SHEAR STRENGTH OF SOILS

Introduction:

One of the most important considerations in the design and construction on earth and earth supporting structures is the stability of soil mass.

The bearing capacity of soil, stability of soils and earth pressure against retaining structure directly depends upon the 'shear strength' of soil.

Due to external loadings or internal stress changes may fail by shear. So it is very important to examine the mechanism of shear failure and the factors affecting the same.

* Concept of Stress:

- Stress is an internal force acting per unit area of a surface. It is a tensor quantity.
- It has two components, one that acts normal to the sectional plane and other that acts along the plane.
- The normal component is called direct stress and the component that acts along the plane is called tangential stress. This tangential stress is responsible for the shearing of the material hence referred as "shearing stress".

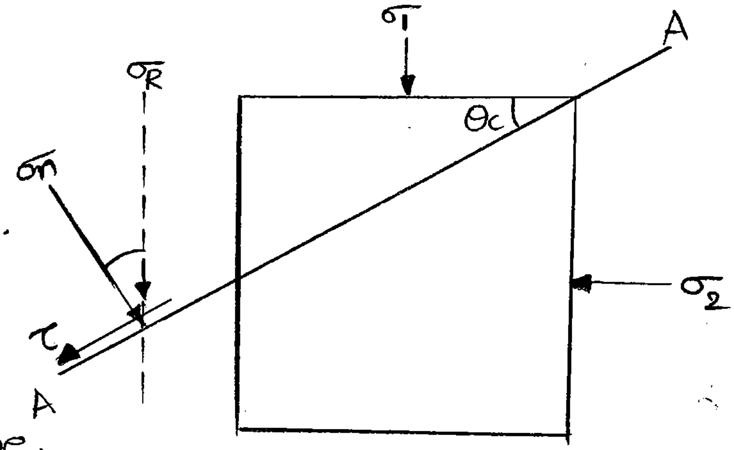
* Shear strength of soil:

Shear strength is the resistance offered by soil against shear deformation, it's value is equal to the shear stress on critical plane (A-A). The critical plane is that plane on which resultant stress has maximum angle of obliquity with the normal of that plane.

Where σ_1 - Major principle stress.

σ_2 - Minor principle stress.

θ_c - Angle of critical plane with the major principal plane.



⇒ on the plane of θ_c (critical plane), σ_R is most inclined [i.e $\beta = \beta_{max}$] for frictional soils $\beta_{max} \simeq$ internal frictional angle of soil (ϕ).

⇒ when angle β is maximum, then the shear stress on plane A-A will be equal to the shear strength of soil.

* **Mechanism of Shear Resistance.**

Shearing resistance of a soil is the property of the soil that enables the soil mass to keep it's equilibrium when it's surface is not level or under any loading situation that is producing shearing stresses.

A soil may derived it's shearing strength from the following parameters.

i) Frictional Resistance.

ii) Interlocking of particles.

iii) Cohesion and adhesion of molecules.

The granular soils derived their shearing strength from friction [Both sliding & Rolling] & interlocking

⇒ fine grained soils derived their strength from friction and cohesion.

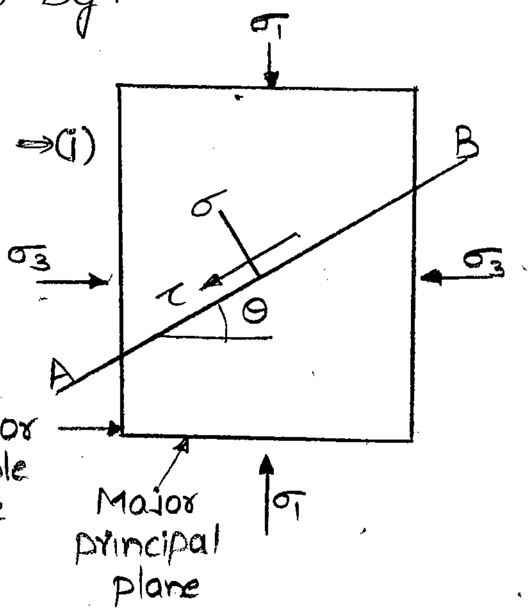
⇒ Highly plastic clay (i.e) pure clays, however only have 'cohesion' as their source of shear strength.

* Stress at a point - Mohr circle of stress:-

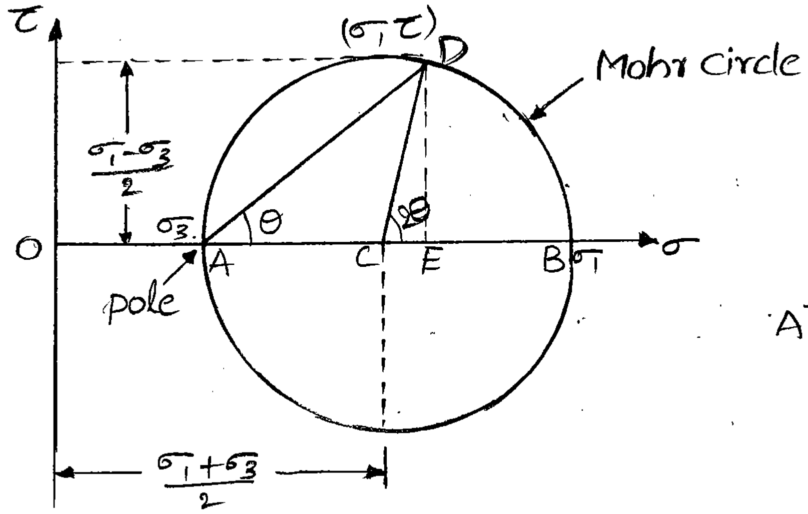
- In a stressed soil mass, shear failure can occur along any plane.
- At any stressed point, there exists three mutually perpendicular planes on which there are no shearing stresses acting. The corresponding planes are respectively designated as the major, minor & intermediate principal planes. However, the critical stress conditions occurs only at σ_1 and σ_3.
- If σ_1 and σ_3 are known it can be shown that on any plane AB inclined at angle θ to the direction of major principal plane, the normal stress σ and the shear stress τ are given by.

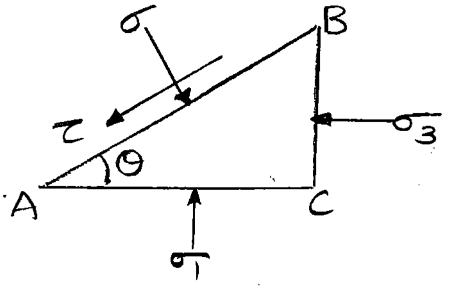
$$\sigma = \left[\frac{\sigma_1+\sigma_3}{2}\right] + \left[\frac{\sigma_1-\sigma_3}{2}\right] \cos 2\theta \Rightarrow \text{(i)}$$

$$\tau = \left[\frac{\sigma_1-\sigma_3}{2}\right] \sin 2\theta \Rightarrow \text{(ii)}$$



- Mohr demonstrated that these equations tend themselves to graphical representation. It can be





Shown that "the locus of stress coordinates (σ, τ) for all planes through a point is a circle, called the "Mohr circle of stresses".

* Important Relationship obtained from the Mohr circle:

1. Maximum shearing stress occurs on planes inclined at 45° to principal planes.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \quad [\text{at } \theta = 45°]$$

2. The normal stresses on plane of maximum shear are equal to each other and they are given by

$$\sigma_{1,2} = \frac{\sigma_1 + \sigma_3}{2}.$$

3. The sum of normal stresses on mutually $\perp^{lar}$ plane is a constant. (i.e) $\sigma_1 + \sigma_3 = \sigma_{n_1} + \sigma_{n_2} = \text{constant}$.

4. When the principal stresses are equal to each other, the radius of the Mohr's circle becomes zero, which means the shear stresses vanish on all planes. Such a point is called "ISOTROPIC" point.

5. The resultant stress at any point is $\sqrt{\sigma^2 + \tau^2}$

and the obliquity β, equal to $\tan^{-1}(\tau/\sigma)$.

6. The maximum angle of obliquity (β_{max}) is obtained from failure envelope and is given by:

$$\theta_{cr} = 45^\circ + \frac{\beta_{max}}{2}.$$

7. The plane of maximum obliquity is most liable to failure and not the plane of maximum shear.

8. Failure becomes incipient when β_{max} approches & equals the angle of internal friction ϕ.
   - For granular soil $\beta_{max} = \phi$.
   - for cohesive soils $\phi = 0$, therefore, $\beta_{max} = 0$ therefore

$$\theta = \frac{\pi}{4} + \frac{\beta}{2} = \frac{\pi}{4}$$

9. In failure plane, $\sigma_n = \sigma_1(1 - \sin\beta_{max})$

$$\sigma_n = \sigma_3(1 + \sin\beta_{max}).$$

$$\frac{\sigma_1}{\sigma_3} = \frac{1 - \sin\beta_{max}}{1 + \sin\beta_{max}}.$$

$$\Rightarrow \sin\beta_{max} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}.$$

- on critical plane at limiting conditions, τ is called shear strength

$$\tau = S = \sigma_n \tan\phi.$$

- Normal stress on the plane of τ_{max} is given by OC.

$$\sigma_n = OC = \frac{\sigma_1 + \sigma_3}{2}$$

Angle of plane of $\tau_{max} = \theta = 45^\circ$ with the major principal planes.

• Resultant stress on the plane of τ_{max} OA,

$$\sigma_R = OA = \sqrt{\sigma_n^2 + \tau_{max}^2} = \sqrt{\left[\frac{\sigma_1+\sigma_3}{2}\right]^2 + \left[\frac{\sigma_1-\sigma_3}{2}\right]^2}$$

$$= \sqrt{\frac{\sigma_1^2+\sigma_3^2}{2}}$$

* Method of drawing Mohr's circle:

1. Normal stress σ is plotted on x-axis.
2. Shear stress τ is plotted on y-axis.
3. Compressive normal stresses are taken as positive.
4. Shear stresses that produce counter-clockwise couples of the element are considered positive.
5. The centre of circle is at $c\left[\frac{\sigma_1+\sigma_3}{2}, 0\right]$ and radius is equal to $\frac{\sigma_1-\sigma_3}{2}$ and the circle cuts the x-axis at two points.

• Now from figure(a) $\angle BCD = 2\theta$, where θ is the angle made by the line joining point $(\sigma_3, 0)$ and parallel to the plane AB of fig (b)

$$\sigma = OE = OC + CE$$

$$= \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2} \cdot \cos 2\theta$$

$$\tau = DE = \frac{\sigma_1-\sigma_3}{2} \cdot \sin 2\theta.$$

• Point A on the Mohr Circle is a unique point

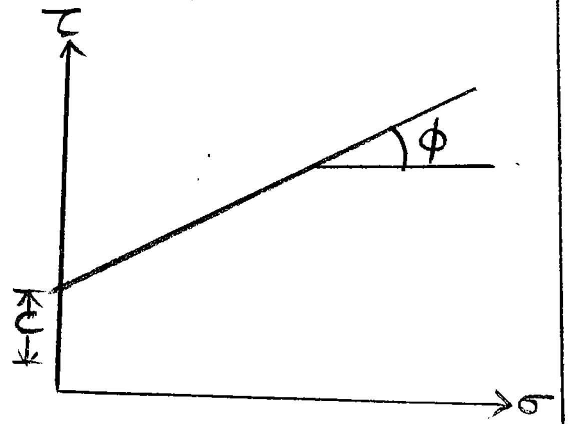
Called the "pole" or "origin of planes".

- The property of pole is : "a line drawn through the pole intersects the Mohr circle at a point which represents the state of stress on a plane which has the same inclination in space as the line itself".
- This property can be utilized in locating the pole in a situation where the state of stress (σ, τ) on a certain plane is known.

* Columb's Equation and Mohr-columb's criterion:

- Coulomb's observed that one component of the shearing strength called intrinsic cohesion is constant for a given soil and is independent of applied stress. The other component, namely the frictional resistance, varies directly as the magnitude of the normal stress on the plane of rupture. Collumb equation is written as.

$$\tau_f = C + \sigma \tan\phi$$



where

τ_f = Shear strength of soil.

C = apparent cohesion.

S = Normal stress on plane of rupture.

ϕ = Angle of internal friction

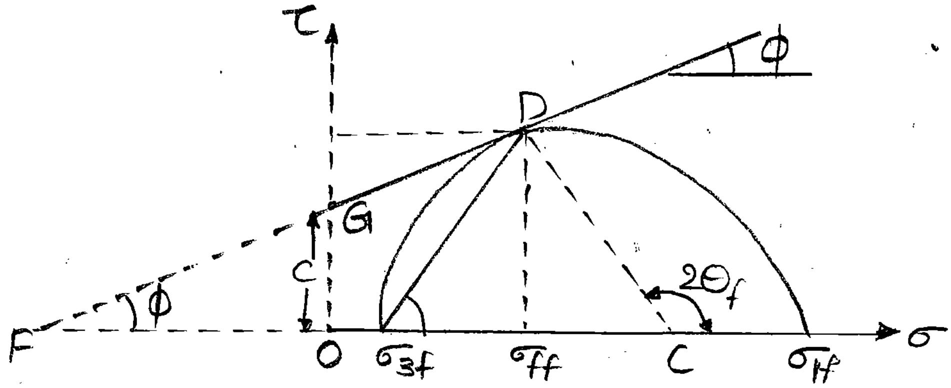
- Mohr-coulomb failure criteria can be expressed in the form of $\tau_{ff} = C + \sigma_{ff} \tan\phi$

• Also, angle of failure can be expressed in the term of angle of shearing resistance(ϕ).

$$\theta_f = 45° + \frac{\phi}{2}.$$

Relationship between c and principal stresses at failure:

$$\sin\phi = \frac{ED}{FC}$$



$$= \frac{ED}{CO+OF}$$

$$= \frac{(\sigma_{1f} - \sigma_{3f})/2}{(\sigma_{1f} + \sigma_{3f})/(2 + \cot\phi)}$$

$$(\sigma_{1f} - \sigma_{3f}) = (\sigma_{1f} + \sigma_{3f})\sin\phi + 2c\cos\phi.$$

* Factors Affecting shear strength:

confining stress:

• If any soil mass is confined conditions (i.e) restrained from lateral movement, then shear resistance of the soil increased because σ increases.

$$\tau = c' + \bar{\sigma}\tan\phi$$

• In confining condition, interlocking of granular soils increase, hence shear strength increases.

* Limitation of Mohr - coulomb theory:

- It neglects the effect of the intermediate principal Stress(σ_2).
- This theory, approx the failure envelope into straight line which may be a little curve for over consolidated soil.
- For some clays, there is no fixed relationship between the normal and shear stresses on the plane of failure. The theory cannot be used for such soils.
- In case of pure clays, according to this theory, shear strength is constant with the depth. However in practice a little increase is observed.

* Terzaghi Modification:

- The original form of coulomb's equation was in terms of total normal stress.
- After terzaghi establishment of effective stress principle, it was found that the shear strength of soil is depends on effective parameters not an total parameters. The shear strength of the soil is accordingly expressed as

$$\tau_f = c' + \bar{\sigma} \tan\phi'.$$

Where $\bar{\sigma} = \sigma - u$.

c' = effective cohesion

ϕ' = Angle of shearing resistance referred to effective stress.

u = pore pressure on the plane of rupture.

Drainage conditions:

- It has been point out earlier, that effective stress which governs the shearing strength of soil.

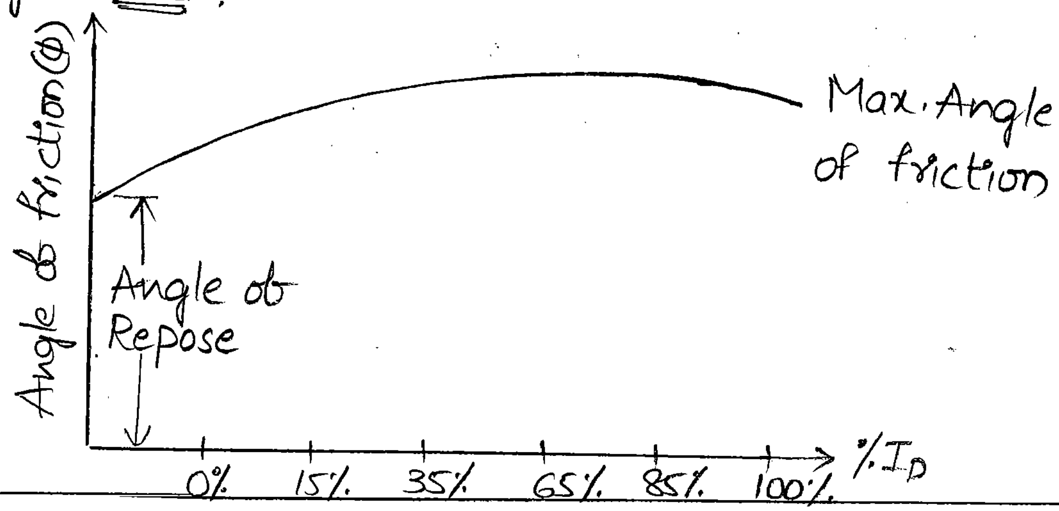
$$\tau = c + \bar{\sigma} \tan\phi'.$$

- Drained condition occurs when the excess pore water pressure developed during loading of a soil. dissipates (i.e) $\Delta u = 0$.
- undrained conditions occurs when the excess pore water pressure cannot drain from the soil. (i.e) $\Delta u \neq 0$.
- The existence of either condition - drained or undrained depends on the soil type, geological formation and the rate of loading.
- The values of c and ϕ depends on the drainage conditions in saturated soils.

$\phi - \phi'$ in case of drain test.

$\phi - 0$ in case of undrained test.

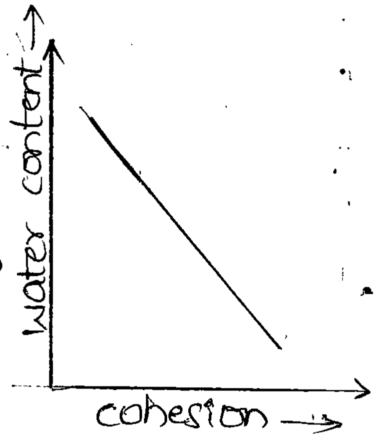
⇒ Density Index:



- The most important factor affecting the shear strength of granular soil is density index.
- For the same composition of the soil, higher the density higher the angle of friction. Hence higher will be the shear strength.

* Water content and saturation:

- In case of fine grained soils, cohesion between soil particles is inversely proportional to water content. The relationship between cohesion and the water content is given in figure.



- The degree of saturation also affects the cohesion and the cohesion increases upto an optimum value above which it decreases with increasing for a given void ratio.
- other hand, in unsaturated soils negative excess pore water pressure increases the effective stress [$\bar{\sigma} = \sigma - u$]. Thus, if the pore water pressure is negative, the effective stress increases.

* Composition & particle characteristics:

- Angle of internal friction depends on the grading of the soil. A well graded soil with high uniformity coefficient has a higher angle of friction as compared to poorly graded soil.
- Similarly sharp angular grains which can interlock well with adjacent grain will show higher friction angles. Hence minerals such as mica and flaky particles will show low angles of internal friction.

problem:

* Compute the shearing strength of soil along a horizontal plane at a depth of 5m in a deposit of sand having the following properties:

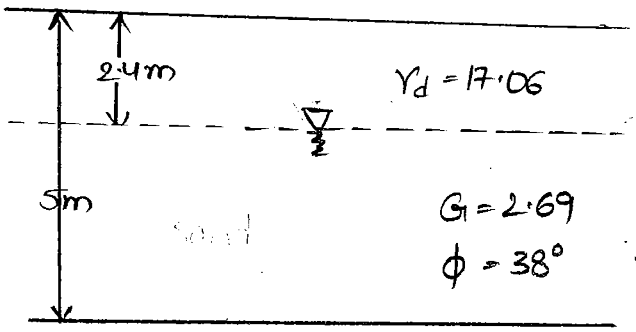
Angle of internal friction, $\phi = 38°$.

Dry unit weight, $\gamma_d = 17.06\ kN/m^3$.

specific gravity, $G = 2.69$.

Assume the ground water table at a depth of 2.4 m from the ground level. Also determine the change in the shear strength, if the water table rises upto the ground level.

Sol:



Shear strength of sand deposit $\tau = c + \bar{\sigma}\tan\phi$

For sand, $c = 0$.

$\therefore \tau = \bar{\sigma}\tan\phi$.

$\bar{\sigma}$ = effective stress at 5m below sand

$= (\gamma_d \times 2.4) + (\gamma \times 2.6)$

For $\gamma_d = \dfrac{G.\gamma_w}{1+e}$

$17.06 = \dfrac{2.69 \times 9.81}{1+e}$.

$\Rightarrow e = 0.546$.

For $\gamma = \left[\dfrac{G-1}{1+e}\right]\gamma_w = \left[\dfrac{2.69-1}{1+0.546}\right] \times 9.81 = 10.723\ kN/m^2$.

Then $\bar{\sigma} = (17.06 \times 2.4) + (10.723 \times 2.6) = 68.82\ kN/m^2$.

The shear strength of soil $\tau = \bar{\sigma}\tan\phi$

$= 68.82 \times \tan 38°$

$= 53.76 \ kN/m^2$.

When water table rises to the ground level, the entire soil below ground level become submerged, then $\bar{\sigma}$ is given by,

$$\bar{\sigma} = \gamma \times 5 \ m = 10.723 \times 5 = 53.615 \ kN/m^2.$$

Now, the shear strength $= \tau = 53.615 \times \tan 38°$

$= 41.89 \ kN/m^2$.

∴ change in shear strength, $\Delta\tau = 53.76 - 41.89$

$= 11.87 \ kN/m^2$.

- A given saturated clay is known to have effective strength parameters of $c' = 10 kpa$ and $\phi' = 28°$. A sample of this clay was brought to failure quickly so that no dissipation of the pore water could occur at failure. It was known that $\bar{\sigma}_1' = 60$ kpa, $\bar{\sigma}_3' = 10$ kpa, and $U_f = 20$ kpa
  a.) Estimate the values of σ_1 and σ_3 at failure.
  b.) what was the effective normal stress on the failure plane?

Sol: Given $c' = 10$ kpa, $\phi' = 28°$.

$\bar{\sigma}_1 = 60$ kpa, $\bar{\sigma}_3 = 10$ kpa

$U_f = 20$ kpa.

a) We know. $\bar{\sigma}_1 = \sigma_1 - U_f$ Similarly $\bar{\sigma}_3 = \sigma_3 - U_f$

$60 = \sigma_1 - 20$ $10 = \sigma_3 - 20$

$\therefore \sigma_1 = 80$ kpa. $\sigma_3 = 30$ kpa

b.) inclination of failure plane with the major principal plane, $\theta = 45° + \frac{\phi'}{2} = 45° + \frac{28°}{2} = 59°$.

The effective normal stress at the failure plane is given by the relation,

$$\bar{\sigma} = \frac{\bar{\sigma}_1 + \bar{\sigma}_3}{2} + \frac{\bar{\sigma}_1 - \bar{\sigma}_3}{2} \cos 2\theta_c$$

$$= \frac{60+10}{2} + \frac{60-10}{2} \cos(2 \times 59^\circ)$$

$$= 35 - 11.74 \quad = 23.26 \text{ kpa.}$$

Measurement of shear strength:

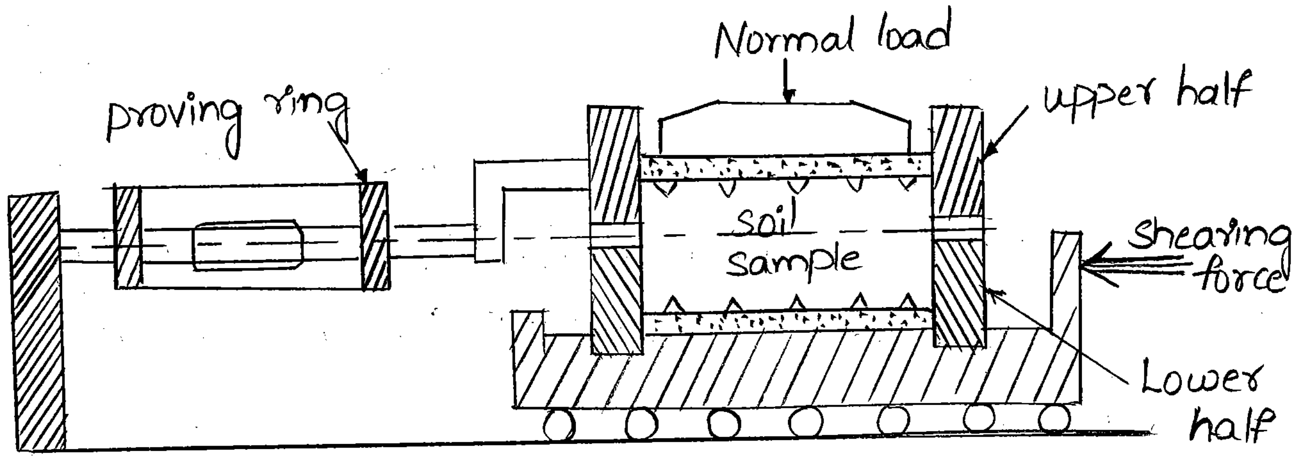
- Determination of shearing strength of a soil involves the plotting of failure envelopes and evaluation of the shear strength parameters for the necessary condition. Following test are carried out for this purpose.

Field tests:

1. Direct shear test.
2. Triaxial test.
3. unconfined compression test.
4. Vane shear test.
5. Torsion test
6. Ring shear test.

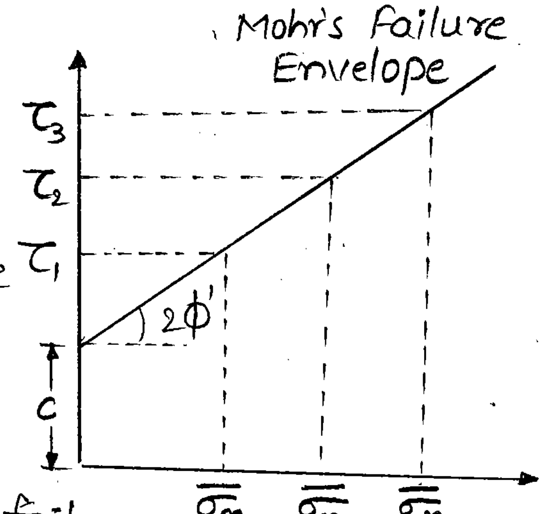
Direct shear test:

- This is the oldest shear test, still in use, quite simple to perform.
- The soil specimen that is to be tested, is confined in a metal box of square cross section that is split into two halves horizontally, a small clearance being maintained between the two halves of the box. This test is also called shear box test.
- There is no control over drainage conditions and no mechanism to measure pore pressure. Hence this test is preferred for drained conditions (CD). Also a Constant shear strain rate of 1.25 mm/min is applied.



- Two types of application of shear are possible: one in which are shear stress is controlled and the other in which the shear strained is controlled. Generally, shear is applied at constant rate of strain

- Normal stress and shear on the failure plane are obtained by dividing the normal force and the shear resistance of the soil is recorded by proving ring dial gauge at failure.



- Values of shear stress at failure are plotted against effective stress for each test. The shear strength parameters c and ϕ are obtained from the best fit straight line through the points.

⇒ Advantages:

- Test is simple and quick.
- Apparatus is cheaper.
- No need of technical skill.
- Thickness of sample is small. Hence, drainage

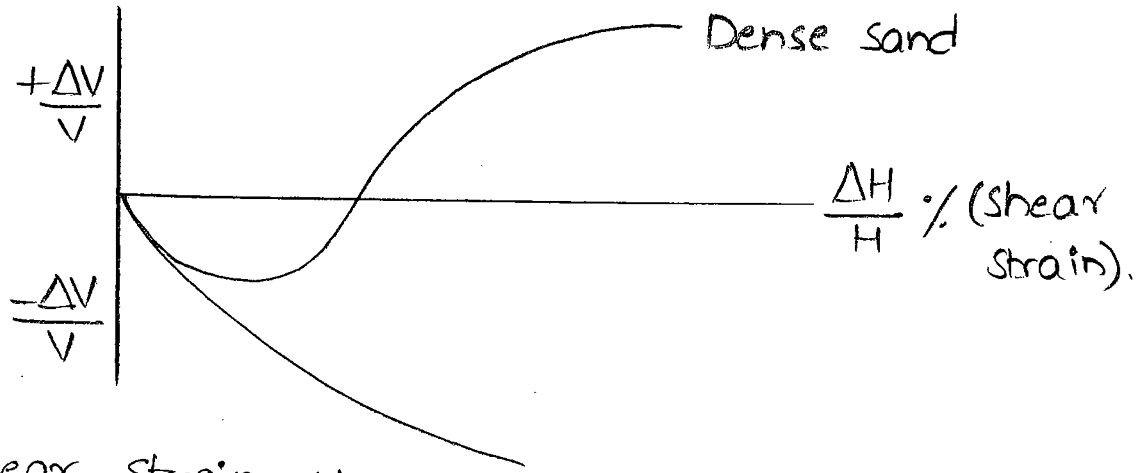
does not take much time.

⇒ Disadvantages:

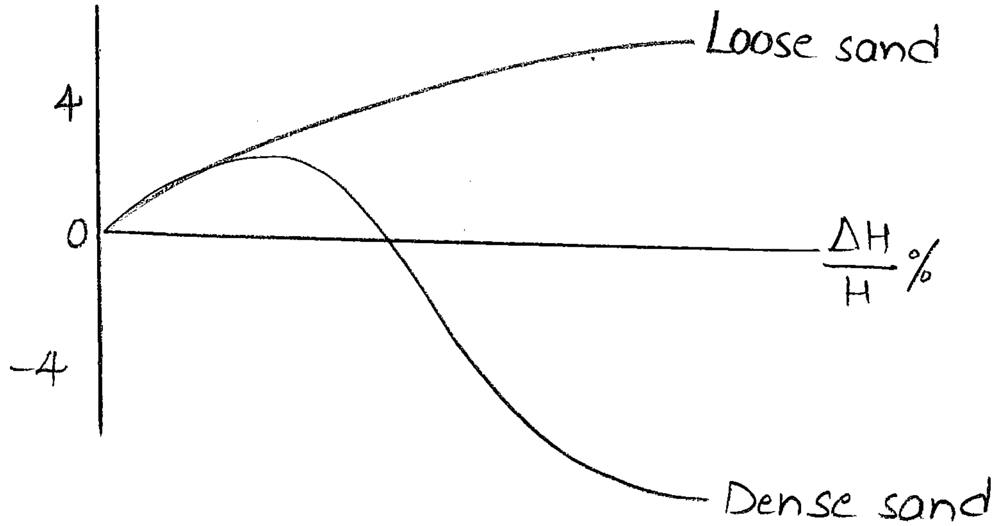
- The failure plane is predetermined, which may not be the weakest plane.
- Drainage conditions cannot be controlled and no mechanism to measure pore water pressure. therefore this test is not suitable for fine grained soils.
- This test is useful only for freely draining soils like sand.
- There are stress concentrations at the sample boundaries, leading to non-uniform shear stress distribution on the failure plane.
- The directions of the principal planes is not known at every stage of the test. It is only after the Mohr failure envelope is known that the magnitude and the direction of the principal stress can be determined. Infact, there is a rotation of the principal planes between the start of the test and failure of the soil.
- As the test progress, the area under the shear gradually decreases. The connected area at failure should be used in determining the values of normal stress and shear stress.

⇒ Representation of results of Direct shear test:

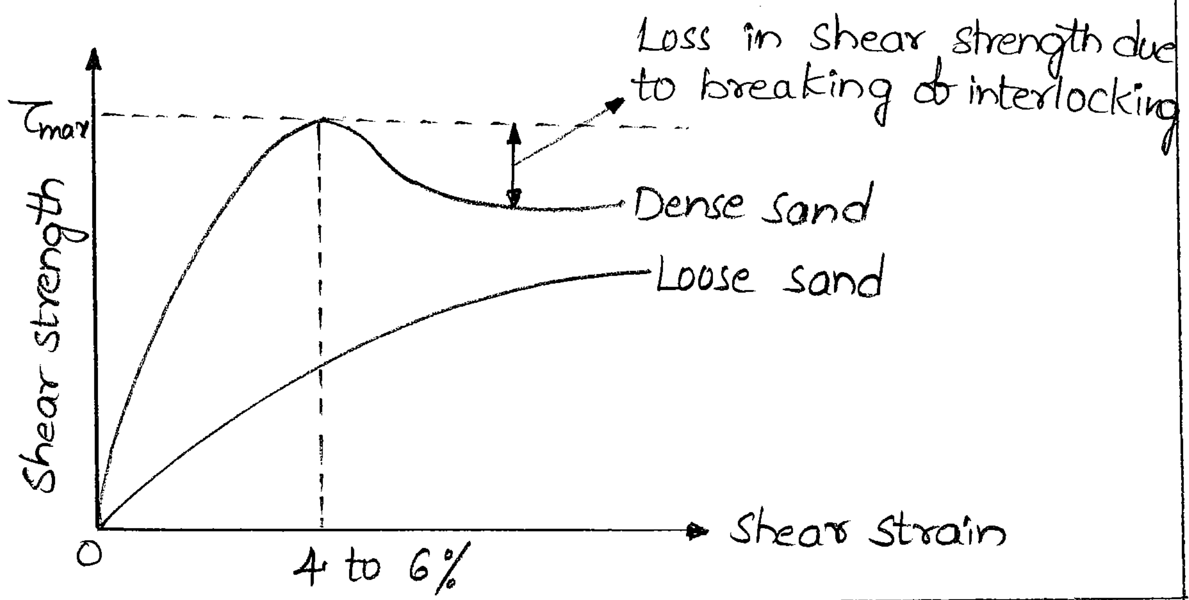
(i) volume changes Vs shear strain curve:



ii) Shear strain Vs pore pressure:



iii) shear strength Vs shear strain:



The shear strength of sand is due to

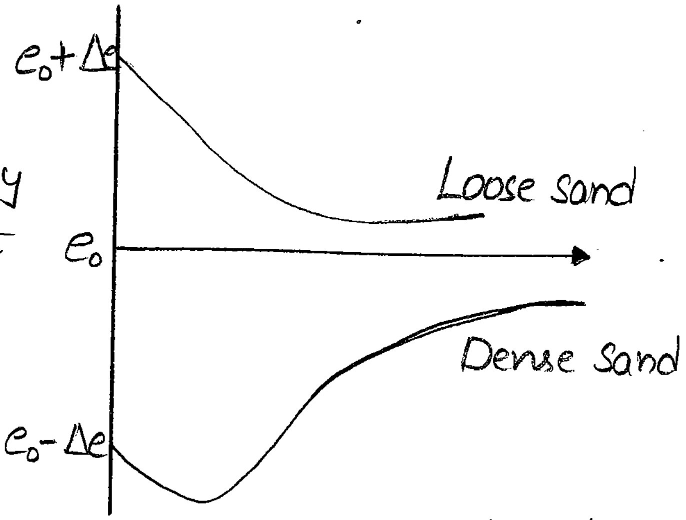
i) Interlocking of molecules.

ii) Friction between molecules.

In dense sand interlocking resistance is 20-25% of total shear strength. On shearing, interlocking of molecules breaks and loss in shear strength is recorded at a strain of about 4-6%. Whereas in case of loose sand interlocking resistance is negligible and peak failure is not recorded. In loose sands, failure is assumed to occur when strain reaches to 20% or more.

iv.) Shear strain Vs void ratio

• At large strain both initially loose and initially dense specimen attain nearly the same void ratio(e_0), at which further strain will not produce may volume changes such a void ratio is usually referred to as the critical void ratio.



e_0 - Critical void ratio

• If void ratio of a soil (i.e) $e > e_0$, the soil is classified as loose soil and if $e < e_0$, then it is called dense soil.

problem:

A shear-box test carried out on a sandy clay gave the following results.

Vertical load (kg)	Division of proving ring dial gauge (1 division - 1μm)
36.8	17
73.5	26
110.2	35
146.9	44

Shear box is 60 mm × 60 mm and the proving ring constant is 20 N/μm. Determine the apparent cohesion and angle of internal friction for this soil.

Sol:

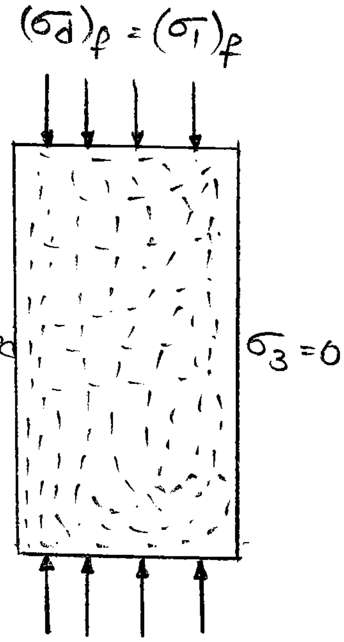
Shear deformation values (μm) gives in column (2) can be converted into shear force values by multiplying them by proving ring constant (i.e) 20 N/μm.

Now shear stress is given by vertical load divided by shear box area. Similarly the shear stress is obtained by shear force divided by shear box area.

calculations of shear force, Normal stress & shear stress are tabulated below:

Vertical load (P) in (kg)	Division of proving ring (1division = 1μm)	Shear force (V) = K × No. of division of dial gauge (N)	Shear stress $\tau = \frac{V}{Area}$ N/mm²	Normal stress $\sigma_n = \frac{P}{A}$
36.8	17	340	$\frac{340}{60\times60} = 0.094$	$\frac{36.8\times9.81}{60\times60} = 0.1$
73.5	26	520	$\frac{520}{60\times60} = 0.144$	$\frac{73.5\times9.81}{60\times60} = 0.2$
110.2	35	700	$\frac{700}{60\times60} = 0.194$	$\frac{110.2\times9.81}{60\times60} = 0.3$
146.9	44	880	$\frac{880}{60\times60} = 0.244$	$\frac{146.9\times9.81}{60\times60} = 0.4$

Unconfined compression Test:



- It is a special case of triaxial test in which confining pressure is zero. It means only deviator or shear stress is applied.
- Since the specimen is laterally unconfined, the test is known as unconfined compression test.
- The axial or vertical compressive stress is the major principal stress and the other two principal stresses are zero.
- Since there is no cell pressure, no rubber membrane required. Therefore without rubber membrane (i.e) [lateral support], sand and dry soils cannot be held in position. Hence this test is applicable for saturated clays and silts.
- If axial force at failure is P, then

$$(\sigma_d)_f = \frac{P}{A_f}$$

- The axial stress at failure is called unconfined compressive strength since confining pressure $\sigma_3 = \sigma_c = 0$, hence

$$\sigma_1 = \sigma_3 + \sigma_d = \sigma_d$$

Let 'q_u' be the unconfined compressive strength,

$$q_u = (\sigma_1)_f = (\sigma_d)_f$$

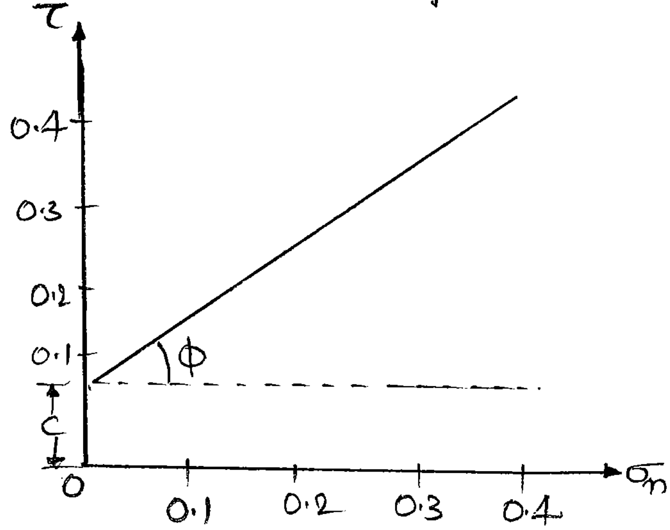
using,

$$q_u = (\sigma_1)_f = (\sigma_d)_f$$

$$\sigma_1 = \sigma_3 \tan^2\left[45 + \frac{\phi}{2}\right] + 2c \tan\left[45 + \frac{\phi}{2}\right]$$

$$\sigma_1 = 2c \tan\left[45 + \frac{\phi}{2}\right] \qquad [\because \sigma_3 = 0]$$

When a shear box test results are plotted between σ and τ_f, A straight line is obtained as shown in figure This line can be used to determine c and ϕ.



$$\tan\phi = \frac{\tau_2-\tau_1}{\sigma_2-\sigma_1} = \frac{0.144-0.094}{0.2-0.1}$$

$$\tan\phi = 0.5$$

$$\phi = 26.5^\circ$$

using equation of line

$$\tau_1 = c + \sigma_1 \tan\phi$$

$$0.094 = c + 0.1 \tan 26.5^\circ$$

$$c = 0.094 - 0.1 \times 0.5$$

$$c = 0.0446\ N/m^2 = 44.6\ kN/m^2.$$

Sol: Given data

Deviator stress, $\sigma_d = 120\ kN/m^2$

Confining pressure, $\sigma_c = \sigma_3 = 0$.

$$\sigma_1 = \sigma_3 + \sigma_d = 0 + 120 = 120\ kN/m^2.$$

Angle of failure plane $= \theta_c = 45 + \frac{\phi}{2} = 50°$, $[\because \phi = 10°]$

using - $\sigma_1 = \sigma_3 \tan^2\left[45 + \frac{\phi}{2}\right] + 2c\tan\left[45 + \frac{\phi}{2}\right]$

$$120 = 0 + 2c \times \tan\left[45 + \frac{10}{2}\right]$$

$$120 = 2c \times \tan 50°$$

$$c = \frac{120}{2 \times \tan 50°} = 50.354\ kN/m^2.$$

2. In an unconfined compression test a sample of clay 100 mm long and 50 mm in diameter fails under a load of 150N at 10% strain. calculate the shearing resistance taking into the account the effect of change in cross section of the sample.

Sol: Given strain, $\epsilon_L = \frac{\Delta L}{L} = 10\%$ or 0.10.

We know, $A_f = \frac{A_0}{1 - \epsilon_L}$

Where $A_0 = \frac{\pi}{4} \times (5)^2 = 19.63\ cm^2$

$$A_f = \frac{19.63}{1 - 0.1} = 21.82\ cm^2.$$

We know, the axial stress at failure is known as

$$q_u = 2c \tan\left[45 + \frac{\phi}{2}\right]$$

For frictionless soils (i.e) for clays ($\phi = 0$)

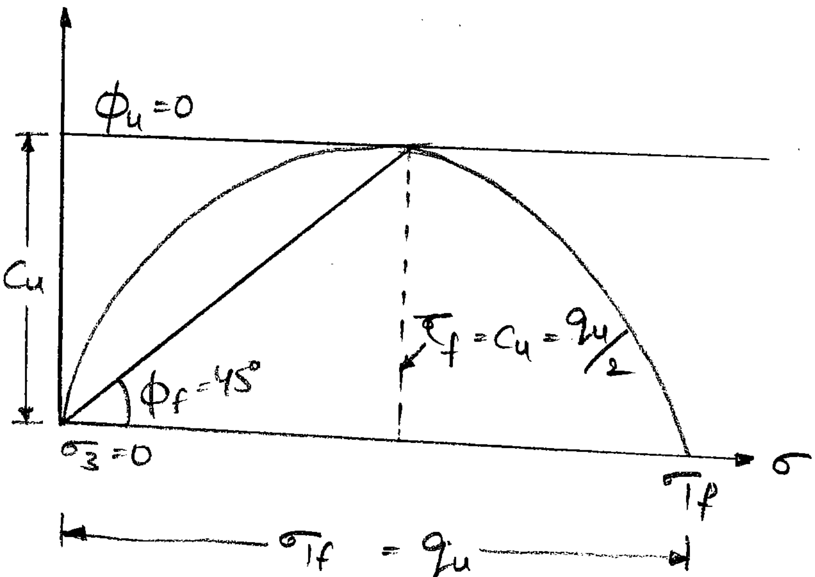
$$q_u = 2c \tan 45^\circ$$

$$c = \frac{q_u}{2}.$$

Hence shear strength for clays,

$$S = c + \sigma \tan\phi$$

$$= c + 0 = \frac{q_u}{2} \quad [\because \phi = 0]$$



- In this test, $\sigma_3 = 0$ and $(\sigma_1)_f = (\sigma_d)_f$. Therefore, there is a unique Mohr circle which passes through origin.

problem:

1. When an unconfined compression test was conducted on a cylindrical soil sample, it failed under an axial stress of 120 kN/m^2. The failure plane makes an angle of 50° with the horizontal. Determine the cohesion and the angle of internal friction of the soil.?

unconfined compressive strength.

$$\therefore q_u = \frac{P_f}{A_f} = \frac{150\ N}{21.82\ cm^2} = 6.87\ N/cm^2.$$

And $c = \frac{q_u}{2} = \frac{6.87}{2} = 3.435\ N/cm^2 = 34.35\ kN/m^2$

$\therefore$ Shear resistance, $s = c + \sigma \tan\phi$

For clay $\phi = 0$, $s = \frac{q_u}{2} = 34.35\ kN/m^2$.

3. An unconfined test was conducted on a soft clay in both in-situ and remoulded samples, the following data was observed:

S.No	Description	In-situ sample	Remoulded Sample
1.	Length of sample	150 mm	150 mm
2.	Initial area, A_0	1500 mm	1500 mm
3.	Extension of spring at failure	20 mm	15 mm.
4.	Spring constant (k)	15 N/mm	15 N/mm
5.	Axial deformation	9 mm	20 mm.

Determine the unconfined compression strength of soil in both conditions. Also determine the sensitivity of soil sample?

Sol: In-situ state (undisturbed):

Failure load = Extension of spring × spring constant

$= 20 \times 15 = 300\ N.$

Corrected area at failure, $A_f = \frac{A_0}{1 + \epsilon_L}$

Where ϵ_L = Axial strain.

$$= \frac{\Delta L}{L} = \frac{9}{150} = 0.06.$$

∴ Corrected area, $A_f = \frac{1500}{1-0.06} = 1595.75\ mm^2$.

∴ Normal stress at failure $= \frac{P_f}{A_f} = \frac{300}{1595.75} = 0.188\ N/mm^2$

∴ unconfined compressive strength, $q_u = 0.188\ N/mm^2$.

Remoulded state:

failure load = Extension of spring × spring constant

$$= 15 \times 15 = 225$$

Corrected area at failure, $A_f = \frac{A_0}{1-\epsilon_L}$

Where, $\epsilon_L = \frac{\Delta L}{L} = \frac{20}{150} = 0.133.$

$$A_f = \frac{1500}{1-0.133} = 1499.13\ mm^2.$$

Normal stress at failure $= \frac{P_f}{A_f} = \frac{225}{1499.13} = 0.15\ N/mm^2$

∴ unconfined compression strength, $q_u = 0.15\ N/mm^2$.

Hence, sensitivity $= \frac{q_u\ (\text{undisturbed})}{q_u\ (\text{remoulded})} = \frac{0.188}{0.150} = 1.25$

The value of sensitivity is lies between 1 and 2, hence the soil is classified as a low sensitivity soil.

Example 10.6 A consolidation-drained triaxial test was conducted on a normally consolidated clay yielding the following data: $\sigma_3 = 250$ kN/m^2: $\sigma_d = 275$ kN/m^2 Determine

(i) The angle of friction

(ii) Angle which the failure plane makes with the major principal plane, and

(iii) Normal stress and shear stress on the failure plane.

(iv) Normal stress on the plane of τ_{max}.

Solution:

We know that, when a drained triaxial test (*C-D*) is conducted on a normally conducted clay, it behave like sand, with no cohesion. Hence, $c = 0$.

and $$\sigma_1 = \sigma_3 + \sigma_d = 250 + 275 = 525 \text{ kN/m}^2$$

(i) Using, $$\sigma_1 = \sigma_3 \tan^2\left(45+\frac{\phi}{2}\right), 2c\tan\left(45+\frac{\phi}{2}\right)$$

or, $$\sigma_1 = \sigma_3 \tan^2\left(45+\frac{\phi}{2}\right) \quad [\because c = 0]$$

or, $$525 = 250\tan^2\left(45+\frac{\phi}{2}\right)$$

$$45+\frac{\phi}{2} = 55.39°$$

$$\phi = 20.78°$$

(ii) Angle made by failure plane with the major principal plane,

$$\theta_C = 45+\frac{\phi}{2} = 55.39°$$

(iii) Normal stress on the failure plane is given by the relation,

$$\sigma_f = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta_C$$

$$= \frac{525+250}{2} + \frac{525-250}{2}\cos(2\times 55.39)$$

$$= 387.5 - 48.78 = 338.72 \text{ kN/m}^2$$

and, shear stress on the failure plane is given by,

$$\tau_f = \frac{\sigma_1-\sigma_3}{2}\sin 2\theta_C = \frac{525-250}{2}\sin(2\times 54.39°) = 128.54 \text{ kN/m}^2$$

(iv) Normal stress on the plane of τ_{max},

$$= \frac{\sigma_1+\sigma_3}{2} = \frac{525+250}{2} = 387.5 \text{ kN/m}^2$$

Example 10.7 A *CU* test was conducted on a soil sample with cell pressure, $\sigma_c = 100$ kN/m^2. The deviator stress at failure was observed to be 60 kN/m^2. The soil is known to have a cohesion $c' = 0$ and the angle of shearing resistance $\phi' = 30°$ (referred to effective stress) and an undrained cohesion $c_u = 0$ and angle of shearing resistance $\phi_u = 13.3°$ (referred to total stress). What was the pore water pressure at failure:

Solution:

Given: Cell pressure, $\sigma_3 = \sigma_C = 100$ kN/m^2

deviator stress, $\sigma_d = 60$ kN/m^2

Major principal stress at failure is given by,

$$\sigma_1 = \sigma_C + \sigma_d = 100 + 60 = 160 \text{ kN/m}^2$$

Let pore water pressure at failure be u

Thus, effective stresses,

$$\bar{\sigma}_3 = \sigma_3 - u = 100 - u$$

and
$$\bar{\sigma}_1 = \sigma_1 - 4 = 160 - u$$

We know that,

$$\sin\phi_u = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \text{ (total parameter)}$$

Similarly,
$$\sin\phi' = \frac{\bar{\sigma}_1 - \bar{\sigma}_3}{\bar{\sigma}_1 + \bar{\sigma}_3}$$

$$\sin 30° = \frac{(160-u)-(100-u)}{(160-u)+(100-u)}$$

$$\frac{1}{2} = \frac{60}{260-2u}$$

$$2u = 260 - 120$$

$$u = 70 \text{ kN/m}^2$$

Example 10.8 **The effective stress shear strength parameters of a completely saturated clay are: $c' = 20$ kN/m^2, $\phi' = 28°$.**

A sample of this clay was tested in a $\overline{UU}$ test under a cell pressure of 180 kN/m^2 and deviator stress at failure was 100 kN/m^2. What was the value of pore water pressure at failure.

Solution:

Given:
$$c' = 20 \text{ kN/m}^2, \phi' = 28°$$

$$\sigma_d = \sigma_1 - \sigma_3 = 100 \text{ kN/m}^2$$

$$\sigma_3 = \sigma_C = 180 \text{ kN/m}^2$$

$\therefore$
$$\sigma_1 = \sigma_3 + \sigma_d = 180 + 100 = 280 \text{ kN/m}^2$$

Using,
$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c\tan\left(45 + \frac{\phi'}{2}\right)$$

$$(\sigma_1 - u) = (\sigma_3 - u)\tan^2\left(45 + \frac{\phi'}{2}\right) + 2c\tan\left(45 + \frac{\phi'}{2}\right)$$

$$(280 - u) = (180 - u)\tan^2\left(45 + \frac{28}{2}\right) + 2 \times 20 \times \tan\left(45 + \frac{28}{2}\right)$$

$$(280 - u) = (180 - u) \times 2.769 + 66.571$$

$$280 - u = 498.42 - 2.769\,u + 66.571$$

$$1.769\,u = 285$$

$$u = 161.10 \text{ kN/m}^2$$

Example 10.9 The results obtained from a series of *CU* tests on a soil gave the following results:

$$c_{cu} = c' = 0$$
$$\phi_{cu} = 17° \quad \text{and } \phi'_{cu} = 34°$$

If sample of this soil was tested in a *CU* test under a cell pressure of 180 kN/m². Determine

(a) deviator stress at failure
(b) pore water pressure at failure
(c) minor principal effective stress at failure
(d) major principal effective stress at failure
(e) the magnitude of A_f.

Solution:

(a) Since:
$$c_u = 0$$

Using,
$$\sigma_1 = \sigma_3 \tan^2\left(45+\frac{\phi_u}{2}\right) + 2c_u \tan\left(45+\frac{\phi_u}{2}\right)$$

$$\sigma_1 = \sigma_3 \tan^2\left(45+\frac{\phi_u}{2}\right)$$

or,
$$\sigma_1 - \sigma_3 = \sigma_3 \tan^2\left(45+\frac{\phi_u}{2}\right) - \sigma_3$$

$$\sigma_d = \sigma_3 \left[\tan^2\left(45+\frac{\phi_u}{2}\right) - 1\right]$$

$$= 180 \left[\tan^2\left(45+\frac{17}{2}\right) - 1\right] = 148.75 \text{ kN/m}^2$$

(b) ∴
$$\sigma_1 = \sigma_3 + \sigma_d = 180 + 148.75 = 328.75 \text{ kN/m}^2$$

Using,
$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45+\frac{\phi'_u}{2}\right) + 2c' \tan\left(45+\frac{\phi'_u}{2}\right)$$

$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45+\frac{\phi'_u}{2}\right) + 0$$

$$(\sigma_1 - u) = (\sigma_3 - u)\tan^2\left(45+\frac{\phi'_{cu}}{2}\right)$$

$$(328.75 - u) = (180 - u) \times 3.537$$
$$2.537u = 307.93$$
$$u = 121.37 \text{ kN/m}^2$$

(c) Minor principal effective stress at failure,

$$\bar{\sigma}_{3f} = \sigma_{3f} - u_f = 180 - 121.37 = 58.63 \text{ kN/m}^2$$

(d) Major principal effective stress at failure,

$$\bar{\sigma}_{1f} = \bar{\sigma}_{df} + \bar{\sigma}_{3f}$$
$$= (\bar{\sigma}_{1f} - \bar{\sigma}_{3f}) + \bar{\sigma}_{3f}$$
$$= [(\sigma_{1f} - u) - (\sigma_3 - u)] + \bar{\sigma}_{3f}$$
$$= (\sigma_{1f} - \sigma_3) + \bar{\sigma}_{3f}$$

$$= \sigma_d + \bar{\sigma}_{3f} = 148.75 + 58.63$$

$$= 207.38 \text{ kN/m}^2$$

(e) $$A_f = \frac{u_f}{(\sigma_1 - \sigma_3)_f} = \frac{121.37}{148.75} = 0.815$$

Unconfined Compression Test:

- It is a special case of triaxial test in which confining pressure is zero. It means only deviator or shear stress is applied.
- Since the specimen is laterally unconfined, the test is known as unconfined compression test.
- The axial or vertical compressive stress is the major principal stress and the other two principal stresses are zero.
- Since there is no cell pressure, no rubber membrane required. Therefore, without rubber membrane (i.e. lateral support), sand and dry soils cannot be held in position. Hence this test is applicable for saturated clays and silts.
- This test cannot be conducted on coarse grained soils such as sands and gravels.
- If axial force at failure is P, then

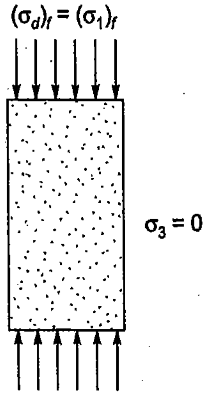


Fig. 10.23

$$(\sigma_d)_f = \frac{P}{A_f}$$

- The axial stress at failure is called unconfined compressive strength since confining pressure $\sigma_3 = \sigma_C = 0$, hence

$$\sigma_1 = \sigma_3 + \sigma_d = \sigma_d$$

Let 'q_u' be the unconfined compressive strength,

$$\therefore \quad q_u = (\sigma_1)_f = (\sigma_d)_f$$

Using, $$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c\tan\left(45 + \frac{\phi}{2}\right)$$

$$\sigma_1 = 2c\tan\left(45 + \frac{\phi}{2}\right) \qquad [\because \sigma_3 = 0]$$

or $$q_u = 2c\tan\left(45 + \frac{\phi}{2}\right)$$

For frictionless soils i.e. for clays ($\phi = 0$)

$$\therefore \quad q_u = 2c\tan 45°$$

$$c = \frac{q_u}{2}$$

Hence shear strength for clays,

$$S = c + \sigma \tan\phi$$

$$= c + 0 \qquad [\because \phi = 0]$$

$$= \frac{q_u}{2}$$

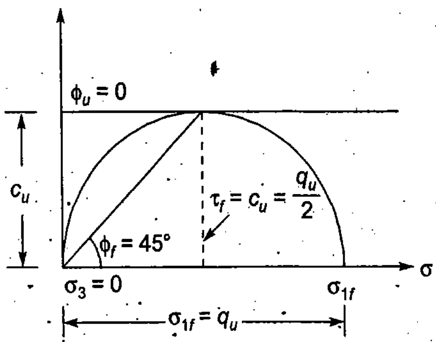


Fig. 10.24

- In this test, $\sigma_3 = 0$ and $(\sigma_1)_f = (\sigma_d)_f$. Therefore, there is a unique Mohr circle which passes through origin.

Pore Pressure Parameters:

- Some time it is not possible to determine pore pressure practically, then theoretical approach given by skempton can be adopted.
- Pore pressure parameters A and B are empirical coefficients which are used to express the response of pore pressure to changes in vertical Pressure and lateral pressure under "undrain conditions".
- These parameters are very useful in field problems where pore pressure that are induced consequent to change in total stress may have to be computed, for e.g. in case of construction of earth embankment over a soft clay deposit.

Parameter B:

- This parameter is defined under cell pressure stage and represents the ratio of change in pore pressure to the change in cell pressure.

$$B = \frac{\Delta u_c}{\Delta \sigma_c}$$

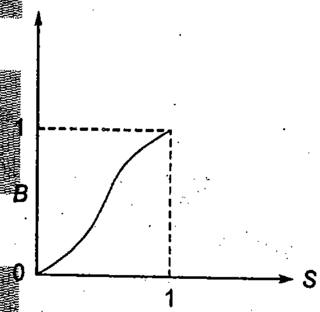
where, Δu_c = change in pore pressure due to change in cell pressure by $\Delta\sigma_3$.

B is also given by, $$B = \frac{1}{1 + n\frac{C_v}{C_c}}$$

where, n is porosity of soil.

C_v and C_c are coefficients of consolidation and curvature respectively.

- B varies from 0 to 1 depending on the degree of saturation.
- B is zero for dry soil and is equals to unity for fully saturate soil.



Parameter A:

- This parameter is valid in deviator stage and is defined in terms of another parameter $\bar{A}$, such that

$$\bar{A} = A \cdot B$$

- The parameter $\bar{A}$ represents the ratio of change in pore pressure to change in deviator stress during shear stage.

$$\bar{A} = \frac{\Delta u_d}{\Delta \sigma_d} = \frac{\Delta u_d}{\Delta(\sigma_1 - \sigma_3)} = \frac{\Delta u_d}{\Delta\sigma_1 - \Delta\sigma_3}$$

where Δu_d = Change in pore pressure due to change in deviator stress.

- The parameter A depends upon strain in soil, degree of saturation, over consolidation ratio, stratification of soil etc. Its value may be as low as –0.5 for over consolidated soil with high O.C.R to as high as 3 or loose saturated sand.
- During loading, cell pressure and deviator stress both changes. Therefore total change in pore pressure will be

$$\begin{aligned}\Delta u &= \Delta u_c + \Delta u_d \\ &= B \cdot \Delta\sigma_3 + \bar{A} \times \Delta\sigma_d \\ &= B \cdot \Delta\sigma_3 + AB\Delta\sigma_d \\ &= B \cdot \Delta\sigma_3 + AB(\Delta\sigma_1 - \Delta\sigma_3)\end{aligned}$$

$\Delta\sigma_1$ = change in major principal stress

$\Delta\sigma_3$ = change in minor principal stress

NOTE

- A and B both are not a constant parameter for a given soil.
- B is a positive parameter, whereas A can be negative.
- For saturated soil,

$$B = 1$$

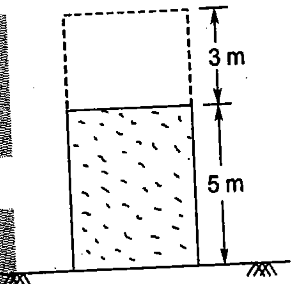
and $\bar{A} = A \cdot B$

$$\therefore \quad A = \bar{A} = \frac{\Delta u_d}{\Delta \sigma_d} \qquad [\because B = 1]$$

Example 10.16 An embankment 5 m high is made up of soil whose effective stress parameters are $c' = 50$ kN/m² and $\phi' = 16°$ and $\gamma = 16.2$ kN/m³. The pore pressure parameters as found from triaxial tests are $A = 0.4$ and $B = 0.49$. Find the shear strength of the soil at the base of the embankment just after the soil has been raised from 5 m to 8 m. Assume that the dissipation of pore pressure during the stage of construction is negligible and that the lateral pressure at any point in one half of the vertical pressure.

Solution:

In-Situ



Change in pore pressure due to increase in height

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$\Delta\sigma_1 = \text{Increase in vertical stress}$$

$$= 3\gamma = 3 \times 16 = 48 \text{ kN/m}^2$$

$$\text{Increase in lateral stress} = \frac{1}{2} \times \Delta\bar{\sigma}_1 = 24 \text{ kN/m}^2$$

$$\Delta u = 0.49[24 + 0.4(48 - 24)] = 16.464 \text{ kN/m}^2$$

Initial pore pressure = 0

Final pore pressure = $U_i + \Delta u$ = 16.464 kN/m

Shear strength at the base

$$S = c' + \bar{\sigma}_n \tan\phi'$$

$$\bar{\sigma}_n = \sigma - u = 8\gamma - u$$

$$= 8 \times 16 - 16.464$$

$$= 111.536 \text{ kN/m}^2$$

$$S = 50 + 111.536 \tan 16°$$

$$= 81.98 \text{ kN/m}^2$$

Stress Path

- Progressive change in the state of a particular load application can be represented by a series of Mohr circle.
- The figure represents successive states as σ_1 is increased with σ_3 constant. Such a diagram with several complete stress circles can appear cluttered. It is convenient to plot only two point of maximum shear stress, and if needed the complete circle can be reconstructed using such a point.
- Thus, the locus of points on the Mohr diagram whose coordinates represent the maximum shear stress and associated stress for the entire stress history is defined as stress path.

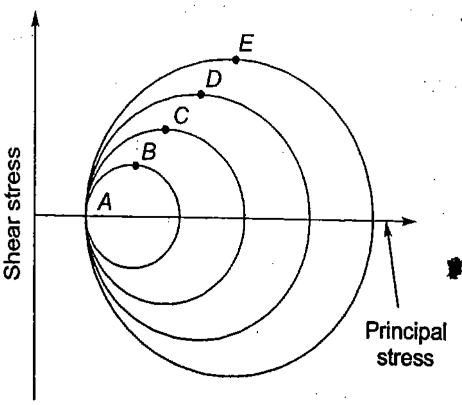


Fig. 10.26 Stress paths

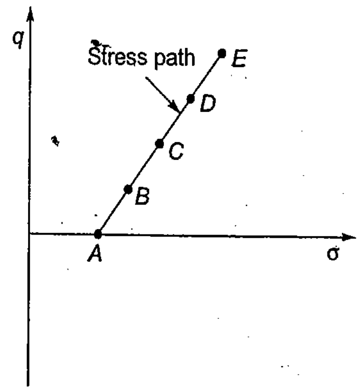


Fig. 10.27 *Stress path*

- For given principal stresses σ_1 and σ_3, the coordinates of a point on the stress path are

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} \quad \text{and} \quad \tau = \frac{\sigma_1 - \sigma_3}{2}$$

The stress path for σ_3 = constant and σ_1 increasing is a 45° line as shown above.

- A stress path diagram may be constructed for total or effective stress conditions.

Liquefaction of soil:

- In loose saturated sand, on shear disturbance, there is decrease in volume as soil molecules comes closer. Hence pore pressure is set, due to which effective stress reduces suddenly. Hence this condition where a soil will undergo continued deformation at constant low residual effective stress or with zero effective stress is known as 'Liquefaction of soil'.
- Liquefaction mostly occurs due to earthquake forces which induce high pore water pressure.
- When soil fails due to liquefaction, the structures founded on such soil sink.
- Increase in pore water pressure results in reduction in shear strength. Complete transfer of intergranular stress from soil grain to water is known as complete liquefaction. In this case, the effective stress reduced to almost zero, and the sand water behaves as a viscous material.

Thixotropy of clays:

- Clays with an initial flocculated structure may loose strength due to disturbance or remoulding. However, with passage of time, the clay may gain the original strength due to thixotropy.
- Thixotropy is defined as an isothermal, reversible, time-dependent process which occurs under constant composition and volume. This phenomenon is attributed to a process of softening caused by remoulding followed by a time-dependent regain of the original hard state.

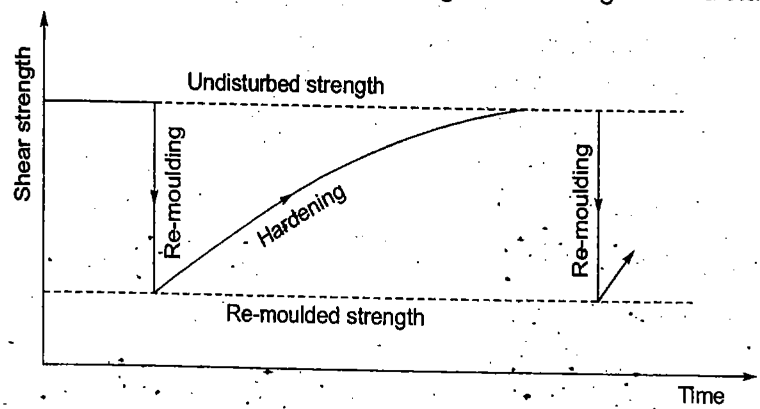


Fig. 10.28 Thixotropy of a material