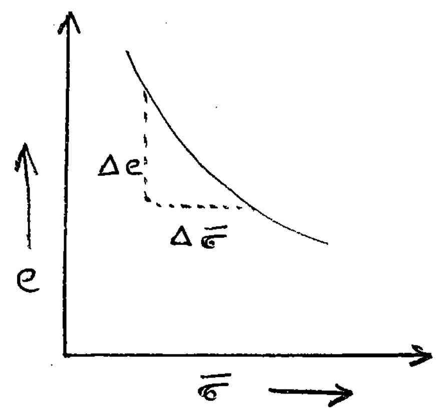
(a)



(b)

Determination of Compressibility Parameters.

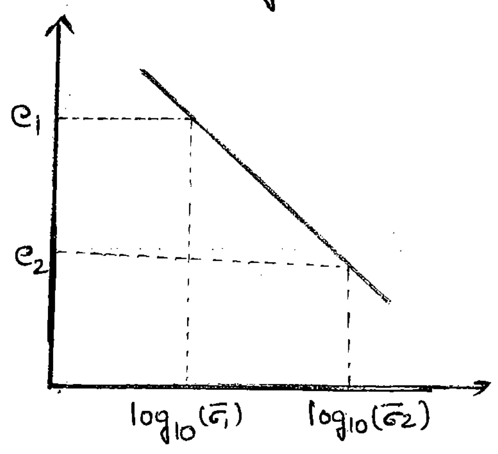
1. Coefficient of compressibility (a_v)



- The slope of effective stress V/s void ratio curve on Arithmetic scale is called "Coefficient of compressibility."
- $a_v = -\left(\frac{\Delta e}{\Delta \bar{\sigma}}\right) m^2/kN$
- The coefficient of compressibility decreases with an increase in the effective stress. Hence soil become stiffer on effective stress increment and curve becomes flatter.

2. Compression Index (C_c):

Voidratio - log σ̄ curve for Normally consolidated clay



- If void ratio is plotted on Y-axis on arithmetic scale and effective stress on log scale on X-axis, then the curve for normally consolidated clays is found to be straightline.
- The slope of this curve is constant, which is called coefficient of compression or compression index.

Coefficient of compression,

$$C_c = \frac{-\Delta e}{\log_{10}(\bar{\sigma}_2) - \log_{10}(\bar{\sigma}_1)} = -\frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma}_2}{\bar{\sigma}_1}\right)}$$

- C_c has significance only for Normally consolidated clay
- C_c does not depends upon effective stress. It has a constant value for a particular type of soil

Determination of C_c:

1. For undisturbed clay of medium sensitivity ($S_t \leq 4$)

$$C_c = 0.009(W_L - 10)$$

Where W_L is liquid limit in %.

2. For remoulded clays of medium to low sensitivity

$$C_c = 0.007(W_L - 10)$$

3. The compression index is also related to in-situ Void ratio as

$$C_c = 0.54(e_0 - 0.35)$$

Where e_0 is initial void ratio at the centre of compressible layer before any compression.

4. In natural water content is W_N, then C_c may be given as

$$C_c = 0.0115\, W_n$$

Where W_n is in %.

Note:

The significance of a_v and C_c is same but a_v changes with effective stress where C_c is constant. Therefore, for the purpose of computation of settlement or compressibility, C_c is widely used.

3. Coefficient of volume change (m_v):

- The coefficient of volume change is defined as the ratio of unit volume change per unit increase in effective stress.

$$m_v = \frac{-\left(\frac{\Delta V}{V}\right)}{\Delta \bar{\sigma}} = \frac{-\left(\frac{\Delta H}{H}\right)}{\Delta \bar{\sigma}} = \frac{-\left(\frac{\Delta e}{1+e_0}\right)}{\Delta \bar{\sigma}}$$

$$= \frac{-\left(\frac{\Delta e}{\Delta \bar{\sigma}}\right)}{1+e_0} = \frac{a_v}{1+e_0}$$

Where a_v = coefficent of compressibility

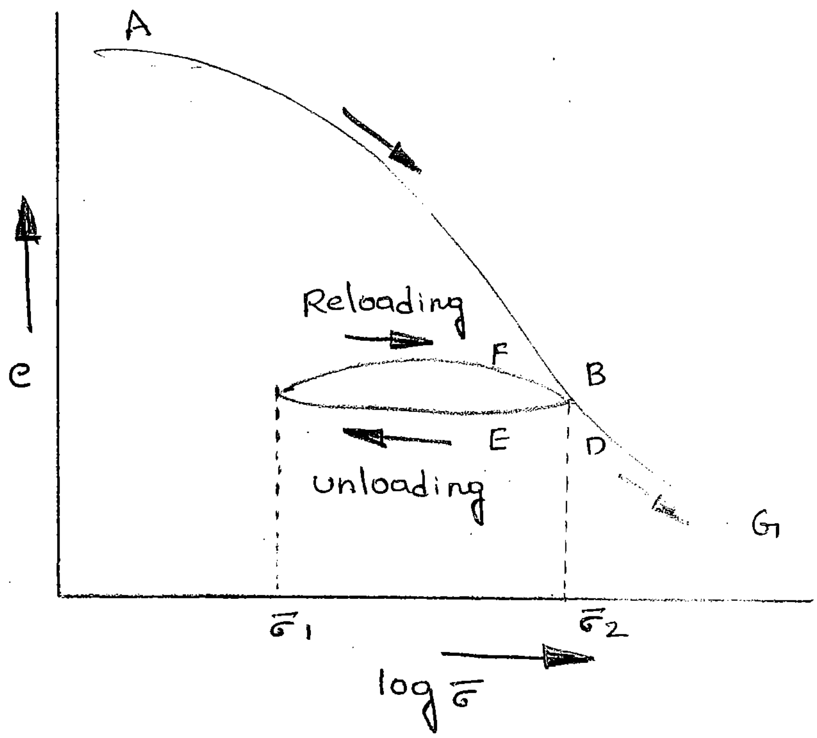
e_0 = intial void ratio of sample before any compression

- m_v and a_v are not constant parameters.

4. Expansion and Recompression Index

- The expansion index or swelling index is the slope of the e-log $\bar{\sigma}$ plot during unloading (see portion BEC)

$$C_e = \frac{\Delta e}{\log_{10}\left(\frac{\bar{\sigma}+\Delta\bar{\sigma}}{\bar{\sigma}}\right)}$$



Loading, unloading and reloading plot.

- Expansion index is much smaller than compression index

- The slope of the compression curve obtained during reloading (Portion CFD) when void ratio is plotted against effective stress on semi-log scale. Thus

$$C_r = -\frac{\Delta e}{\log\left(\frac{\bar{\sigma}+\Delta\bar{\sigma}}{\bar{\sigma}}\right)}$$

- The recompression index is $\frac{1}{5}$ to $\frac{1}{10}$ times the compression index.

Settlement Analysis:

The total settlement of a loaded soil can be grouped into two broadly, components

(a) Immediate settlement (S_i)

(b) Consolidation settlement (S_{con})

(a) Immediate Settlement (S_i)

- It occurs almost immediately after the load is imposed, as a result to distortion of the solid without any volume change
- It is due to compression, expulsion of pore air, elastic deformation of solids and squeezing of water.
- For cohesionless soils

$$S_i = \frac{H_o}{C_s} \log_{10}\left(\frac{\bar{\sigma}_o + \Delta\bar{\sigma}}{\bar{\sigma}_o}\right); \quad C_s = 1.5\frac{C_r}{\bar{\sigma}_o}$$

Where C_r = Static cone resistance in kN/m^2

$\bar{\sigma}_o$ = Initial effective stress due to over burden pressure at the center of layer

H_o = Total thickness of soil layer initially.

$\Delta\bar{\sigma}$ = Increase in effective stress at the centre of layer due to application of load

* For cohesive soils

- In case of saturated clays, immediate settlement is insignificant. However, small elastic settlement below the corners may occur due to elastic deformation to molecules and squeezing of water.

- The immediate elastic deformation below the corner of a rectangular base foundation is given by

$$S_i = \frac{q \cdot B (1-\mu^2)}{E_s} \cdot I_t$$

Where q = pressure at the base of foundation

B = Width of foundation (dimension)

μ = Poisson's ratio of soil

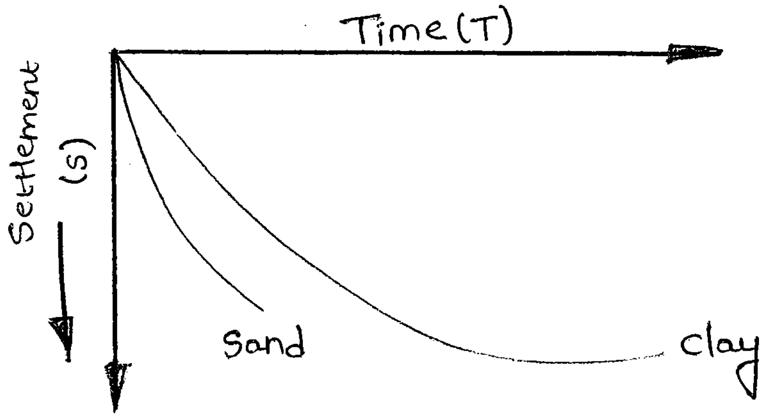
0.3 to 0.45

E_s = Young's Modulus of soil

I_t = Influence factor | shape factor of foundation which depends upon $\frac{B}{L}$ ratio.

(b) Consolidation Settlement (S_{con})

The total consolidation settlement of soil can be further divided into two parts.



(i) Settlement due to primary consolidation (S_c)

(ii) Settlement due to Secondary consolidation (S_s)

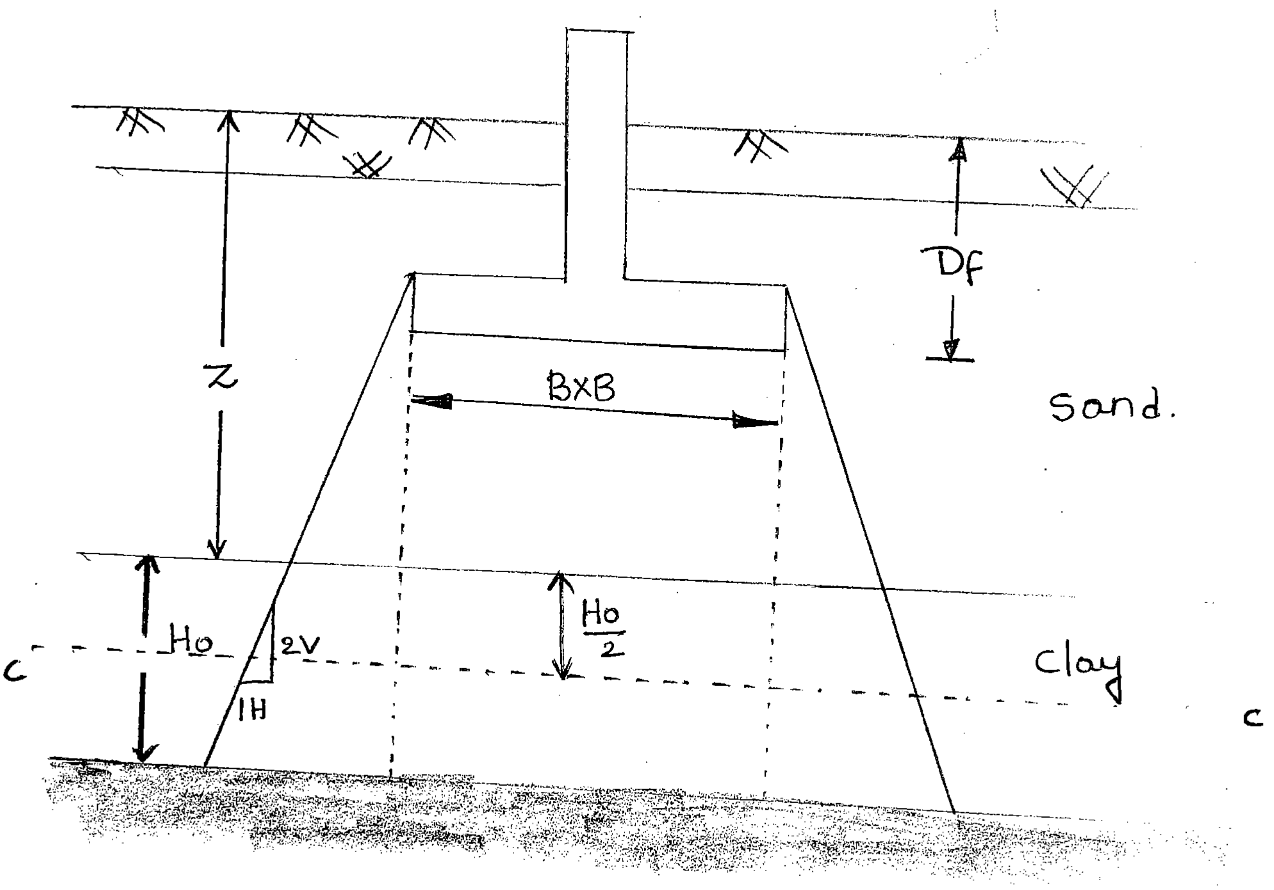
The total consolidation settlement and the time of consolidation in clays is much greater than that of sand. Hence clays are more compressible than sand.

* Remember:

The behaviour of highly over consolidated clay is similar to that of dense sand which are more stable and less compressible.

(i) Computation Of Settlement due to Primary Consolidation:

- It occurs due to expulsion of pore water from a loaded saturated soil mass
- In primary consolidation, the rate of flow is controlled by pore pressure, the permeability and the compressibility of the soil.



Method - 1 :

- If Δe is change in void ratio due to increase in effective stress on soil layer.

As per change in void ratio method,

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$$

$$S_c = \Delta H = H_0 \frac{\Delta e}{1+e_0}$$

Where e_0 = Initial void ratio at beginning of consolidation

H_0 = Initial thickness of compression layer

- This method is suitable for both over consolidated and normally consolidated soils.

Method - 2 When a_v is known

- We know $a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$

Hence $\Delta e = a_v \Delta \bar{\sigma}$

- On substituting Δe in equation (i) we get

$$S_c = H_0 \frac{\Delta e}{1+e_0} = \frac{H_0 a_v . \Delta \bar{\sigma}}{1+e_0}$$

$$S_c = \left(\frac{a_v}{1+e_0}\right) H_0 \Delta \bar{\sigma}$$

Where

$\Delta \bar{\sigma}$ = increase in effective stress at c-c due to application of load on foundation.

Method -3 : When Coeff. of Compressibility (Mv) is known

- We know, $m_v = \frac{a_v}{1+e_o}$

  Hence $a_v = (1+e_o) m_v$

- Now substitute a_v in equation (ii) we get

$$S_c = \frac{(1+e_o) m_v}{1+e_o} \times H_o \Delta \bar{\sigma}$$

or $$S_c = H_o m_v \Delta \bar{\sigma} \quad \text{---- (iii)}$$

- Since m_v and $\Delta\bar{\sigma}$ vary with the depth and Variation of m_v with the depth is not linear. Hence computation of m_v and $\Delta\bar{\sigma}$ at centre does not gives accurate results

- For more precise computation, the total thickness should be divided in more number of layers say 'n' layers and settlement of each layer should be computed separately by making computation at the centre of each layer for m_v and $\Delta\bar{\sigma}$

- Total settlement may be computed as

$$S_c = \sum_{i=1}^{n} H_{oi} m_{vi} \Delta\bar{\sigma}_i$$

Method -4 : When C_c is known

- We know $C_c = \dfrac{-\Delta e}{\log_{10}\left(\dfrac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)}$

Hence $\Delta e = C_c \log_{10}\left(\dfrac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$

- Substituting "Δe" in equation (i) we get

$$S_c = \frac{H_o C_c}{1+e_o} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

Where $\bar{\sigma}_0$ = initial effective overburden pressure at the centre of layer c-c.

Note :

- If the compressible layer is over consolidated, then in above equation, use C_r in place of C_c i.e

$$S_c = \frac{H_o C_r}{1+e_o} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

- If pre-consolidation stress $\bar{\sigma}_c$ is greater than $\bar{\sigma}_0$ but less than $(\bar{\sigma}_0 + \Delta\bar{\sigma})$ i.e $\bar{\sigma}_c < (\bar{\sigma}_0 + \Delta\bar{\sigma})$ the settlement is computed in two parts.

(i) Settlement for pressure $\bar{\sigma}_0$ to $\bar{\sigma}_c$

(ii) Settlement for pressure $\bar{\sigma}_c$ to $(\bar{\sigma}_0 + \Delta\bar{\sigma})$.

- For the first part, the recompression index is applicable, whereas for the second part, the Compression index is used

Thus, $$S_c = \frac{H_o C_r}{1+e_o} \log\left(\frac{\sigma_c}{\sigma_o}\right) + \frac{H_o C_c}{1+e_o} \log\left(\frac{\sigma_o + \Delta\bar{\sigma}}{\bar{\sigma}_c}\right)$$

(ii) Settlement due to secondary settlement

- It is due to plastic readjustment of solids. It occurs at very slow rate and is negligible in granular soils
- The secondary settlement, after time 't' from the completion of primary settlement is given by

$$S_s = \frac{C_s H_{100}}{1+e_{100}} \log_{10}\left(\frac{t}{t_{100}}\right)$$

Where C_s = Secondary Compression index

H_{100} = Thickness of compressible layer after primary Consolidation. In absence of data $H_{100} \simeq H_o$.

e_{100} = Void ratio at the centre of layer after primary consolidation In absence of data $e_{100} \simeq e_o$

t_{100} = Time required to complete primary Consolidation.

t = Time at which secondary settlement is computed after completion of primary Consolidation

- Thus, the total settlement of soil is given by

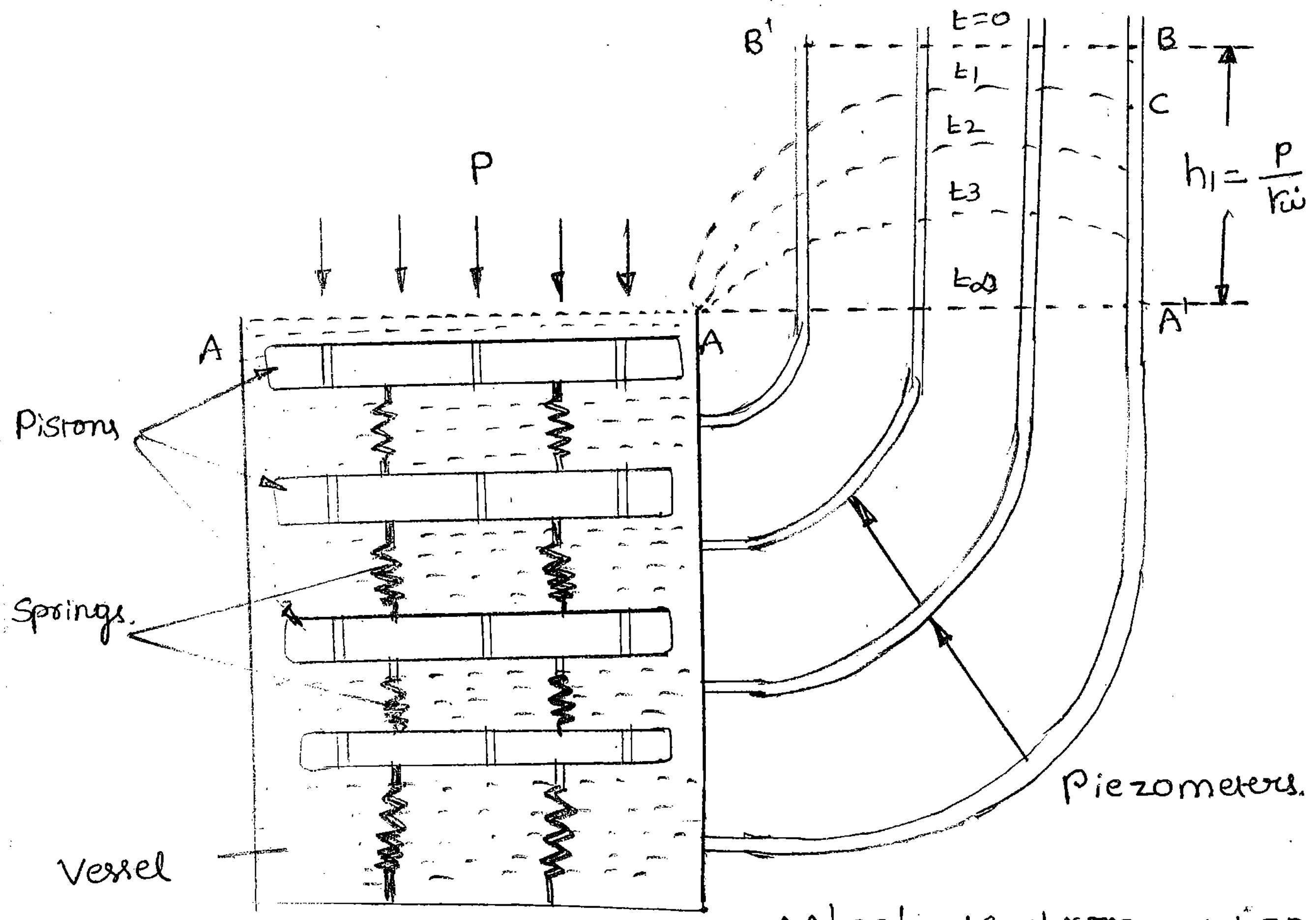
$$S_t = S_i + S_c + S_s$$

* Time Rate of Consolidation.

Terzaghi's Experiment:

- Terzaghi showed that the consolidation process in a clay subjected to loading is analogous to the behaviour of the spring piston model

- Springs represents soil skeleton, Spring surrounded by water represents saturated soil, perforations in the pistons are analogous to the voids that impart permeability to soil.



$AA'BB'$ = Isochrome at $t=0$.

$AA'C$ = Isochrome at $t=t_1$

Mechanisom:

- Initially the load is taken by the springs.
- When extraload is applied, water being more stiffer than the spring bears the extraload which develops the excess hydrostatic pressure which is dissipated from the perforations.
- This allows the pistons to move furtherdown thus compressing the spring and relieving some of the load from water.
- This process represents the gradual transfer of stress from the pore water to soil solids. This process continues until the spring has compressed sufficiently to accommodate the original effective stress plus the additional stress. Now the pore water pressure is zero (i.e no excess pressure). So no dissipation, but spring is lowered showing the vertical strain in soil

Terzaghi's Theory of One Dimensional Consolidation

Assumptions:

1. The soil is homogenous and isotropic
2. Soil is fully saturated and flow is laminar i.e Darcy's law is valid.
3. The soil remain saturated during the process of consolidation
4. The strain produced due to stress applied is small. It means there will be no change in soil structure.
5. The flow is essentially one-dimensional and no change in area cross-sectional occurs.
6. The hydrodynamic lag is considered while plasticlag is ignored. However, it is known to exit.

   It mean this theory is applicable only for Primary consolidation not for secondary consolidation.
7. There is a unique relationship between Void ratio and effective stress, independent of time i.e

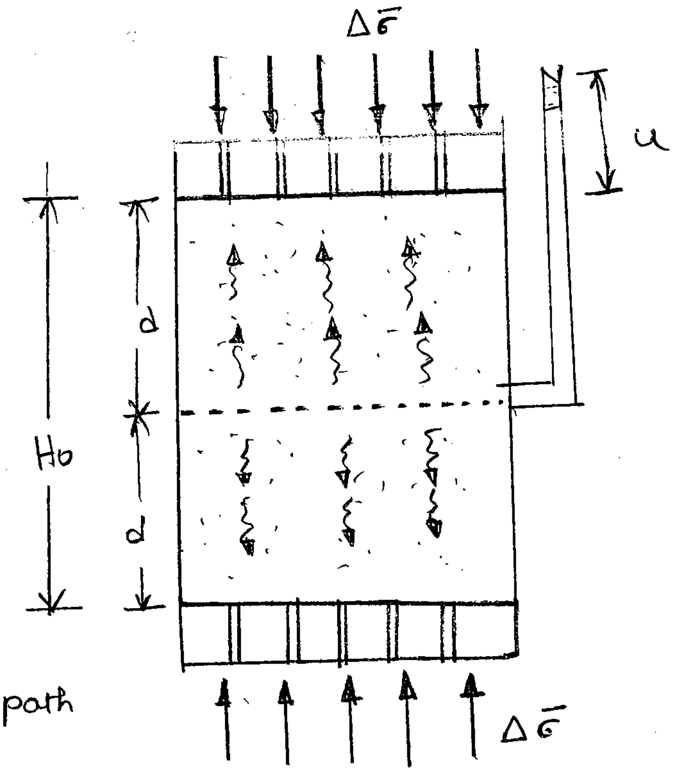
$$\Delta e = -a_v \Delta \bar{\sigma}$$

While a_v is assumed constant over the stress increment

(15)

Note:

- Inverse of mv is called Compression modulus of soil
- mv is not a constant parameter. It changes with the depth of soil



Length of drainage path:

- In case of double drainage, there are two drainage surface one at top another at bottom of consolidating layer.

Where d = length of drainage path

Length of drainage path $= \frac{Ho}{2}$ ....(For 2 way)

- In case of single or one-way drainage, drainage surface to provided either at top or bottom.

Length of drainage path = Ho ....(For one way)

- During the process of consolidation, thickness of soil layer decreases. Hence length of drainage path also changes. So, for accurate computation average drainage length must be taken

Let d_i = initial drainage length

d_f = final drainage length after consolidation

$$d_{avg} = \frac{d_i + d_f}{2}$$

Degree of Consolidation:

- It refers to the percentage of primary consolidation at any instant of time. At any instant it is represented by U.
- At the beginning of consolidation, degree of consolidation is 0 or 0%.
- At the end of consolidation, when expulsion of pore water stops, degree of consolidation is 1 or 100%.

$$0\% \leq U \leq 100\%$$

- Degree of consolidation can be computed as follows:

(i) In terms of Settlement:

Let ΔH be ultimate consolidation settlement corresponding to $U = 100\%$.

degree of consolidation at any time 't' when settlement Δh

$$U\% = \frac{\Delta h}{\Delta H} \times 100.$$

(ii) In terms of void ratio:

Let e_0 is initial void ratio at the centre of soil when $U = 0\%$.

Let e_{100} is final void ratio after completion of consolidation at $U = 100\%$.

at any intermediate time 't', void ratio is 'e', then degree of consolidation is given by

$$U\% = \frac{e_0 - e}{e_0 - e_{100}} \times 100$$

(iii) In terms of excess pore pressure:

Let initially excess pore pressure is u_i and let after any time 't' excess pore pressure is u. Then degree of consolidation is given by

$$U\% = \frac{u_i - u}{u_i} \times 100.$$

Note: Theoretically, infinite time is required for 100% degree of consolidation but practically consolidation complete in certain finite time and hence, for all practical purpose of computation consolidation is assumed to be completed when $U \geq 90\%$.

Time Factor (T_v):

This is a non-dimensional parameter, which directly proportional to time elapsed 't' for a particular soil and inversely proportional to the square of length of drainage path. This is given by

$$T_v = \frac{C_v \cdot t}{d^2}$$

Where C_v = Coefficent of consolidation

d = length of drainage path.

Taylor gave the following approximate relationship between T_v and U which are widely adopted.

(i) $T_v = \frac{\pi}{4} U^2$, when $U \leq 0.6$

(ii) $T_v = 1.781 - 0.933 \log_{10} (100 - U\%)$

Some important values of T_v for different U are tabulated below.

U%	0	50	60	90
T_v	0	0.197	0.287	0.848.

Determination of Cv:

Cv is not a constant parameter which is a function of stress increment. It also depends on the type of soil and its value decreases with increases in liquid limit it value can be given by following relation indirectly.

$$C_v = \frac{K_z (1+e_0)}{a_v \cdot \gamma_w} \quad \text{Unit } m^2/s \text{ (or) } m^2/\text{day}$$

Generally, the coefficient of consolidation, Cv is determined empirically on the generally basis of comparison between the experimental time-compression curve with the theoretical curve. These methods are known as time-fitting methods.

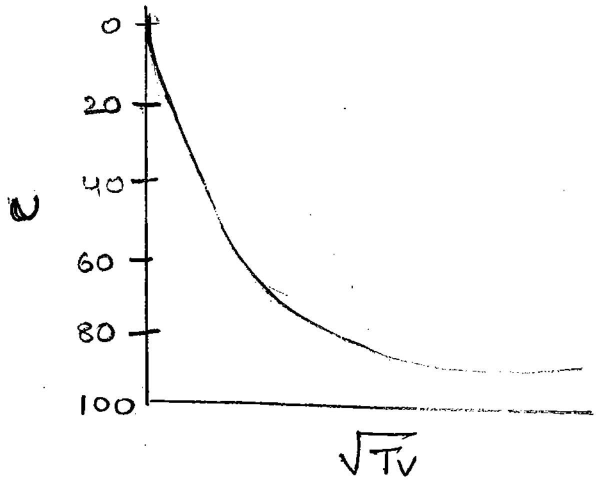
There are two method in general use:

(a) The square root of time fitting method.

(b) The logarithm of time fitting method.

(a) Square root of time fitting method:

- It is given by Taylor and is also called "Taylor's method."



- If a graph is plotted between U and $\sqrt{T_v}$, then it is found straight when $U \leq 0.6$ but thereafter graph is asymptotic.
- As per this method, the C_v is obtained from the relation

$$C_v = \frac{T_{90}\, d^2}{t_{90}}$$

Where

T_{90} = Time factor for 90% consolidation
= 0.848.

t_{90} = Time required to achieve 90% consolidation

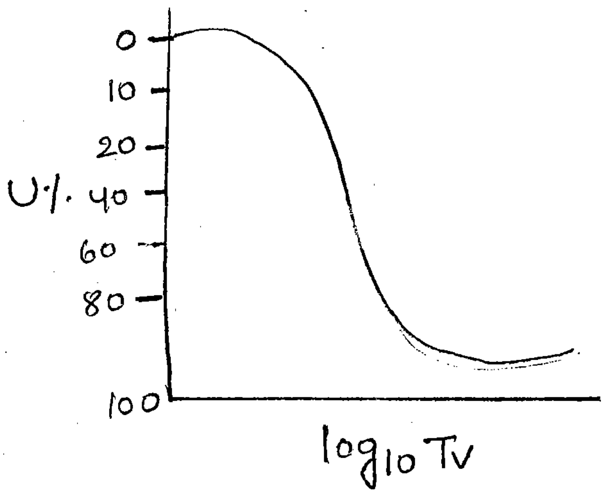
d = length of drainage path

$$C_v = 0.848 \frac{d^2}{t_{90}}$$

(b) Logarithm time fitting method:

- This approach was suggested by A. Casagrande.

- In this method, a graph is plotted between percent consolidation against logarithm of time factor.



- As per this method, the C_v is obtained from the relation

$$C_v = \frac{T_{50} \cdot d^2}{t_{90}}$$

Where T_{50} = Time factor for 50% consolidation

= 0.197.

T_{50} = Time required to achieve 90% consolidation.

d = length of drainage path

$$\therefore \quad C_v = 0.197 \frac{d^2}{t_{50}}.$$