UNIT - I

TWO-HINGED ARCHES

Introduction:-

→ Mainly three types of arches are used in practice: three-hinged, two-hinged and hingeless arches.

→ In the early part of the nineteenth century, three-hinged arches were commonly used for the long span structures as the analysis of such arches could be done with confidence.

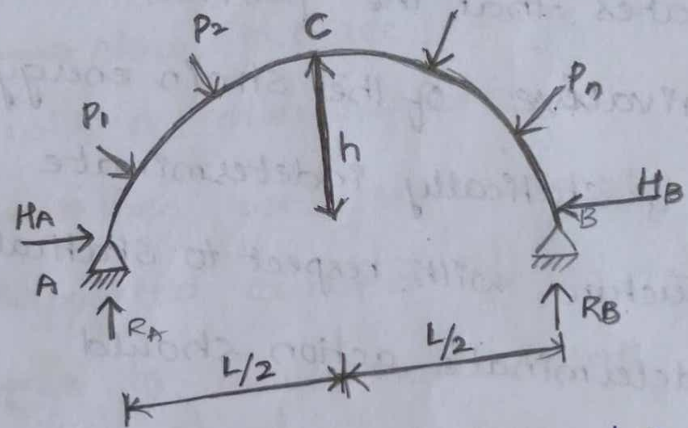
→ However, with the development in structural analysis, for long span structures starting from late nineteenth century engineers adopted two-hinged and hingeless arches.

→ Two-hinged arch is the statically indeterminate structure to degree one.

→ Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work.

→ In this lesson, the analysis of two-hinged arches is discussed and few problems are solved to illustrate the procedure for calculating the internal forces.

Analysis of Two-Hinged Arch:-



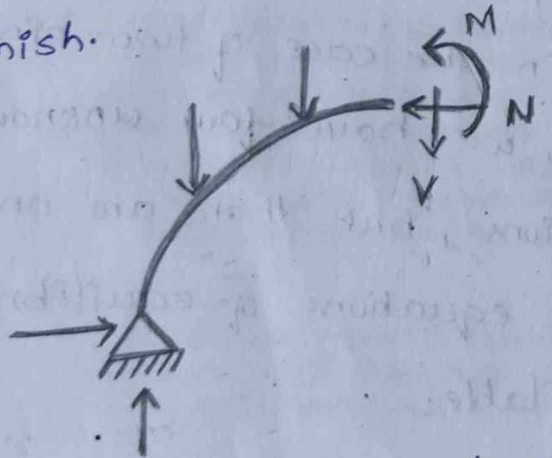
→ In the case of two-hinged arch, we have four unknowns reactions, but there are only three equations of equilibrium available.

→ Hence the degree of staticcal indeterminancy is one for two hinged arch.

→ The fourth equation is written considering deformation of the arch.

→ The unknown redundant reaction is calculated by noting that the horizontal displacement of hinge HB is 'zero'.

→ In general the horizontal reaction in the two hinged arch is evaluated by straight forward application of the theorem of least work which states that the partial derivative of the strain energy of a statically indeterminate structure with respect to statically indeterminate action should vanish.



→ Hence to obtain, horizontal reaction, one must develop an expression for strain energy.

→ Typically any section of the arch is subjected to shear force 'v', bending moment 'M', the axial compression 'N'.

→ The strain energy due to bending U_b is calculated from the expression.

$$U_b = \int_0^s \frac{M^2}{2EI}\, ds \quad \text{—— (1)}$$

→ The above expression is similar to the one used in the case of straight beams.

→ However, in this case, the integration needs to be evaluated along the curved arch length.

→ In the above equation, s is the length of the centreline of the arch, I is the moment of inertia of the arch cross section, E is the Young's Modulus of the arch material.

→ The strain energy due to shear is small as compared to the strain energy due to bending and is usually neglected in the analysis.

→ In the case of flat arches, the strain energy due to axial compression can be appreciable and is given by

$$U_a = \int_0^s \frac{N^2}{2AE}\,ds \quad \text{— (2)}$$

→ The total strain energy of the arch is given by,

$$U = \int_0^s \frac{M^2}{2EI}\,ds + \int_0^s \frac{N^2}{2AE}\,ds \quad \text{— (3)}$$

→ Now, according to the principle of least work

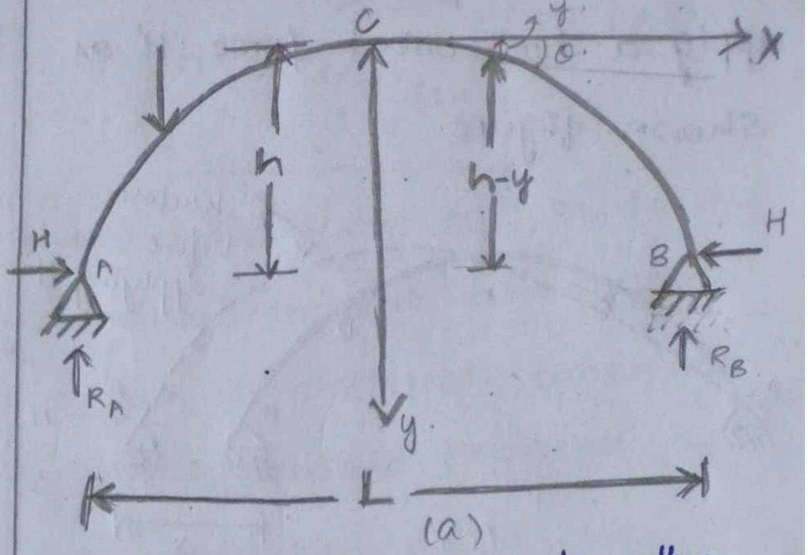
$\frac{\partial U}{\partial H} = 0$, where H is chosen as the redundant reaction.

$$\frac{\partial U}{\partial H} = \int_0^s \frac{M}{EI}\frac{\partial M}{\partial H}\,ds + \int_0^s \frac{N}{AE}\frac{\partial N}{\partial H}\,ds = 0 \quad \text{— (4)}$$

by solving eq^n (4), the horizontal reaction H is evaluated.

Symmetrical two-hinged arch:

Consider a symmetrical two-hinged arch as shown in figure.

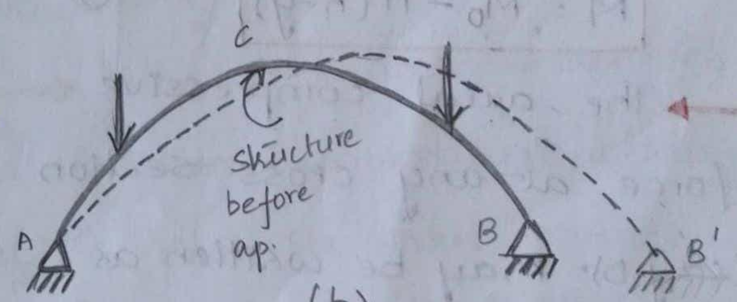


(a)

→ Let 'C' at crown be the origin of co-ordinate axes.

→ Now, replace hinge at B with a roller support.

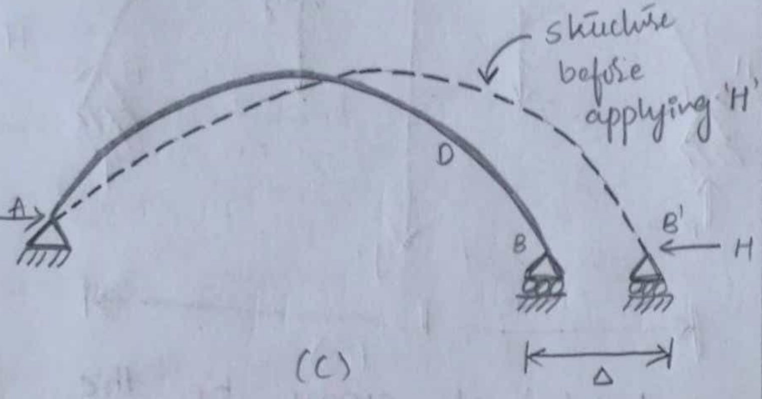
→ Then we get a simply supported curved beam is free to move horizontally, it will do so as shown by dotted lines in figure.



(b)

→ Let M_0 & N_0 be the bending moment and axial force at any cross section of the simply supported curved beam.

→ Since, in the original arch structure, there is no horizontal displacement, now apply a horizontal force 'H' as shown figure.



(C)

→ The horizontal force 'H' should be of such magnitude, that the displacement at 'B' must vanish.

→ From (B) & (C), the bending moment at any cross section of the arch (say D), may be written as,

$$\boxed{M = M_0 - H(h-y)} \quad \text{— (5)}$$

→ The axial compressive force at any cross section (say D) may be written as

$$\boxed{N = N_0 + H\cos\theta} \quad \text{— (6)}$$

where 'θ' is the angle made by the tangent at 'D' with horizontal substituting the value of M and N in the equation (4)

$$\frac{\partial U}{\partial H} = 0 = -\int_0^s \frac{M_0 - H(h-y)}{EI}(h-y)\,ds + \int_0^s \frac{N_0 + H\cos\theta}{EA}\cos\theta\,ds \quad \text{— (7)}$$

Let $\bar{y} = h - y$

$$\Rightarrow -\int_0^s \frac{M_0 - H\bar{y}}{EI}\bar{y}\,ds + \int_0^s \frac{N_0 + H\cos\theta}{EA}\cos\theta\,ds = 0 \quad \text{— (8)}$$

Solving for H, yields.

$$-\int_0^s \frac{M_0}{EI}\bar{y}\,ds + \int_0^s \frac{H\bar{y}^2}{EI}ds + \int_0^s \frac{N_0}{EA}\cos\theta\,ds + \int_0^s \frac{H\cos^2\theta}{EA}ds = 0$$

$$H = \frac{\int_0^s \frac{M_0}{EI}\bar{y}\,ds - \int_0^s \frac{N_0}{EA}\cos\theta\,ds}{\int_0^s \frac{\bar{y}^2}{EI}ds + \int_0^s \frac{\cos^2\theta}{EA}ds} \quad \text{— (9)}$$

→ Using the above eq^n, the horizontal reaction 'H' for any two-hinged symmetrical arch may be calculated.

→ The above equation, there is valid for any general type of loading.

→ Usually the above equation is further simplified.

→ The second term in the numerator is small compared with the first terms and is neglected in the analysis.

→ Only in case of very accurate analysis second term considered.

→ Also for flat-arched,

$\cos\theta = 1$ as θ is small.

→ The eq^n (9) is

$$H = \frac{\int_0^s \frac{M_0}{EI}\bar{y}\,ds}{\int_0^s \frac{\bar{y}^2}{EI}ds + \int_0^s \frac{ds}{EA}} \quad \text{— (10)}$$

→ As axial rigidity is very high, the second term in the denominator may also be neglected.

→ Finally the horizontal reaction is calculated by the equation.

$$H = \frac{\int_0^s \frac{M_0}{EI}\bar{y}\,ds}{\int_0^s \frac{\bar{y}^2}{EI}\,ds} \quad \text{— (11)}$$

→ For an arch with uniform cross section EI as is constant and hence.

$$\boxed{H = \frac{\int_0^s M_0 \bar{y}\,ds}{\int_0^s \bar{y}^2\,ds}} \quad \text{— (12)}$$

→ In the above eq^n, M_0 is the bending moment at any cross section of the arch when one of the hinges is replaced by a roller support.

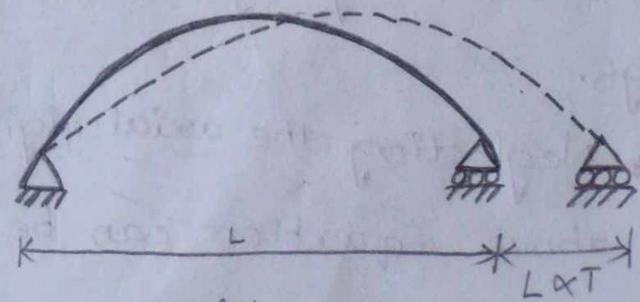
→ $\bar{y}$ is the height of the arch as shown in the figure.

→ If the moment of inertia of the arch rib is not constant, then eq^n (10) must be used to calculate the horizontal reaction 'H'.

Temperature effect:-

→ Consider an unloaded two-hinged arch of span L.

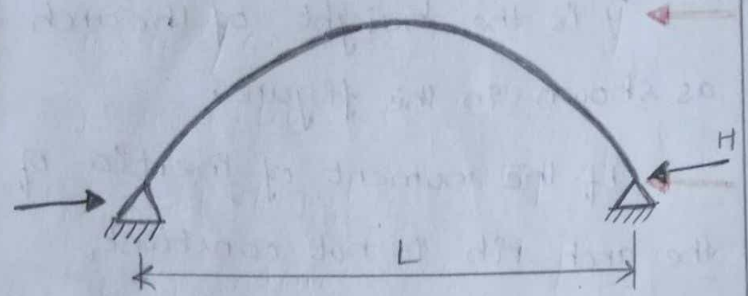
→ When the arch undergoes a uniform temperature change of $T^\circ C$, then its span would increase by αLT if it were allowed to expand freely. (fig(a))



(a)

α is the co-efficient of thermal expansion of the arch material.

→ Since the arch is restrained from the horizontal movement, a horizontal force is induced at the support as the temperature is increased.



→ Now applying the Castigliano's first theorem,

$$\frac{\partial U}{\partial H} = \alpha L T = \int_0^s \frac{H\bar{y}^2}{EI} ds + \int_0^s \frac{H\cos^2\theta}{EA} ds \quad \text{—— (1)}$$

solving for H,

$$H = \frac{\alpha L T}{\int_0^s \frac{\bar{y}^2}{EI} ds + \int_0^s \frac{\cos^2\theta}{EA} ds} \quad \text{—— (2)}$$

→ The second term in the denominator may be neglected, as the axial rigidity is quite high.

→ Neglecting the axial rigidity, the above equation can be written as

$$H = \frac{\alpha L T}{\int_0^s \frac{\bar{y}^2}{EI} ds} \quad \text{—— (3)}$$

[Now M_0 the bending moment at any cross section of the arch when one of the hinges is replaced by a roller support is given by,

$$M_0 = R_{ay} x = R_{ay} R(1-\cos\theta)$$

$$0 \le \theta \le \theta_c \quad \text{and,}$$

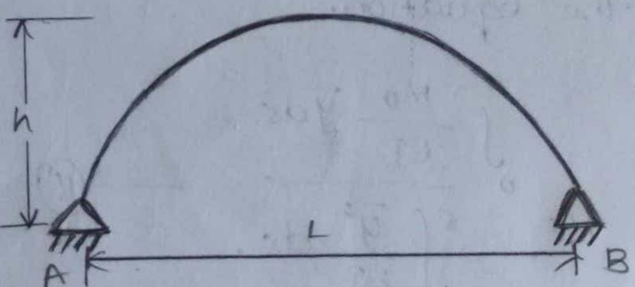
$$M_0 = R_{ay} R(1-\cos\theta) - 40(x-8)$$

$$= R_{ay} R(1-\cos\theta) - 40\{R(1-\cos\theta) - 8\}$$

$$\theta_c \le \theta \le \pi \quad \text{—— (4)}$$]

Secondary stresses in two hinged arches due to temperature.

Consider the two-hinged arch shown in figure.(a)



(a) Typical two-hinged arch.

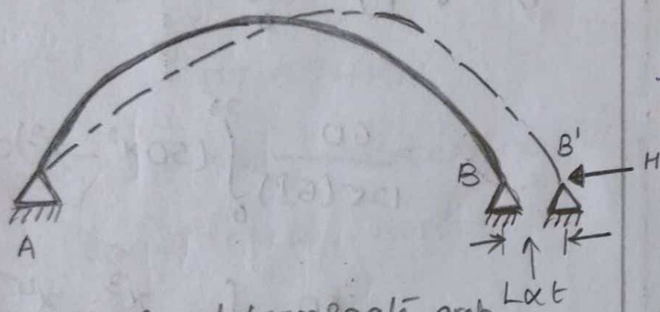
→ Let the temperature of the arch increase by 't' degrees.

→ If the hinge at 'B' is replaced by a roller as shown in figure (b), let end B move to the position B'. Then

$$\boxed{BB' = L\alpha t} \quad \text{--- (1)}$$

where, L - span of the arch

α - coefficient of thermal expansion



(b) Basic determinate arch.

→ Let H be the force required to bring back end B' to B, which infers that H is the horizontal thrust developed in the two-hinged arch.

Then, from castigliano's theorem,

$$\frac{dU}{dH} = L\alpha t$$

in this case,

$$M = -Hy$$

$$L\alpha t = \frac{dU}{dH} = \int_0^a \left(\frac{d}{dH}\right)\left(\frac{M^2}{2EI}\right) ds$$

$$= \int \left(\frac{M}{EI}\right)\left(\frac{dM}{dH}\right) ds$$

$$= \int -Hy(-y) \frac{ds}{EI}$$

$$= H \int y^2 \frac{ds}{EI}$$

$$H = \frac{L\alpha t}{\int y^2 \left(\frac{ds}{EI}\right)} \quad \text{--- (2)}$$

→ The above expression is for horizontal thrust due to temperature changes only.

→ If it is combined with loading and settlement of supports, total horizontal thrust,

$$H = \frac{\int M' y \left(\frac{ds}{EI}\right) + L\alpha t - \Delta}{\int y^2 \left(\frac{ds}{EI}\right)} \quad \text{--- (3)}$$

→ If effect of rib shortening is also to be accounted, then

$$H = \frac{\int M' y \left(\frac{ds}{EI}\right) + L\alpha t - \Delta}{\int y^2 \left(\frac{ds}{EI}\right) + \left(\frac{L}{EA_m}\right)} \quad \text{--- (4)}$$

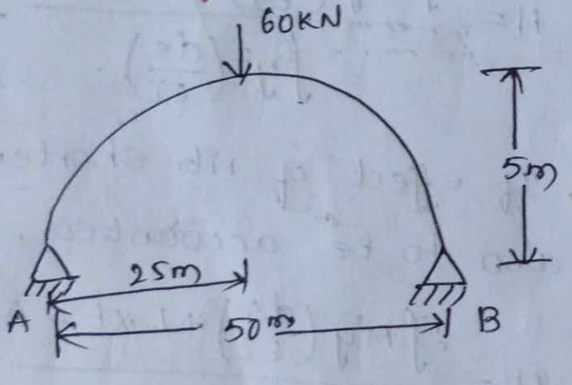
→ If it is an elastic support with yielding of 'K' per unit horizontal force, then,

$$H = \frac{\int M' y \left(\frac{ds}{EI}\right) + L\alpha t}{\int y^2 \left(\frac{ds}{EI}\right) + \left(\frac{L}{EA_m}\right) + K} \quad \text{--- (5)}$$

Problem ①: A two-hinged parabolic arch of span 50m and rise 5m is subjected to a central concentrated load of 60KN. It has an elastic support which yields by 0.0001 mm/KN. Taking $E = 200\ KN/mm^2$, $I = 5\times10^9 mm^4$, Average area $A_m = 10000\ mm^2$, $\alpha = 12\times10^{-6}/°C$ and assuming secant variation, calculate the horizontal thrust developed when the temperature rises by 20°C.

(i) neglecting rib shortening

(ii) considering rib shortening.



Sol: Taking left springing 'A' as the origin,

$$y = \frac{4hx(L-x)}{L^2}$$

$$= \frac{4(5)\times x(L-x)}{50^2}$$

$$= \frac{x(50-x)}{125}$$

$$\int_0^{50} y^2 dx = \int_0^{50} \frac{x^2(50-x)^2}{125^2}$$

$$= \int_0^{50}\left(\frac{2500x^2 - 100x^3 + x^4}{125^2}\right)dx$$

$$= \frac{1}{125^2}\left[2500\left(\frac{x^3}{3}\right) - 100\left(\frac{x^4}{4}\right) + \left(\frac{x^5}{5}\right)\right]_0^{50}$$

$$= 666.67$$

Beam moment is $M' = 30x$

$$\int M'y\frac{dx}{EI} = 2\int_0^{25} 30x\left(\frac{x}{125}\right)(50-x)\frac{dx}{EI}$$

$$= \frac{60}{125(EI)}\int_0^{25}(50x^2 - x^3)dx$$

$$= \frac{60}{125(EI)}\left[50\frac{x^3}{3} - \frac{x^4}{4}\right]_0^{25}$$

$$= \frac{78125}{EI}$$

converting E, I, A_m to the metre unit,

$$EI = 200\times10^6\times5\times10^9\times10^{-12}$$

$$= 1000000\ kNm^2$$

$$EA_m = 200\times10^6\times10000\times10^{-6}$$

$$= 2000000$$

$$\int M'y\left(\frac{dx}{EI}\right) = 0.078125$$

$$L\alpha t = 50\times12\times10^{-6}\times20$$

$$= 0.012$$

Yielding of support per unit thrust,

$$k = 0.0001$$

∴ neglecting the rib shortening, horizontal thrust developed is,

$$H = \frac{\int M'y\left(\frac{dx}{EI}\right) + L\alpha t}{\int y^2\left(\frac{ds}{EI}\right) + 0 + K}$$

$$= \frac{0.078125 + 0.012}{\left(\frac{666.67}{1000000}\right) + 0.0001}$$

$$= \frac{0.090125}{7.6667 \times 10^{-4}}$$

$$= 117.554\,KN$$

→ If the rib shortening is also considered, horizontal thrust is given by

$$H = \frac{\int M'y\left(\frac{ds}{EI}\right) + L\alpha t}{\int y^2\left(\frac{ds}{EI}\right) + \left(\frac{L}{EA_m}\right) + K}$$

$$= \frac{0.078125 + 0.012}{7.6667 \times 10^{-4} + \left(\frac{50}{2000000}\right)}$$

$$= \frac{0.090125}{7.6667 \times 10^{-4} + \left(\frac{50}{(2000000)}\right)}$$

$$= \frac{0.090125}{7.9167 \times 10^{-4}}$$

$$= 113.842\,KN.$$

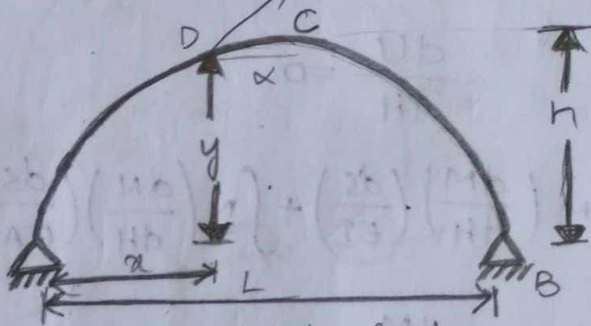
Effect of shortening of Rib:-

The cross-section of the arch is subjected to normal thrust also.

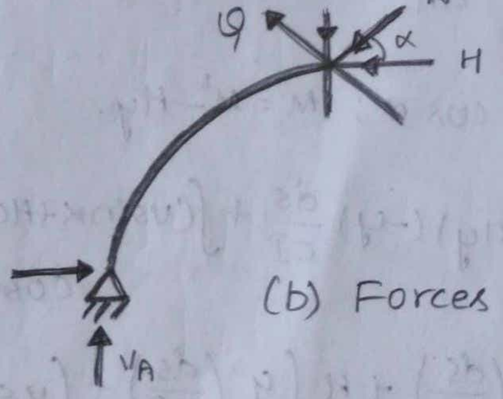
→ The arch, being made up of elastic material, shortening of the rib takes place.

→ This shortening reduces the horizontal thrust developed.

→ Now, we aim at getting an expression for the horizontal thrust, if rib shortening is considered.



(a) typical two-hinged arch



(b) Forces at D.

→ Referring to figure, normal thrust on section x is

$$\boxed{N = V\sin\alpha + H\cos\alpha}$$

$$M = M' - Hy$$

where V = beam shear

α = slope of arch at 'x'.

→ Considering strain energy due to bending as well as normal thrust, the strain energy is given by

$$U = \int \frac{M^2 ds}{2EI} + \int \frac{N^2}{2EI} ds$$

where, A – cross sectional area at 'α'.

→ If the support is unyielding, the horizontal displacement of the arch is zero.

$$\frac{dU}{dH} = 0$$

$$\int M\left(\frac{dM}{dH}\right)\left(\frac{ds}{EI}\right) + \int N\left(\frac{dN}{dH}\right)\left(\frac{ds}{EA}\right) = 0$$

but $\frac{dM}{dH} = -y$.

$\frac{dN}{dH} = \cos\alpha$; $M = M' - Hy$.

$$\therefore \int (M' - Hy)(-y)\frac{ds}{EI} + \int (V\sin\alpha + H\cos\alpha)\cos\alpha \frac{ds}{EA} = 0$$

$$\boxed{-\int M'y\left(\frac{ds}{EI}\right) + H\int y^2\left(\frac{ds}{EI}\right) + \int V\sin\alpha\cos\alpha\left(\frac{ds}{EA}\right) + H\int \cos^2\alpha\left(\frac{ds}{EA}\right) = 0}$$

$$H = \frac{\int M'y\left(\frac{ds}{EA}\right) - \int V\sin\alpha\cos\alpha\left(\frac{ds}{EA}\right)}{\int y^2\left(\frac{ds}{EI}\right) + \int \cos^2\alpha\left(\frac{ds}{EA}\right)}$$

Neglecting the effect of shear

$$H = \frac{\int M'y\left(\frac{ds}{EI}\right)}{\int y^2\left(\frac{ds}{EI}\right) + \int \cos^2\alpha\left(\frac{ds}{EA}\right)}$$

→ Now, consider the term $\int \cos^2\alpha\left(\frac{ds}{EA}\right)$.

→ At the crown, $\cos\alpha = 0$ and at the springing it has some definite value.

→ Usually cross-sectional area is small at the crown and large at the springing.

→ Hence, assuming $\frac{A}{\cos\alpha}$ as a constant, say A_m,

$$\int \cos^2\alpha\left(\frac{ds}{EA}\right) = \int \frac{\cos\alpha\, ds}{EA_m}$$

$$= \int_0^L \frac{dx}{EA_m} = \frac{L}{EA_m}$$

where, L – span of the arch.

$$\therefore H = \frac{\int M'y\left(\frac{ds}{EI}\right)}{\int y^2\left(\frac{ds}{EI}\right) + \frac{L}{EA_m}} \quad \text{—— (1)}$$

→ If both yielding of support and rib shortening are to be considered.

$$H = \frac{\int M'y\left(\frac{ds}{EI}\right) - \Delta}{\int y^2\left(\frac{ds}{EI}\right) + \left(\frac{L}{EA_m}\right)}$$

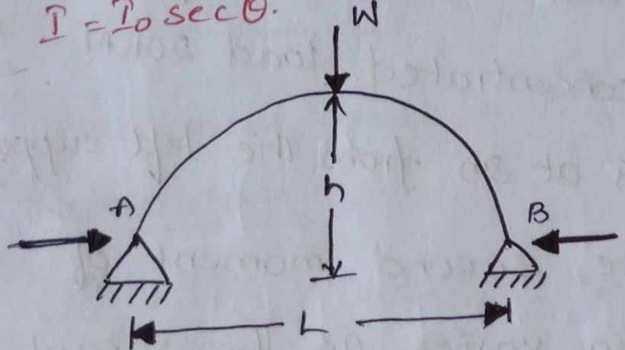
$$H = \frac{\int M' y \left(\frac{ds}{EI}\right) - KH}{\int y^2 \left(\frac{ds}{EI}\right) + \left(\frac{L}{EA_m}\right)}$$

$$H = \frac{\int M' y \left(\frac{ds}{EI}\right)}{\int y^2\left(\frac{ds}{EI}\right) + \frac{L}{EA_m} + K} \quad \text{---(2)}$$

where,

K = yielding of support per unit horizontal force.

Problem no. ②:- Derive horizontal thrust for two hinged arch with centrally applied point load. Assume $I = I_0 \sec\theta$.



Sol: In this arch

L = span

h = Rise

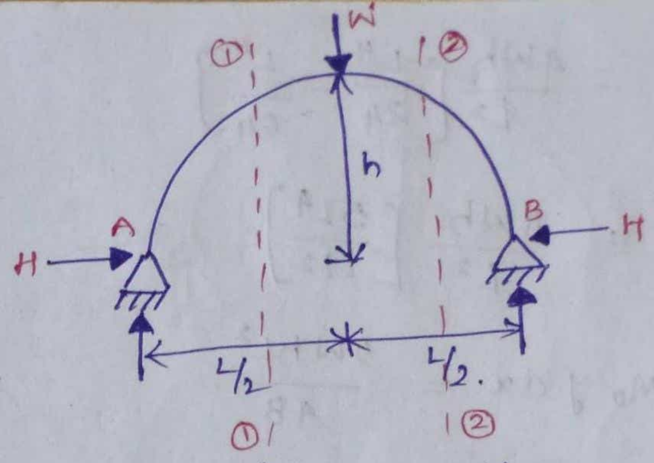
H = Horizontal Thrust

⇒ As the arch is symmetrical

the $V_A = V_B = \frac{W}{2}$

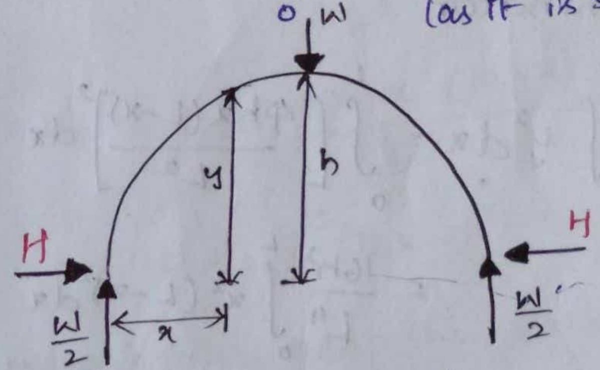
Horizontal thrust, H

$$H = \frac{\int M_0 y\, dx}{\int y^2 dx}$$



$$\int M_0 y\, dx = \underbrace{\int_0^{L/2} M_{01} y_1\, dx}_{1^{st}\ section} + \underbrace{\int^{L/2} M_{02} y_2\, dx}_{2^{nd}\ section}$$

$$= 2\int_0^{L/2} M_{01}\, y_1\, dx \quad \text{(as it is symmetrical)}$$



R.H.S ↻ +ve ↺ -ve.

$$M_{01} = \left(\frac{W}{2}\right)x$$

$$y = \frac{4hx(L-x)}{L^2}$$

$$\int M_0 y\, dx = 2\int_0^{L/2} M_{01} y_1\, dx$$

$$= \cancel{2}\int_0^{L/2} \left(\frac{W}{\cancel{2}}\right) x \cdot \frac{4hx(L-x)}{L^2} dx$$

$$= \frac{4Wh}{L^2}\int_0^{L/2} (Lx^2 - x^3) dx$$

$$= \frac{4Wh}{L^2}\left[\frac{Lx^3}{3} - \frac{x^4}{4}\right]_0^{L/2}$$

$$= \frac{4Wh}{L^2}\left[\frac{L^4}{24} - \frac{L^4}{64}\right]$$

$$= \frac{4Wh}{L^2}\left[\frac{5L^4}{192}\right]$$

$$\int M_0\, y\, dx = \frac{5WhL^2}{48}$$

→ In horizontal thrust we have calculated Numerator now will calculate the denominator.

$$\int y^2 dx = \int_0^L \left[\frac{4hx(L-x)}{L^2}\right]^2 dx$$

$$= \frac{16h^2}{L^4}\int_0^L x^2 (L-x)^2 dx$$

$(a-b)^2 = a^2 - 2ab + b^2$

$$= \cancel{\frac{16h^2}{L^4}\left[\frac{L^5}{30}\right]}$$

$$= \frac{16h^2}{L^4}\int_0^L x^2(L^2 - 2Lx + x^2)dx$$

$$= \frac{16h^2}{L^4}\int_0^L (L^2x^2 - 2Lx^3 + x^4)dx$$

$$= \frac{16h^2}{L^4}\left[\frac{L^2x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5}\right]_0^L$$

$$= \frac{16h^2}{L^4}\left[\frac{L^5}{3} - \frac{L^5}{2} + \frac{L^5}{5}\right]$$

$$= \frac{16h^2}{L^4}\left[\frac{L^5}{30}\right]$$

$$\int_0^L y^2 dx = \frac{8Lh^2}{15}$$

Horizontal thrust,

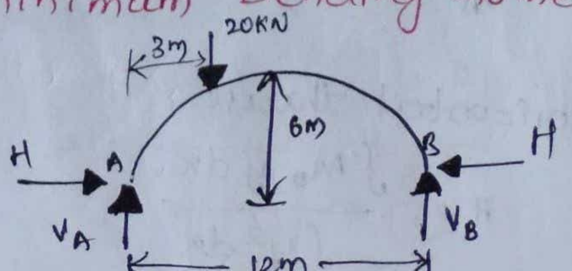
$$H = \frac{\int M_0\, y\, dx}{\int y^2 dx}$$

$$= \frac{\frac{5WhL^2}{48}}{\frac{8Lh^2}{15}}$$

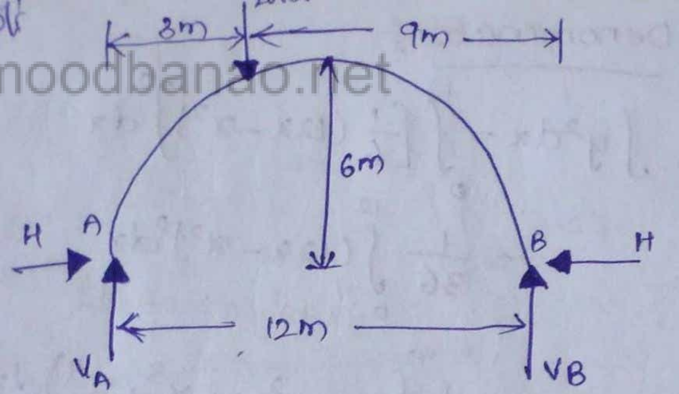
$$= \frac{5WhL^2}{48} \times \frac{15}{8Lh^2}$$

$$\boxed{H = \frac{25}{128}\left(\frac{WL}{h}\right)}$$

Problem no. 3 :- A two hinged parabolic arch has a span of 12m and a rise of 6m. A concentrated load 20KN acts at 3m from the left support. The second moment of area varies as the secant of the inclination of the arch axis. Calculate the reactions and horizontal thrust at the hinge. Also calculate the maximum and minimum bending moments.



Soln



R.H.S $\circlearrowright$ +ve $\circlearrowleft$ -ve.

Take moment about B

$\circlearrowright$ +ve $\circlearrowleft$ -ve

$12V_A - 20\times 9 = 0$

$V_A = \frac{180}{12}$

$\boxed{V_A = 15KN}$

$V_A + V_B = 20KN$

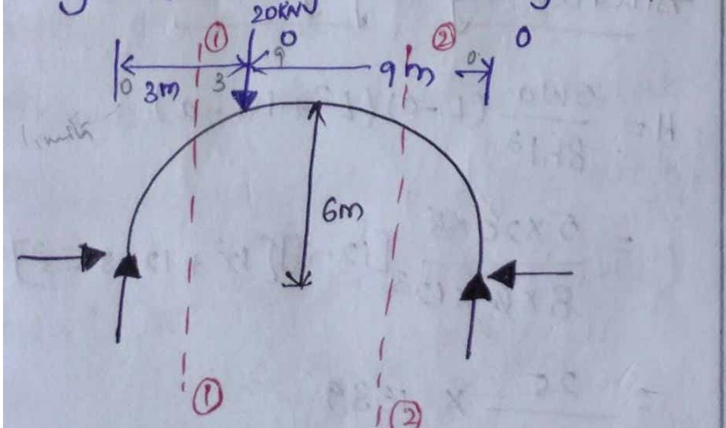
$V_B = 20 - 15$

$\boxed{V_B = 5KN}$

Horizontal thrust, $H = \frac{\int M_0 y\, dx}{\int y^2 dx}$

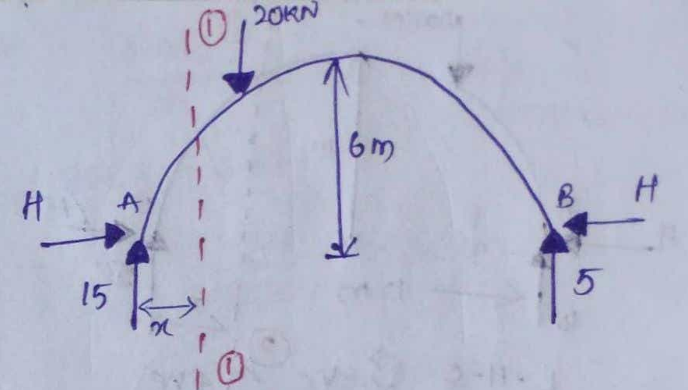
Numerator:-

$\int M_0 y\, dx = \int_0^3 M_{01} y_1\, dx + \int_0^9 M_{02} y_2\, dx$



→ As we are having two sections.

first we will consider one section.



$$\int_0^3 M_{01} y_1\, dx = \int_0^3 15x \times \frac{1}{6}(12x - x^2)\, dx$$

$M_{01} = 15x$

$$y = \frac{4hx(L-x)}{L^2}$$

$$= \frac{4\times 6x\,(12-x)}{12^2}$$

$$y = \frac{1}{6}(12x - x^2)$$

Now will keep the values of 'M_{01}' & 'y'

$$\int_0^3 M_{01} y_1\, dx = \int_0^3 15x \times \frac{1}{6}(12x - x^2)\, dx$$

$$= \frac{15}{6}\int_0^3 x(12x - x^2)\, dx$$

2.5

$$= 2.5\int_0^3 (12x^2 - x^3)\, dx$$

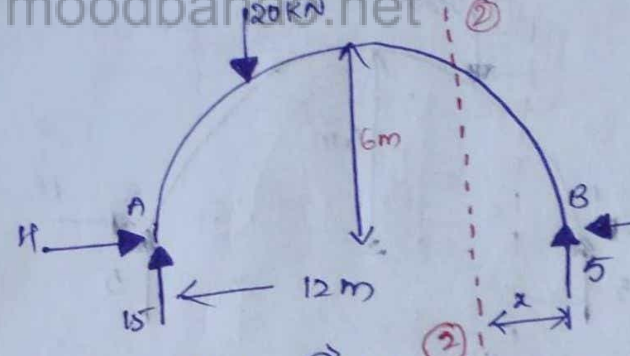
$\left\{ \int x^n dx = \frac{x^{n+1}}{n+1} \right\}$

$$= 2.5\left[\frac{12x^3}{3} - \frac{x^4}{4}\right]_0^3$$

$$= 2.5\left[\frac{12\times 3^3}{3} - \frac{3^4}{4}\right]$$

$$\int_0^3 M_{01} y_1\, dx = 219.375$$

Now will take the second section.



L.H.S ↻ -ve ↺ +ve

$M_{02} = 5x$

$y = \frac{4hx(L-x)}{L^2}$

$= \frac{4\times 6x(12-x)}{12^2}$

$y = \frac{1}{6}(12x-x^2)$

$\int_0^9 M_{02} y_2 \, dx = \int_0^9 5x \times \frac{1}{6}(12x-x^2)\,dx$

$= \frac{5}{6}\int_0^9 x(12x-x^2)\,dx$

$= \frac{5}{6}\int_0^9 (12x^2-x^3)\,dx$

$\left\{\int x^n dx = \frac{x^{n+1}}{n+1}\right\}$

$= \frac{5}{6}\left[\frac{12x^3}{3} - \frac{x^4}{4}\right]_0^9$

$= \frac{5}{6}\left[\frac{12\times 9^3}{3} - \frac{9^4}{4}\right]$

$\int_0^9 M_{02} y_2 \, dx = 1063.125$

Now we need to add two sections.

$\int M_0 y \, dx = \int_0^3 M_{01} y_1 \, dx + \int_0^9 M_{02} y_2 \, dx$

$= 219.375 + 1063.125$

$\int M_0 y \, dx = 1282.5$

Denominator:-

$\int y^2 dx = \int_0^{12} \left[\frac{1}{6}(12x-x^2)\right]^2 dx$

$= \frac{1}{36}\int_0^{12} (12x-x^2)^2 dx$

$(a-b)^2 = a^2-2ab+b^2$

$= \frac{1}{36}\int_0^{12} (144x^2-24x^3+x^4)\,dx$

$\left\{\int x^n dx = \frac{x^{n+1}}{n+1}\right\}$

$= \frac{1}{36}\left[\frac{144x^3}{3} - \frac{24x^4}{4} + \frac{x^5}{5}\right]_0^{12}$

$= \frac{1}{36}\left[\frac{144\times 12^3}{3} - \frac{24\times 12^4}{4} + \frac{12^5}{5}\right]$

$\int y^2 dx = 230.4$

Now, horizontal thrust,

$H = \frac{\int M_0 y \, dx}{\int y^2 dx}$

$= \frac{1282.5}{230.4}$

$\boxed{H = 5.57 \text{ KN}}$

Alteratively by formula

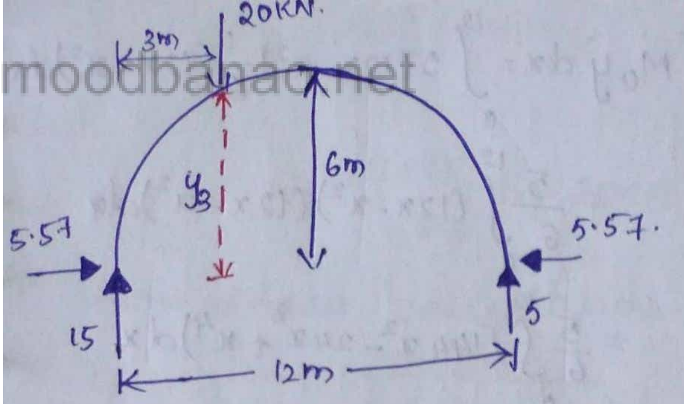
$H = \frac{5Wa}{8hL^3}(L-a)(L^2+La-a^2)$

$= \frac{5\times 20\times 3}{8\times 6\times 12^3}[12-3][12^2+12\times 3-3^2]$

$= \frac{25}{6912}\times 1539$

$\boxed{H = 5.57 \text{ KN}}$

The maximum positive Bending moment occurs under the point load.



→ We have already an equation to calculate 'y'

so,

$$y = \frac{1}{6}(12x - x^2)$$

$$y_3 = \frac{1}{6}(12\times 3 - 3^2)$$

$$= 4.5m$$

→ Now will take the BM from 'A'

R.H.S ↻ +ve ↺ −ve.

Max^m +ve BM

$$= 15\times 3 - 5.57\times 4.5$$

$$= 19.935 \text{ KNm.}$$

The maximum negative Bending moment occurs at the right side of the arch

$$M_x = 5x - 5.57\times\frac{1}{6}(12x - x^2)$$

L.H.S ↻ −ve ↺ +ve.

$$= 5x - 11.14x + 0.928x^2$$

$$M_x = 0.928x^2 - 6.14x$$

To find the value of 'x'

For maximum negative BM,

$$\frac{dM_x}{dx} = 0 \text{ (or)}$$

In calculator d/dx ⇒ $2(0.928)x - 6.14 = 0$ $x = 3.30m$

$$1.856x - 6.14 = 0$$

$$x = \frac{6.14}{1.856} \Rightarrow 3.31m.$$

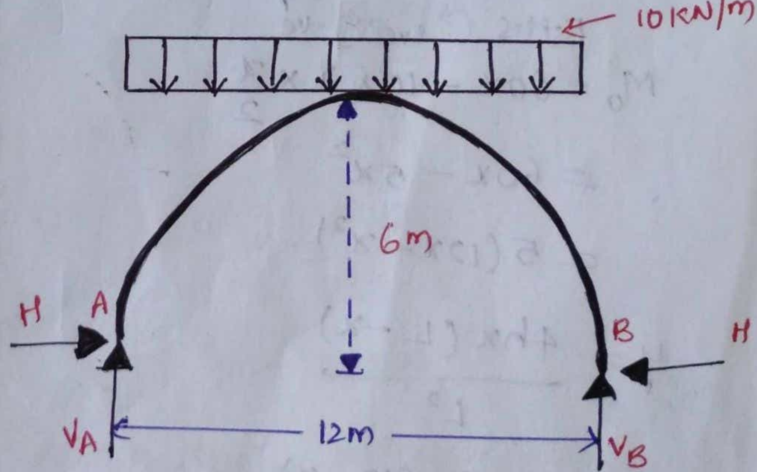
maximum negative BM

$$= 0.928\times 3.31^2 - 6.14\times 3.31$$

$$= -10.15 \text{ KNm.}$$

Problem no. ④ :- A two hinged parabolic arch with span of 12m carries a UDL of 10KN/m over the entire span. The rise of the arch is 6m. Calculate the horizontal thrust. Also find the normal thrust and radial shear at 3m from the left support.

Sol^r

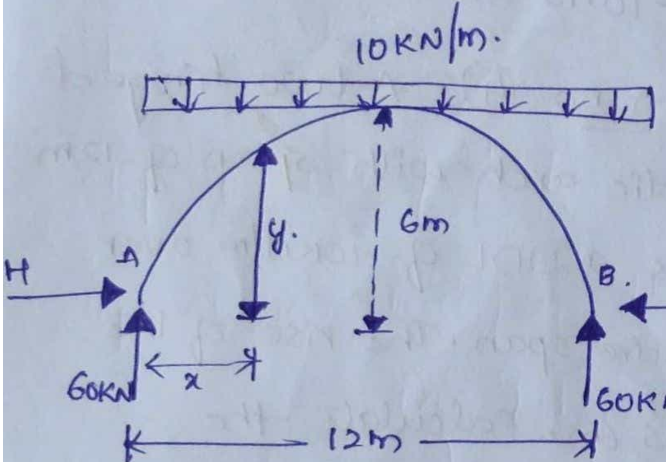


$$V_A = V_B = \frac{10\times 12}{2} = 60KN$$

Horizontal thrust $H = \dfrac{\int M_o y\,dx}{\int y^2 dx}$

Initially will make arch into two sections, as UDL is acting on entire span and it is symmetric, so calculation of one section is enough.

→ Section can be made anywhere on the arch.



→ Now will find Mo, where in Mo calculation 'H' is not considered (or required.

R.H.S ↻ +ve ↺ -ve

$M_0 = 60x - 10 \times x \times \frac{x}{2}$

$= 60x - 5x^2$

$= 5(12x - x^2)$

$y = \frac{4hx(L-x)}{L^2}$

$= \frac{4 \times 6x(12-x)}{12^2}$

$y = \frac{1}{6}(12x - x^2)$

→ Now will place the 'Mo' & 'y' values in the equation & Integrating them.

$\int M_0 y\, dx = \int_0^{12} 5(12x-x^2)\frac{1}{6}(12x-x^2)dx$

$= \frac{5}{6}\int_0^{12}(12x-x^2)(12x-x^2)dx$

$= \frac{5}{6}\int_0^{12}[144x^2 - 24x^3 + x^4)dx$

$= \frac{5}{6}\left[\frac{144x^3}{3} - \frac{24x^4}{4} + \frac{x^5}{5}\right]_0^{12}$ ⓪ not required

$= \frac{5}{6}[48 \times 12^3 - 6 \times 12^4 + 0.2 \times 12^5]$

$\int M_0 y\, dx = 6912$

Denominator $\int y^2 dx$

$\int y^2 dx = \int_0^{12}\left[\frac{1}{6}\,12x - x^2\right]dx$

$= \frac{1}{36}\int_0^{12}(12x-x^2)^2 dx$

$= \frac{1}{36}\int_0^{12}[144x^2 - 24x^3 + x^4]dx$

$= \frac{1}{36}\left[\frac{144x^3}{3} - \frac{24x^4}{4} + \frac{x^5}{5}\right]_0^{12}$

$= \frac{1}{36}[48 \times 12^3 - 6 \times 12^4 + 0.2 \times 12^5]$

$\int y^2 dx = 230.4$

Horizontal thrust,

$H = \frac{\int M_0 y\, dx}{\int y^2 dx}$

$= \frac{6914}{230.4}$

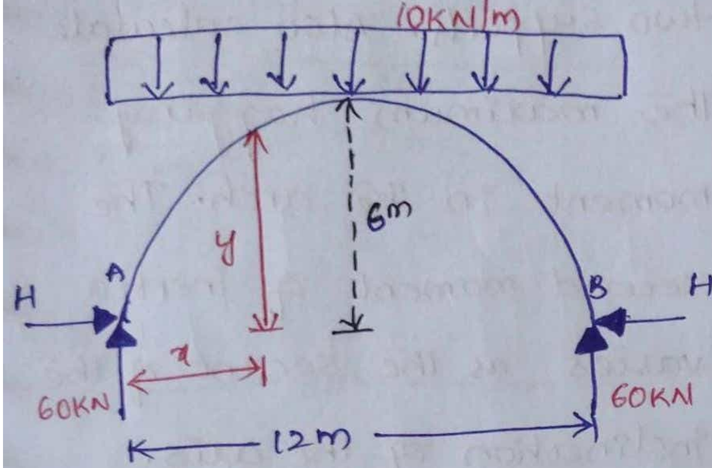
$H = 30KN$

Alternatively using the formula,

$H = \frac{WL^2}{8h}$

$= \frac{10 \times 12^2}{8 \times 6} \Rightarrow 30KN$

Denominator calculation.



$\int M_0 y dx = \int_0^{12} 5(12x - x^2) \frac{1}{6}(12x - x^2) dx$

$M_0 = 60x - 10 \times x \times \frac{x}{2}$

$= 60x - 5x^2$

$= 5(12x - x^2)$

$y = \frac{4hx(L-x)}{L^2}$

$= \frac{4 \times 6x(12-x)}{12^2}$

$y = \frac{1}{6}(12x - x^2)$

$\int M_0 y dx = \int_0^{12} 5(12x - x^2)\frac{1}{6}(12x - x^2) dx$

$= \frac{5}{6}\int_0^{12} (12x - x^2)(12x - x^2) dx$

$= \frac{5}{6}\int_0^{12} [144x^2 - 24x^3 + x^4] dx$

$= \frac{5}{6}\left[\frac{144}{3}x^3 - \frac{24x^4}{4} + \frac{x^5}{5}\right]_0^{12}$

$= \frac{5}{6}[48 \times 12^3 - 6 \times 12^4 + 0.2 \times 12^5]$

$\int M_0 y dx = 6912$

Horizontal thrust, $H = \frac{\int M_0 \cdot y dx}{\int y^2 dx}$

$\int y^2 dx = \int_0^{12} \left[\frac{1}{6}(12x - x^2)\right]^2 dx$

$= \frac{1}{36}\int_0^{12} (12x - x^2)^2 dx$

$= \frac{1}{36}\int_0^{12} [144x^2 - 24x^3 + x^4] dx$

$= \frac{1}{36}\left[\frac{144x^3}{3} - \frac{24x^4}{4} + \frac{x^5}{5}\right]_0^{12}$

$= \frac{1}{36}[48 \times 12^3 - 6 \times 12^4 + 0.2 \times 12^5]$

$= 230.4$

Horizontal thrust

$H = \frac{\int M_0 y dx}{\int y^2 dx}$

$= \frac{6912}{230.4}$

$H = 30KN$

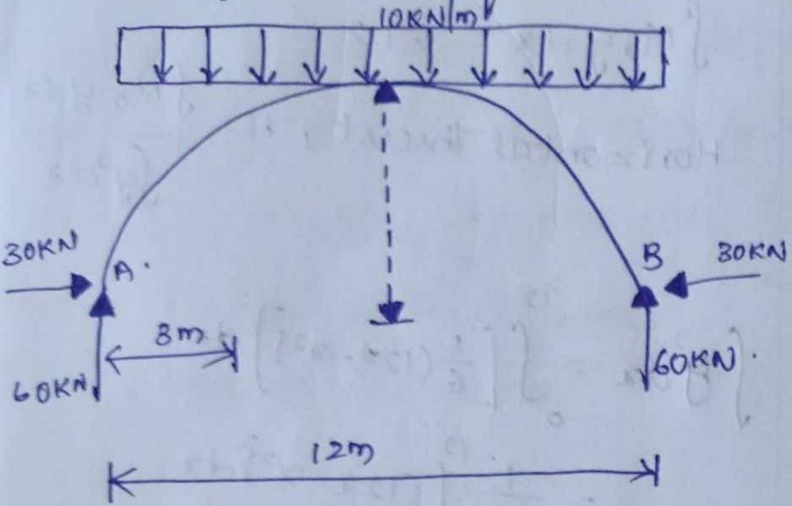
Alternatively using the formula,

$H = \frac{WL^2}{8h}$

$= \frac{10 \times 12^2}{8 \times 6}$

$= 30KN$

Normal thrust and radial shear at 3m from the left support



slope $\theta = \tan^{-1}\left[\frac{4h}{L^2}[L-2x]\right]$

Slope at 3m $= \tan^{-1}\left[\frac{4\times6}{12^2}[12-2\times3]\right]$

$= \tan^{-1}[1]$

$\boxed{\theta = 45}$

R.H.S ↑ +ve ↓ -ve

$V_x = 60 - 10\times3$

$\boxed{V_x = 30KN}$

Normal Thrust $N_x = V_x \sin\theta + H\cos\theta$

$= 30\sin45 + 30\cos45$

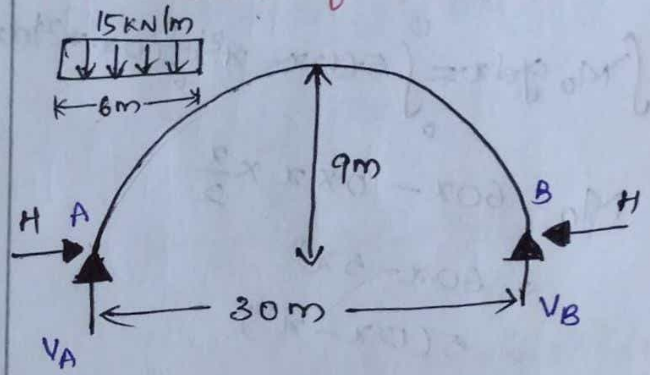
$\boxed{N_x = 42.43\ KN}$

Radial shear $R_x = V_x\cos\theta - H\sin\theta$

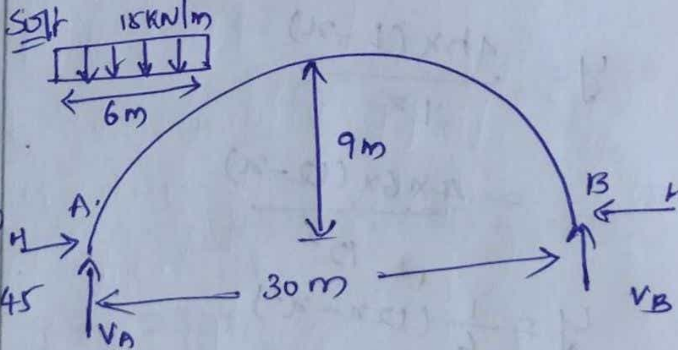
$= 30\cos45 - 30\sin45$

$\boxed{R_x = 0}$

Problem no. ⑤ :- A two hinged parabolic arch of span 30m and rise 9m carries a UDL of 15KN/m covering a distance of 6m from left end. Find the horizontal reaction and vertical reactions at the two supports. Also calculate the maximum hogging moment in the arch. The second moment of inertia varies as the secant of the inclination of the axis.



Soln



Take moment about A.

L.H.S ↻ -ve ↺ +ve

$30V_B - 15\times6\times\frac{6}{2} = 0$

$30V_B - 270 = 0$

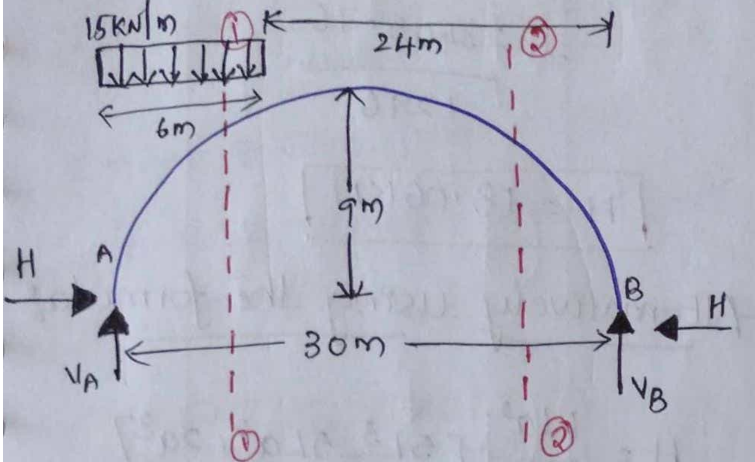
$V_B = \frac{270}{30}$

$\boxed{V_B = 9KN}$

Total load $= 15\times6 = 90KN$

$V_A = 90 - 9$; $\boxed{V_A = 81KN}$

→ To find horizontal thrust we need to make the structure into two sections, till UDL 1-section remaining another section into 2 parts as shown



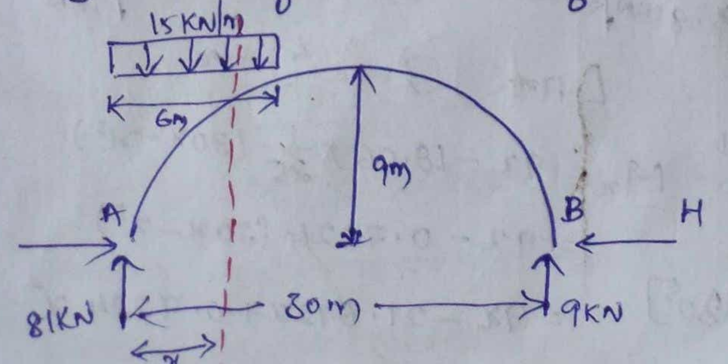
Horizontal thrust, $H = \dfrac{\int M_0 y\,dx}{\int y^2 dx}$

in which we will calculate the Numerator first.

→ In Numerator we will consider two sections with two integrations.

→ For the first integration we need consider 1st section & for 2nd integration 2nd section.

$$\int M_0 y\,dx = \int_0^6 M_{01} y_1\,dx + \int_0^{24} M_{02} y_2\,dx$$



R.H.S ↻ +ve ↺ -ve.

$$M_{01} = 81x - 15 \times x \times \frac{x}{2}$$

$$= 81x - 7.5x^2$$

$$= 7.5(10.8x - x^2)$$

$$y = \frac{4hx(L-x)}{L^2}$$

$$= \frac{4 \times 9x(30-x)}{30^2}$$

$$= \frac{1}{25}(30x - x^2)$$

→ Now will keep these two values in integration.

$$\int M_{01} y_1\,dx = \int_0^6 7.5(10.8x - x^2) \times \frac{1}{25}(30x - x^2)\,dx$$

$$= 0.3 \int_0^6 (10.8x - x^2)(30x - x^2)\,dx$$

$$= 0.3 \int_0^6 [324x^2 - 10.8x^3 - 30x^3 + x^4]\,dx$$

$$= 0.3 \int_0^6 [324x^2 - 40.8x^3 + x^4]\,dx$$

$$\left\{ \int x^n dx = \frac{x^{n+1}}{n+1} \right\}$$

$$= 0.3 \int_0^6 [324x^2 - 40.8x^3 + x^4]\,dx$$

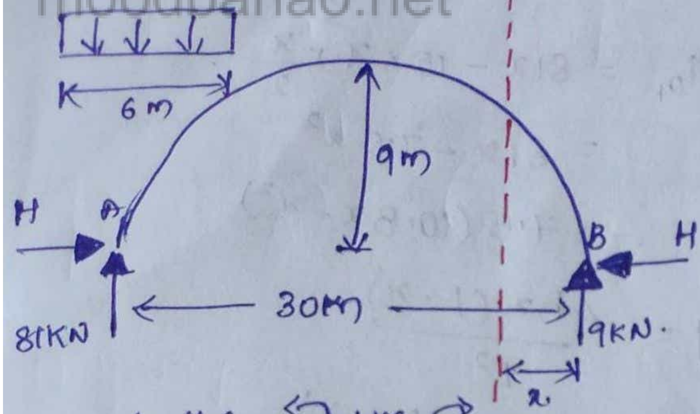
$$= 0.3 \left[\frac{324x^3}{3} - \frac{40.8x^4}{4} + \frac{x^5}{5} \right]_0^6$$

$$= 0.3 [108 \times 6^3 - 10.2 \times 6^4 + 0.2 \times 6^5]$$

$$= 0.3 [11664]$$

$$\int M_{01} y_1\,dx = 3499.2$$

→ Now Let us make the second integration.



L.H.S ↺ +ve ↻ −ve

$M_{O_2} = 9x$

$y = \frac{1}{25}(30x - x^2)$

$$\int M_{O_2} y_2\, dx = \int_0^{24} 9x \times \frac{1}{25}(30x - x^2)\, dx$$

$$= \frac{9}{25}\int_0^{24}(30x^2 - x^3)\, dx$$

$$= \frac{9}{25}\left[\frac{30x^3}{3} + \frac{x^4}{4}\right]_0^{24}$$

$$= \frac{9}{25}\left[\frac{30 \times 24^3}{3} - \frac{24^4}{4}\right]$$

$$\int M_{O_2} y_2\, dx = 19906.56$$

$$\int M_O y\, dx = \int_0^6 M_{O_1} y_1\, dx + \int_0^{24} M_{O_2} y_2\, dx$$

$$= 3499.2 + 19906.56$$

$$\boxed{\int M_O y\, dx = 23405.76}$$

Denominator

$$\int y^2 dx = \int_0^{30}\left[\frac{1}{25}(30x - x^2)\right]^2 dx$$

$$= \frac{1}{625}\int_0^{30}(30x - x^2)^2\, dx$$

$(a-b)^2 = a^2 + b^2 - 2ab$

$$= \frac{1}{625}\int_0^{30}\left[900x^2 - 60x^3 + x^4\right] dx$$

$\left(\int x^n dx = \frac{x^{n+1}}{n+1}\right)$

$$= \frac{1}{625}\left[\frac{900x^3}{3} - \frac{60x^4}{4} + \frac{x^5}{5}\right]_0^{30}$$

$$= \frac{1}{625}\left[300 \times 30^3 - 15 \times 30^4 + 0.2 \times 30^5\right]$$

$$\int y^2 dx = 1296$$

Horizontal thrust,

$$H = \frac{\int M_O y\, dx}{\int y^2 dx}$$

$$= \frac{23405.76}{1296}$$

$$\boxed{H = 18.06\,KN}$$

Alternatively using the formula:

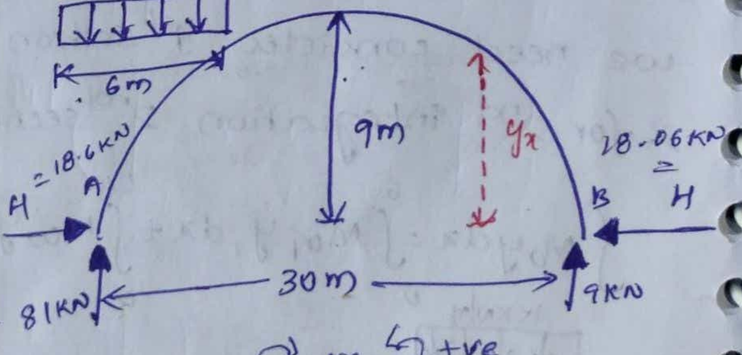
$$H = \frac{Wa^2}{16hL^3}\left[5L^3 - 5La^2 + 2a^3\right]$$

$$= \frac{15 \times 6^2}{16 \times 9 \times 30^3}\left[5 \times 30^3 - 5 \times 30 \times 6^2 + 2 + 6^3\right]$$

$$= \frac{1}{7200} \times 130032$$

$$\boxed{H = 18.06\,KN}$$

The maximum hogging (negative) BM occurs at the right side of the arch.



L.H.S ↻ −ve ↺ +ve

$$M_x = 9x - 18.06 \times \frac{1}{25}(30x - x^2)$$

$$= 9x - 0.7224(30x - x^2)$$

$$= 9x - 21.672x + 0.7224x^2$$

$M_x = 0.7224x^2 - 12.67x$.

For maximum hogging (-ve) BM, $\frac{dM_x}{dx} = 0$.

$1.4448x - 12.672 = 0$

$x = \frac{12.672}{1.4448} \Rightarrow \boxed{8.77m = x}$

BM at 8.77m from right

$= 0.7224 \times 8.77^2 - 12.672 \times 8.77$

$= -55.57$ kNm.

MOMENT DISTRIBUTION METHOD

Analysis of continuous beams with and without settlement of supports

→ Moment distribution is a convenient method to deal with certain types of settlement problems.

→ Settlement problems require fixed end moment due to lateral movement.

Basic concepts:-

- In MDM, counter clockwise beam end moments are taken as positive.
- The counterclockwise beam end moments produce clockwise moments on the joint.

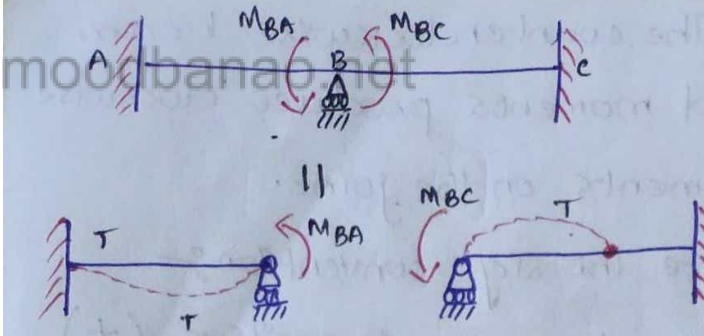
Note the sign convention:-

Anti-clockwise is positive (+)

clockwise is negative (−)

Assumptions in moment distribution method:-

- All the members of the structures are assumed to be fixed & FEM due to external loads are obtained.
- All the hinged joints are released by applying an equal and opposite moment.
- The joints are allowed to deflect (rotate) one after the other by releasing them successively.
- The unbalanced moment at the joint is shared by the members connected at the joint when it is released.
- The unbalanced moment at a joint is distributed into the two spans with their distribution factor.



■ Hardy cross method makes use of the ability of various structural members at a joint to sustain moments in proportional to their relative stiffness.

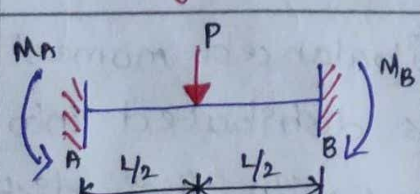
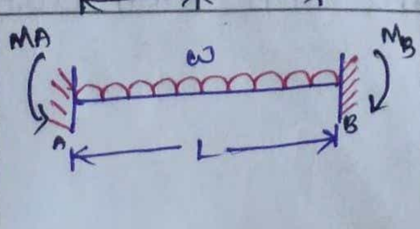
Fixed End Moments:-

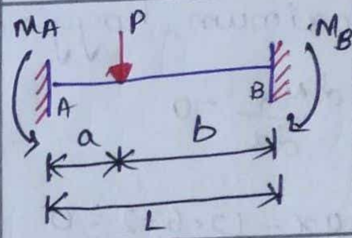
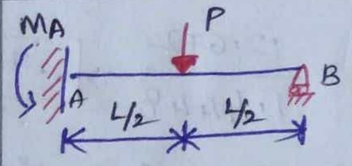
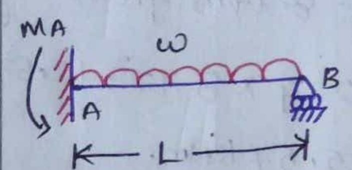
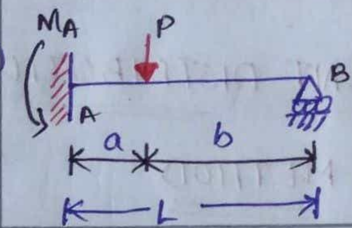
■ All members of a given frame are initially assumed fixed at both ends.

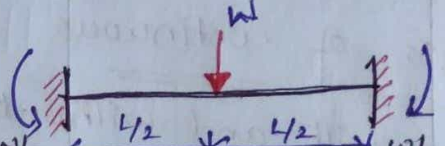
■ The loads acting on these fixed beams produce fixed end moments at the ends.

■ FEM are the moments exerted by the supports on the beam ends.

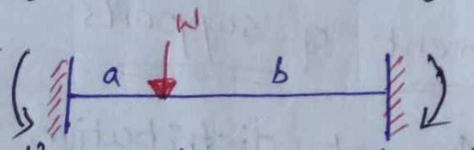
■ These (non-existent) moments keep the rotations at the ends of each member zero.

M_A	Configuration	M_B
$+\frac{PL}{8}$		$-\frac{PL}{8}$
$+\frac{wL^2}{12}$		$-\frac{wL^2}{12}$

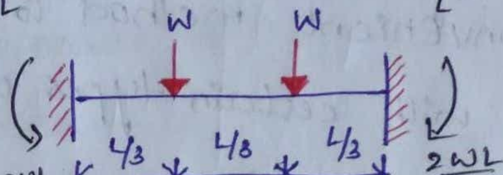
M_A	Configuration	M_B
$+\frac{Pab^2}{L^2}$		$-\frac{Pab}{L^2}$
$+\frac{3PL}{16}$		–
$+\frac{wL^2}{8}$		–
$+\frac{Pab(2L-a)}{2L^2}$		–

1


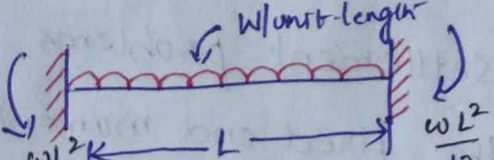
$\frac{WL}{8}$ $\frac{WL}{8}$

2


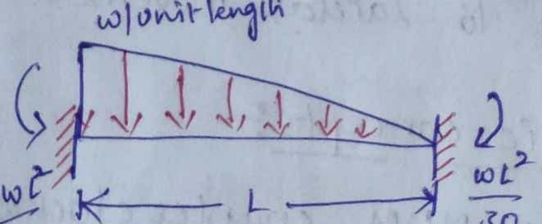
$\frac{wab^2}{L^2}$ $\frac{wa^2b}{L^2}$

3


$\frac{2WL}{9}$ $\frac{2WL}{9}$

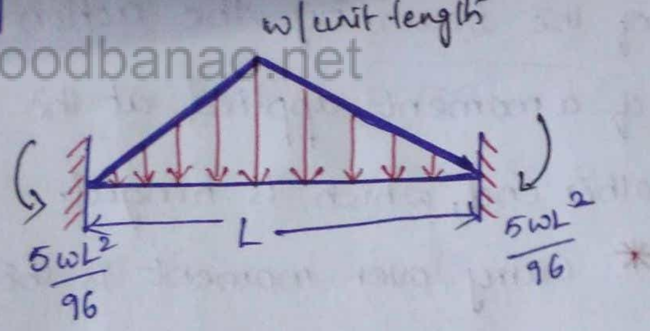
4


$\frac{wL^2}{12}$ $\frac{wL^2}{12}$

5


$\frac{wL^2}{20}$ $\frac{wL^2}{30}$

6



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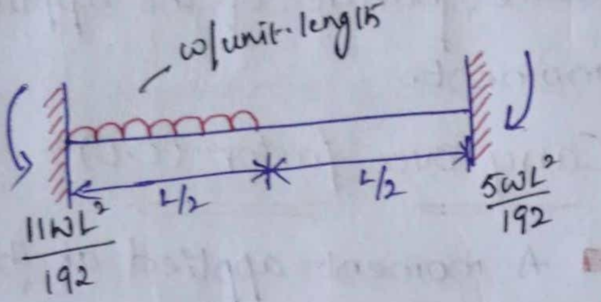

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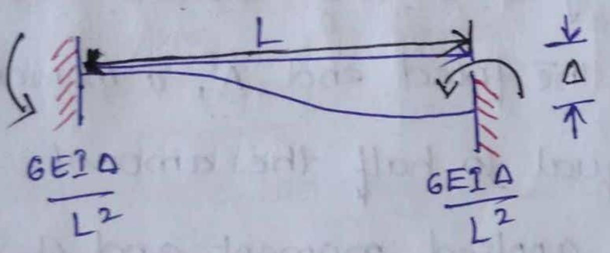

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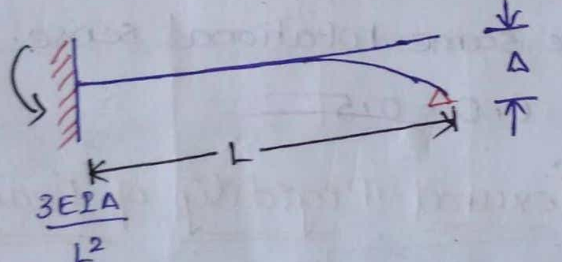


Relative (or) Beam stiffness (or) stiffness factor.

→ When a structural member of uniform section is subjected to a moment at one end, then the moment required so as to rotate that end to produce unit slope is called the stiffness of the member.

→ stiffness is the member of force required to produce unit deflection.

→ It is also the moment required to produce unit rotation at a specified joint in a beam (or) a structure. It can be extended to denote the torque needed to produce unit twist.

→ It is the moment required to rotate the end while acting on it through a unit rotation, without translation of the far end being.

→ Beam is hinged (or) simply supported at both ends.

$$\boxed{K = \frac{3EI}{L}}$$

→ Beam is hinged (or) simply supported at one end and fixed at other end.

$$\boxed{K = \frac{4EI}{L}}$$

→ Stiffness of members in continuous beams and rigid frames

→ Stiffness of all intermediate members.

$$\boxed{K = \frac{4EI}{L}}$$

→ Stiffness of edge members,

1) If edge support is fixed

$$\boxed{K = 4EI/L}$$

2) If edge support is hinged (or) roller

$$\boxed{K = 3EI/L}$$

where, E = Young's modulus of the beam material

I = M.of I. of the beam.

L = Beam's span length.

Distribution factor:-

→ When several members meet at a joint and a moment is applied at the joint to produce rotation without translation of the members, the moment is disturbed among all the members meeting at that joint proportionate to their stiffness.

→ Distribution factor = $\dfrac{\text{Relative stiffness}}{\text{Sum of relative stiffness at the joint}}$

→ If there is 3 members,

Distribution factors

$= K_1/(K_1+K_2+K_3),\ K_2/(K_1+K_2+K_3),\ K_3/(K_1+K_2+K_3)$

Carry over moment:

* Carry over moment : It is defined as the moment induced at the fixed end of the beam by the action of a moment applied at the other end, which is hinged.

* Carry over moment is the same nature of the applied moment.

Carry over factor (C.O):-

- A moment applied at the hinged end B "carries over" to the fixed end 'A', a moment equal to half the amount of applied moment and of the same rotational sense.

$C.O = 0.5$

Flexural Rigidity of Beams:-

- The product of Young's modulus (E) and moment of inertia (I) is called Flexural Rigidity (EI) is of beams. The unit is N.mm

Constant strength beam:-

- If the flexural Rigidity (EI) is constant over the uniform section, it is called constant strength beam.

Sway:

* Sway is the lateral movement of joints in a portal frame due to the ~~of~~ unsymmetrical dimensions, loads, moments of inertia, end conditions etc.

What are the situations where in sway will occur in portal frames?

» Eccentric (or) unsymmetrical loading.

» Unsymmetrical geometry.

» Different end conditions of the columns.

» Non-uniform section of the members.

» Unsymmetrical settlement of supports.

» A combination of the above.

What are symmetric and antisymmetric quantities in structural behaviour?

» When a symmetrical structure is loaded with symmetrical loading, the bending moment and deflected shape will be symmetrical about the same axis.

» Bending moment and deflection are symmetrical quantities.

Steps involved in Moment Distribution method:-

1. Calculate fixed end moments due to applied loads following the same sign convention and procedure, which was adopted in the slope-deflection method.

2. Calculate relative stiffness.

3. Determine the distribution factors for various members framing into a particular joint.

4. Distribute the net fixed end moments at the joints to various members by multiplying the net moment by their respective distribution factors in the first cycle.

5. In the second and subsequent cycles, carry-over

moments from the far ends of the same member.

[carry over moment will be half of the distributed moment]

6. Consider this carry over moment as a fixed end moment and determine the balancing moment

→ This procedure is repeated from second cycle onwards till convergence.

Advantages of Fixed ends (or) Fixed supports:-

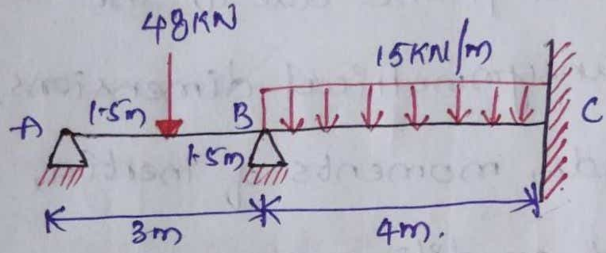
1. Slope at the ends is zero.

2. Fixed beams are stiffer, stronger and more stable than SSB.

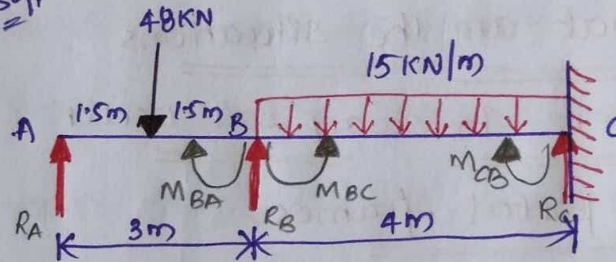
3. In case of fixed beams, fixed end moments will reduce the BM in each section.

4. The maximum defection is reduced.

Problem no (6):- Analyse the beam using Moment distribution method.

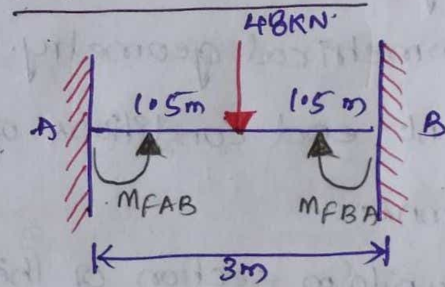


Sol:



$M_{AB} = 0$.

Fixed End Moments:-



$$M_{FAB} = \frac{-WL}{8}$$

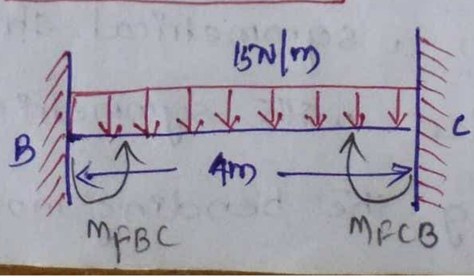
$$= \frac{-48 \times 3}{8}$$

$$\boxed{M_{FAB} = -18 KN/m}$$

$$M_{FBA} = \frac{WL}{8}$$

$$= \frac{48 \times 3}{8}$$

$$\boxed{M_{FBA} = 18 KN/m}$$



$M_{FBC} = \frac{-WL^2}{12}$

$= -\frac{15 \times 4^2}{12}$

$\boxed{M_{FBC} = -20 KNm}$

$M_{FCB} = \frac{WL^2}{12}$

$= \frac{15 \times 4^2}{12}$

$\boxed{M_{FCB} = 20 KNm}$

How to calculate stiffness at the joints

Farend is fixed $= \frac{4EI}{L}$

Farend is hinged (or) with roller support $= \frac{3EI}{L}$

Farend is continuous $= \frac{4EI}{L}$

Joint	Members	K	ΣK	DF
B	BA	$\frac{3EI}{3} = EI$	$EI + EI = 2EI$	$\frac{EI}{2EI} = 0.5$
	BC	$\frac{4EI}{4} = EI$		$\frac{EI}{2EI} = 0.5$

$DF = \left(\frac{K}{\Sigma K}\right)$

Member	A	B		C
	AB	BA	BC	CB
Distribution Factor	1	0.5	0.5	0
Fixed End Moments (FEM)	-18	18	-20	20

Release AB and C.O to BA	18 → 9			
Adjusted FEM		27	-20	20
1st distribution at B		-3.5	-3.5	-
Carryover to CB from BC		-	-	-1.75
Final Moments	0	23.5	-23.5	18.25

→ By adding

$(27-20) \times 0.5 = 3.5$

$(27-20) \times 0.5 = 3.5$

$\boxed{\frac{18}{2} = 9}$

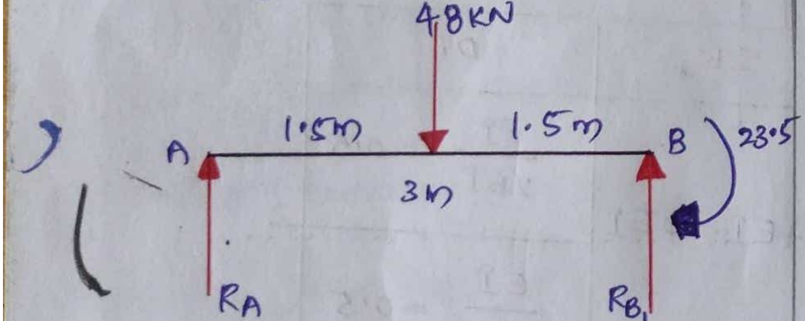
$\frac{-3.5}{2} \Rightarrow -1.75$

$M_{AB} = 0$

$M_{BA} = 23.5\,KNm$

$M_{BC} = -23.5\,KNm$

$M_{CB} = 18.25\,KNm.$



To find out Reactions:

R.H.S +ve -ve

$3R_A - 48 \times 1.5 + 23.5 = 0$

$R_A = \frac{48.5}{3}$

$\boxed{R_A = 16.17\,KN}$

$\Sigma V = 0$

$R_A + R_{B_1} - 48 = 0$

$R_{B_1} = 48 - 16.17$

$\boxed{R_{B_1} = 31.83\,KN}$

To find out Reactions:

R.H.S. +ve -ve

Take moment about C,

$4R_{B_2} - 15 \times 4 \times \frac{4}{2} - 23.5 + 18.25 = 0$

$R_{B_2} = \frac{125.25}{4}$

$\boxed{R_{B_2} = 31.31\,KN}$

$\Sigma V = 0$

$R_{B_2} + R_C - 15 \times 4 = 0$

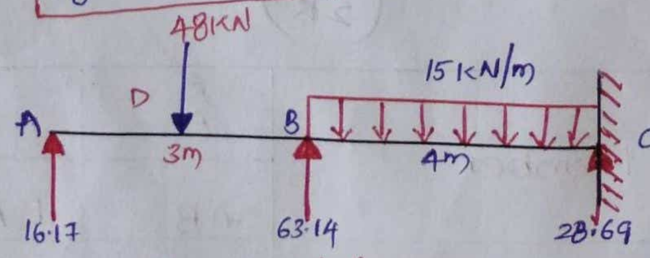
$R_C = 60 - 31.31$

$\boxed{R_C = 28.69\,KN}$

$R_B = R_{B_1} + R_{B_2}$

$= 31.83 + 31.31$

$\boxed{R_B = 63.14\,KN}$



Shear Forces Values:

R.H.S ↑+ve ↓-ve

S.F at A = 16.17KN

S.F at just left of D = 16.17KN

S.F at just right of D

$= 16.17 - 48 = -31.88\,KN$.

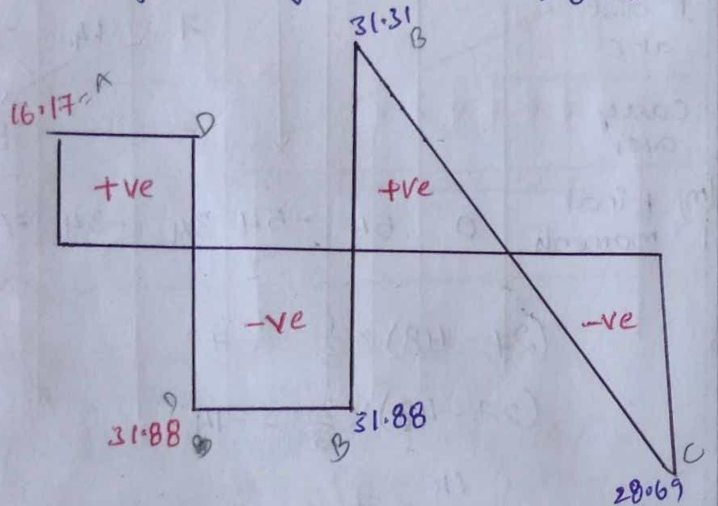
S.F at just left of B $= -31.88\,KN$

S.F at just right of B $= -31.88 + 63.14 = 31.31\,KN$

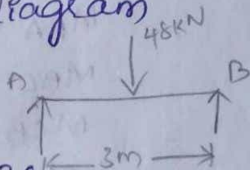
S.F at just left of C $= 31.31 - 15 \times 4$

$= -28.69\,KN$.

S.F at just right of C $= -28.69 + 28.69 = 0$.



For free moment diagram



For span AB

$= \frac{WL}{4} = \frac{48 \times 3}{4} = 36$

For span BC

$= \frac{WL^2}{8} = \frac{15 \times 4^2}{8} = 30$

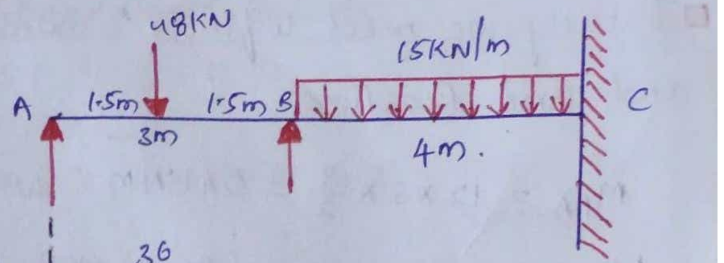
For End Moment diagram

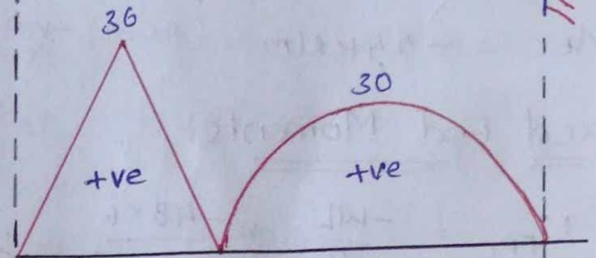
$M_{AB} = 0$

$M_{BA} = 23.5\,KN/m$

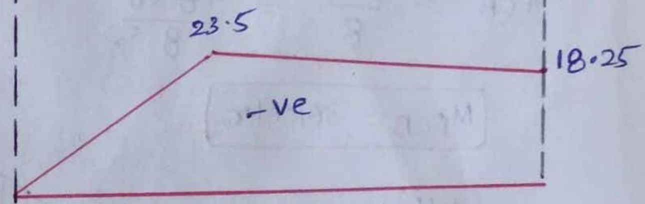
$M_{BC} = -23.5\,KN/m$

$M_{CB} = 18.25\,KN/m$

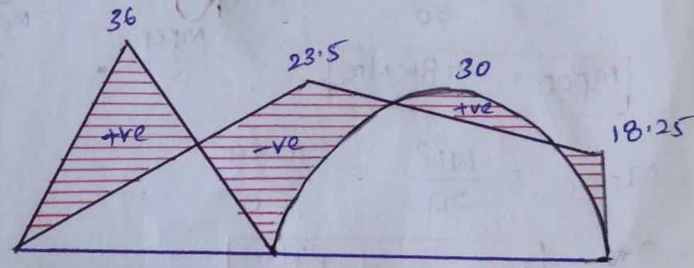




Free Moment Diagram

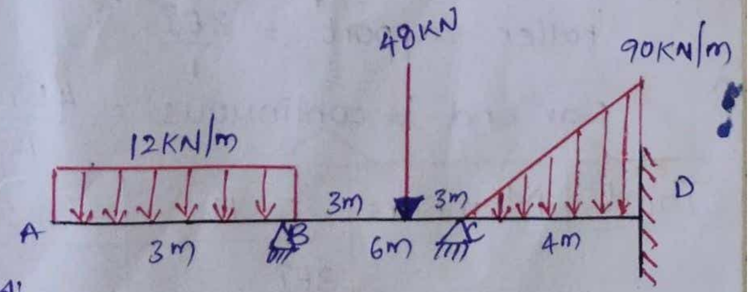


End Moment Diagram.

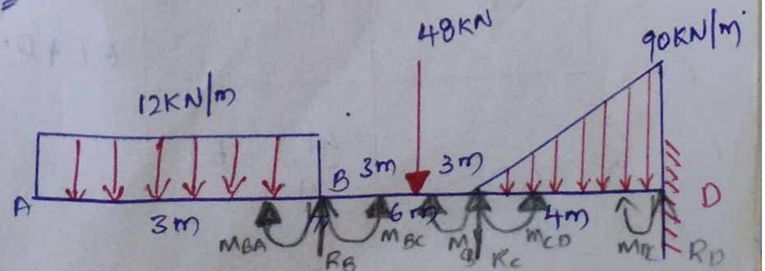


Bending Moment diagram.

Problem no. 7 :- Analyse this beam using Moment distribution method.



Sol:



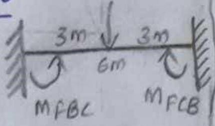
- Totally we need to find '5' moments and three Reactions

$M_{BA} = 12 \times 3 \times \frac{3}{2} = 54\,KNm$. (CW +ve)

$M_{BC} = -54\,KNm$ (ACW -ve)

Fixed End Moments:

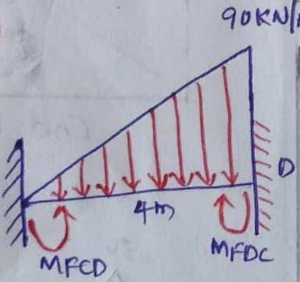
$M_{FBC} = \frac{-WL}{8} \Rightarrow \frac{-48 \times 6}{8}$



$\boxed{M_{FBC} = -36\,KNm}$

$M_{FCB} = \frac{WL}{8} = \frac{48 \times 6}{8}$

$\boxed{M_{FCB} = 36\,KNm}$



$M_{FCD} = \frac{-WL^2}{30}$

$= \frac{-90 \times 4^2}{30}$

$\boxed{M_{FCD} = -48\,KNm}$

$M_{FDC} = \frac{WL^2}{20} = \frac{90 \times 4^2}{20}$

$\boxed{M_{FDC} = 72\,KNm}$

How to calculate stiffness at the joints

Far end is fixed $= \frac{4EI}{L}$

Far end is hinged (or) with roller support $= \frac{3EI}{L}$

Far end is continuous $= \frac{4EI}{L}$

Member	A	B		C		D
	AB	BA	BC	CB	CD	DC
Distribution factor	0	0	1	1/3	2/3	0
Fixed End Moment (FEM)	0	54	-36	36	-48	72
Release BC and C.O to CB			-18	-9		
Adjusted FEM				27	-48	72
1st distn at C				7	14	-
Carry over				-	-	7
Final moments	0	54	-54	34	-34	79

$(27-48) \times \frac{1}{3} = -7$

$(27-48) \times \frac{2}{3} = -14$

$\left\{ \frac{14}{2} = 7 \right\}$

$M_{BA} = 54\,KNm$

$M_{BC} = -54\,KNm$

$M_{CB} = 34\,KNm$

$M_{CD} = -34\,KNm$

$M_{DC} = 79\,KNm$

To find out Reactions:-

Take moment about C,

R.H.S ↻ +ve ↺ -ve

$-12 \times 3 \times \left(\frac{3}{2} + 6\right) + 6R_B - 48 \times 3 + 34 = 0$

$R_B = \frac{380}{6} \Rightarrow \boxed{R_B = 63.33\,KN}$

Joint	Members	K	ΣK	DF
C	CB	$\frac{3EI}{6} = 0.5EI$	$EI + 0.5EI = 1.5EI$	$\frac{0.5EI}{1.5EI} = \frac{1}{3}$
	CD	$\frac{4EI}{4} = EI$		$\frac{EI}{1.5EI} = \frac{2}{3}$

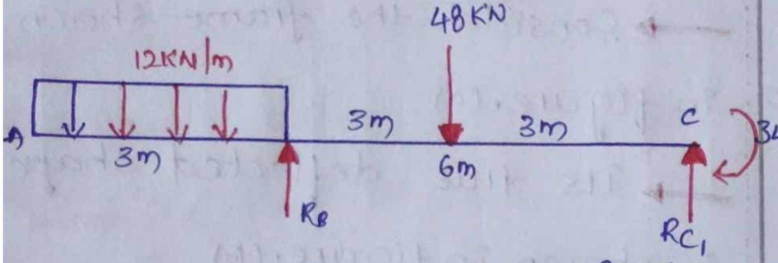
$\boxed{DF = \frac{K}{\Sigma K}}$

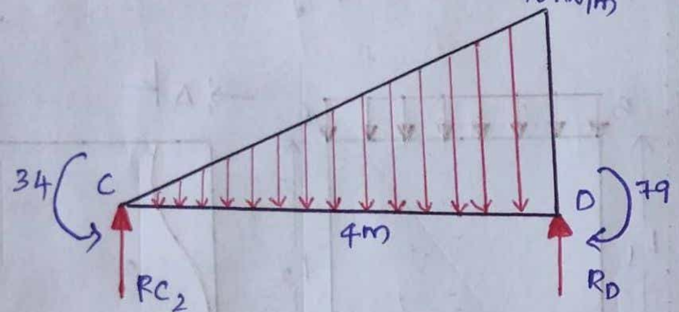
$\Sigma V = 0$

$R_B + R_{C_1} - 12 \times 3 - 48 = 0$

$R_{C_1} = 84 - 63.33$

$R_{C_1} = 20.67 KN.$





To find out Reactions:-

Take moment about D,

R.H.S. ↻ +ve ↺ -ve

$4R_{C_2} - \frac{1}{2} \times 4 \times 90 \times \frac{1}{3} \times 4 - 34 + 79 = 0$

$R_{C_2} = \frac{195}{4}$

$\boxed{R_{C_2} = 48.75 KN}$

$\Sigma V = 0$

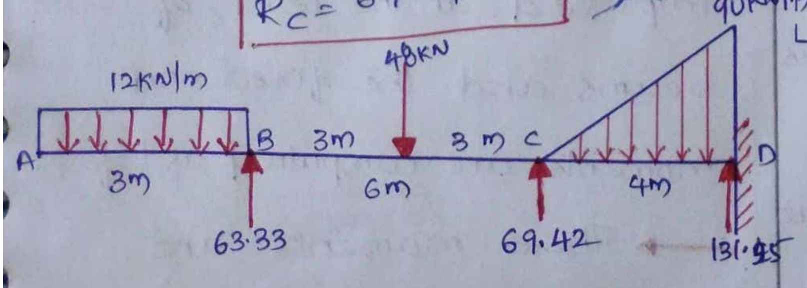
$R_{C_2} + R_D - \frac{1}{2} \times 4 \times 90 = 0$

$R_D = 180 - 48.75$

$\boxed{R_D = 131.25 KN}$

$R_C = R_{C_1} + R_{C_2} = 20.67 + 48.75$

$\boxed{R_C = 69.42 KN}$



Shear Force Values:-

R.H.S ↑+ve ↓-ve

S.F at A = 0

S.F at just left of B $= -12 \times 3 = -36 KN$

S.F at just right of B $= -36 + 63.33$
$= 27.33 KN$

S.F at just left of E $= 27.33 KN.$

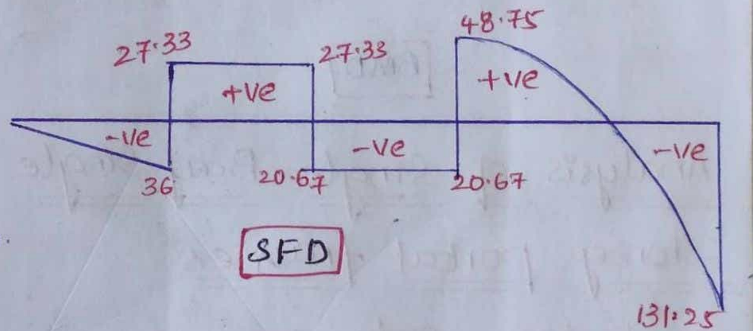
S.F at just right of E $= 27.33 - 48$
$= -20.67 KN.$

S.F at just left of C $= -20.67 KN$

S.F at just right of C $= -20.67 + 69.42$
$= 48.75 KN$

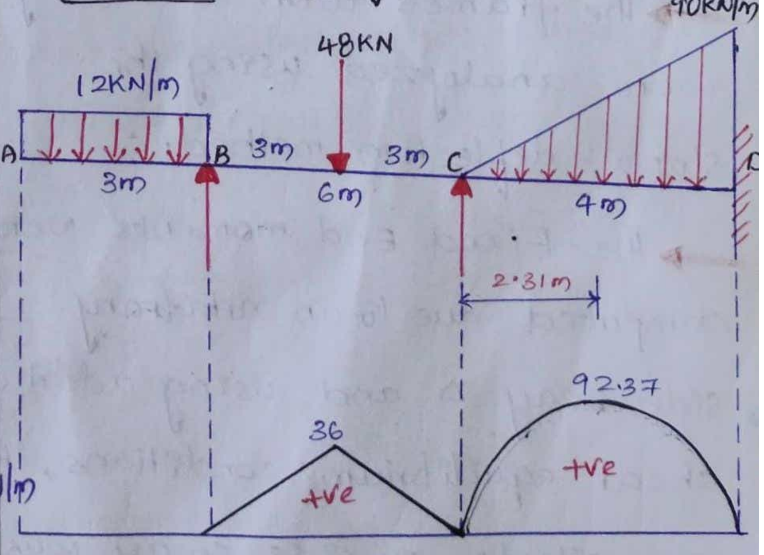
S.F at just left of D $= 48.75 - \frac{1}{2} \times 4 \times 90$
$= -131.25$

S.F at D $= -131.25 + 131.25 = 0.$



SFD

Free moment diagram:-

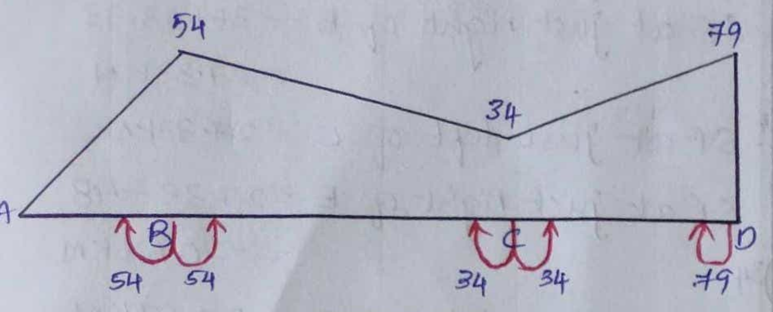


For span BC $= \frac{WL}{4} = \frac{48 \times 6}{8} = 36 KNm$

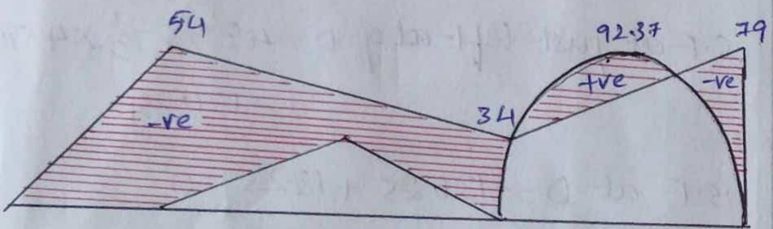
For span CD $= \frac{WL^2}{9\sqrt{3}} = \frac{90 \times 4^2}{9\sqrt{3}}$
$= 92.37 KNm.$

It occurs at $\frac{L}{\sqrt{3}}$ $\left(\frac{4}{\sqrt{3}} = 2.31m\right)$ from C.

» By using end moments we can draw the End moment diagram



END M.D



BMD

Analysis of Single Bay Single storey portal frames including side sway:-

→ The frames with side sway were analyzed using the slope-deflection method.

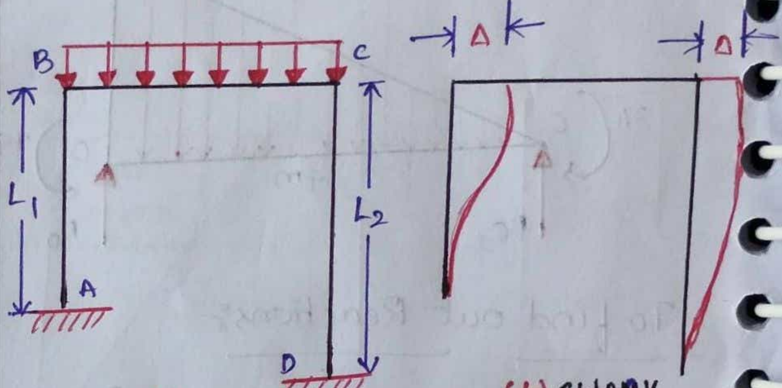
→ The Fixed End moments were computed due to an arbitrary side sway Δ and using additional shear equilibrium conditions, the magnitude of side sway was determined.

→ It is not possible to repeat this procedure in the moment distribution method which is iterative in nature.

→ The moment distribution method is carried out in two stages.

→ Consider the frame shown in figure.(a)

→ Its true deflected shape is shown in figure.(b)



(a) (b) SWAY

→ In stage 1, an imaginary support is provided at the level of the beams(s) to prevent the side sway.

→ This problem can be solved using the moment distribution procedure. as ~~d~~

→ In stage 2, an arbitrary lateral displacement Δ is imposed at the level of beams and the fixed end moments are computed as before.

→ These moments are written in terms of the arbitrary displacement Δ.

→ The external loads are withdrawn from the frame for stage 2 analysis.

→ The moment distribution procedure is carried out for the arbitrary values of a sway.

→ The true situation is obtained by superposition of the results of stage 1 and 2.

→ The equation of shear equilibrium is written assuming that the correct moments in stage 2 are k times those obtained due to the arbitrary sway Δ.

→ The factor k tells us how much to scale the moments of stage 2 to obtain correct moments.

$$\frac{M'_{AB}+M'_{BA}}{L_1}+\frac{M'_{DC}+M'_{CD}}{L_2}+k\left[\frac{M^2_{AB}+M^2_{BA}}{L_1}+\frac{M^2_{DC}+M^2_{CD}}{L_2}\right]=0$$

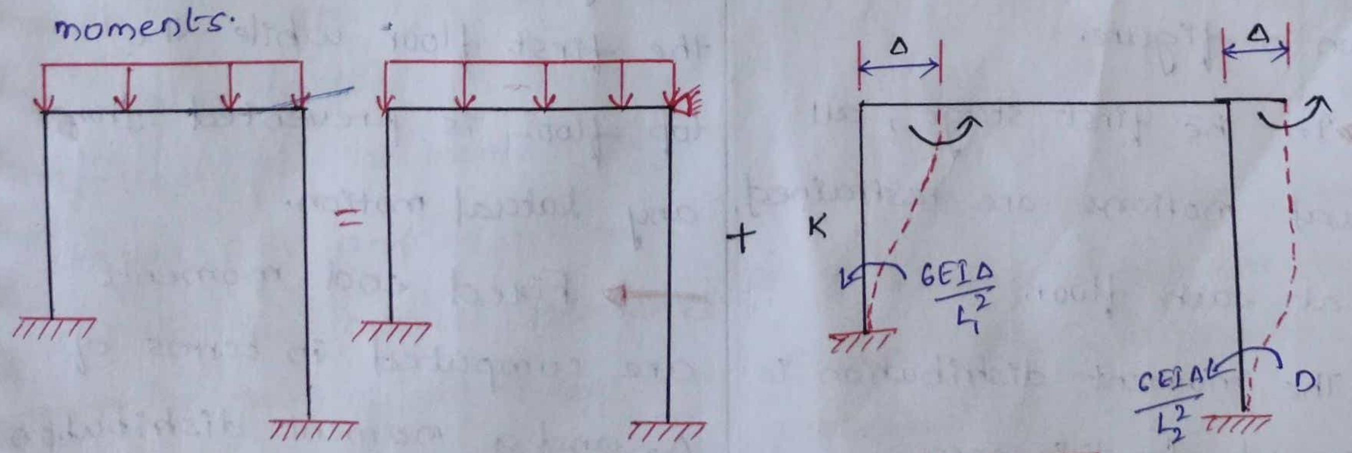
→ Where, the superscript 1 refers to the first stage and 2 refers to the second stage.

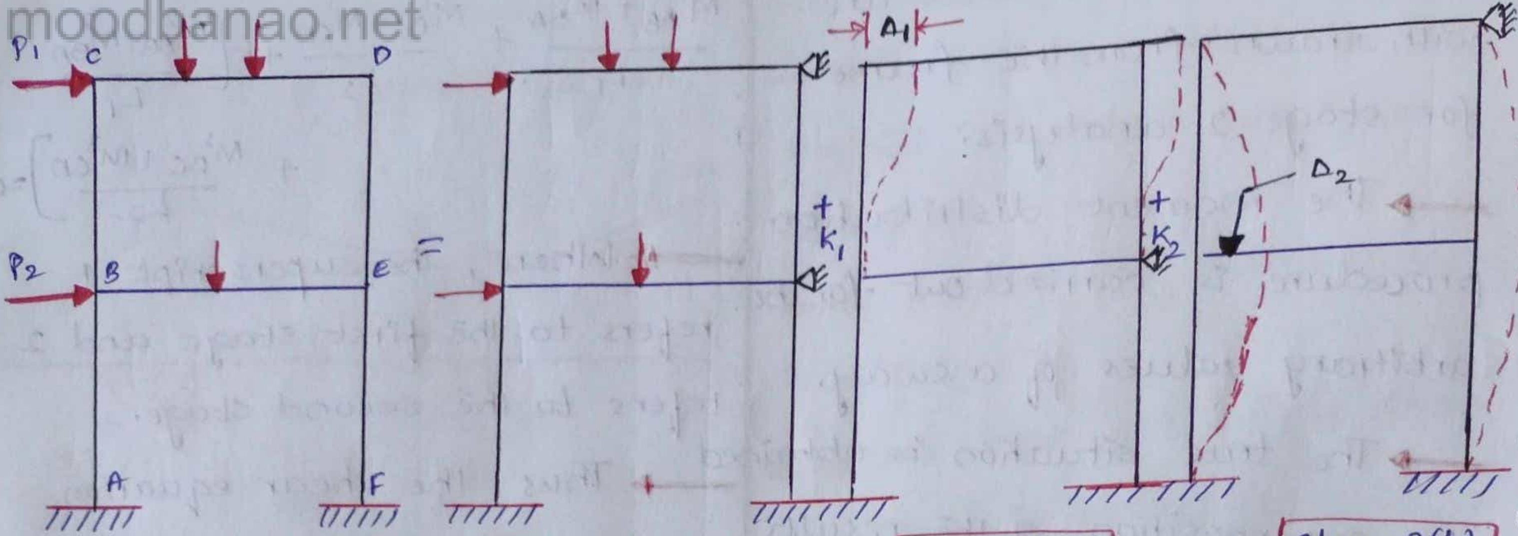
→ Thus, the shear equation provides the value of the factor K.

→ The scaled moments of stage 2 are then added to the moments of the stage 1 to obtain the actual moments in the structure.

→ The true sway Δ' is equal to 'K' times the arbitrary sway. Δ.

→ Consider a two storey portal frame shown in figure →





Stage 1

Stage 2 (a)

Stage 2(b)

→ It is a typical situation in which there is more than one degree of freedom in sidesway.

→ They key to the solution is again the principle of superposition.

→ The actual equilibrium and kinematic state of the structure is considered to be the superposition of the three stages shown in figure.

→ In the first stage, all lateral motions are restrained, one at each floor.

→ The moment distribution is performed for this case.

→ In stage 2(a), a lateral sway of displacement Δ_1 is imposed on the top floor while the other floor is prevented from any lateral motion.

→ Fixed end moments are computed in terms of Δ_1 and a moment distribution is performed.

→ In stage 2(b), a lateral sway of Δ_2 is imposed on the first floor while the top floor is prevented from any lateral motion.

→ Fixed end moments are computed in terms of Δ_2 and a moment distribution is performed.

→ The principle of superposition requires that

$Q_1^1 + K_1 Q_1^{2a} + K_2 Q_1^{2b} + P_1 = 0$

& $Q_2^1 + K_1 Q_2^{2a} + K_2 Q_2^{2b} + P_1 + P_2 = 0$

→ Where Q_1 and Q_2 are the storey shears in the top storey and first storey, respectively, and computed as:

$$Q_1^i = \left[\frac{M^i_{BC} + M^i_{CB}}{L_1} + \frac{M^i_{ED} + M^i_{DE}}{L_1} \right]$$

$$Q_2^i = \left[\frac{M^i_{AB} + M^i_{BA}}{L_2} + \frac{M^i_{FE} + M^i_{EF}}{L_2} \right]$$

where,

i = stage 1, stage 2(a) stage 2(b)

L_1 = length BC,

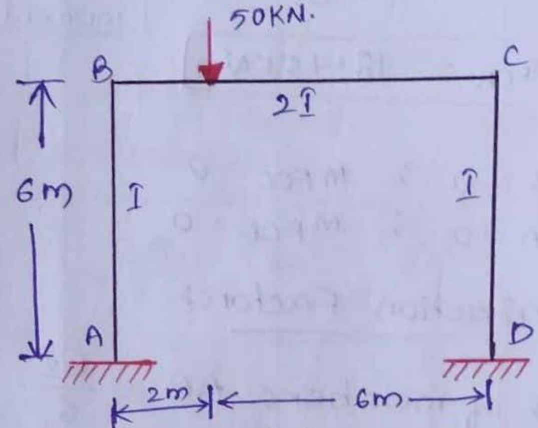
L_2 = length AB.

→ These equations are now solved for K_1 and K_2.

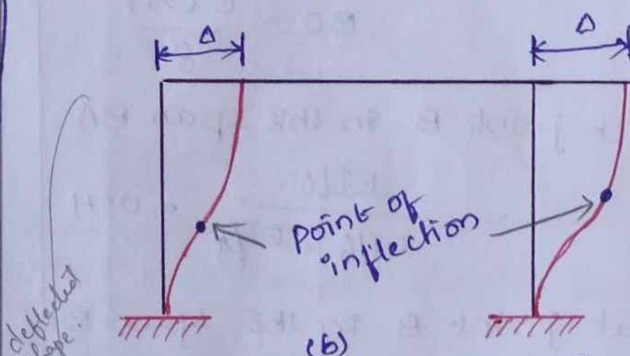
→ These are the correction (or) scale factors to be applied to the moments of stages 2(a) and 2(b) to obtain the correct moments.

→ The sum of all three cases give the correct moments in the real structure.

problem no ⑧:- Analyze the portal frame shown in figure(a) by the moment distribution method and sketch the deflected shape.

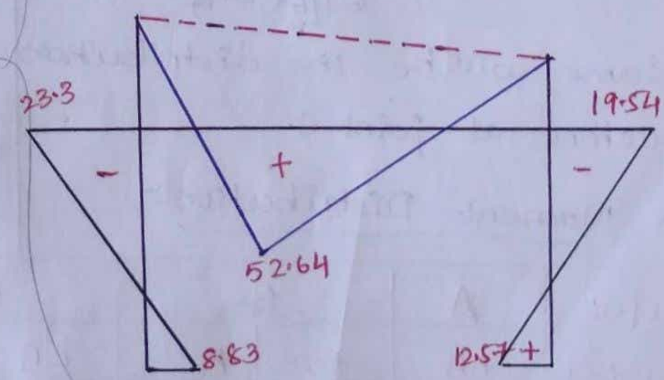


(a)



(b)

FRAME UNDER SWAY



(c)

BENDING MOMENT, KNm

Sol: I. Moment distribution for the applied loads with sidesway prevented.

(a) Fixed End Moment

This frame has only one sway, i.e., to the right. If this sway is restrained, the FEM is

$$M_{FBC} = -\frac{50 \times 2 \times 6^2}{8^2}$$

$$\boxed{M_{FBC} = -56.25\,KNm}$$

$$M_{FCB} = \frac{50 \times 2^2 \times 6}{8^2}$$

$$\boxed{M_{FCB} = 18.75\,KNm}$$

$M_{FAB} = 0$; $M_{FDC} = 0$

$M_{FBA} = 0$; $M_{FCD} = 0$

(b) Distribution Factors:-

Stiffness of members $AB = \frac{EI}{6}$

$BC = \frac{E(2I)}{8}$

DF at joint B in the span BA

calculator

$$= \frac{EI/6}{EI/6 + EI/4} = 0.4$$

DF at joint B in the span BC

calculator

$$= \frac{EI/4}{EI/6 + EI/4} = 0.6$$

Same will be the distribution factors at joint C.

(c) Moment Distribution:-

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	—	0.4	0.6	0.6	0.4	—
FEM	0	0	-56.25	18.75	0	0
Balance	0	+22.50	+33.75	-11.25	-7.5	0
carryover	+11.25	—	-5.62	16.87	—	-3.75
Balance	0	+2.25	+3.37	-10.12	-6.75	—
carryover	+1.12	—	-5.06	+1.68	—	3.37
Balance	0	+2.02	+3.03	-1.00	-0.67	0
carryover	—	—	-0.50	+1.50	—	-0.34
Balance	+1.01	+0.20	+0.30	-0.90	-0.60	0
carryover	0.10	—	-0.45	+0.15	—	-0.30
Balance	0	0.18	+0.27	-0.09	-0.06	0
carryover	+0.09	—	-0.05	+0.14	—	[illegible]
Balance	0	+0.02	+0.03	-0.08	-0.06	0
carryover	+0.01	—	-0.04	+0.02	—	-0.03
Balance	0	+0.02	+0.02	-0.01	-0.01	0
Total moment	+13.57	+27.18	-27.18	+15.66	-15.66	-7.83

$5.06 \times 0.4 \Rightarrow 2.02$

$5.06 \times 0.6 \Rightarrow 3.03$

$1.68 \times 0.6 \Rightarrow 1.00$

$1.68 \times 0.4 \Rightarrow 0.67$

$56.25 \times 0.6 = 33.75$

$56.25 \times 0.4 = 22.5$

$18.75 \times 0.6 = 11.25$

$18.75 \times 0.4 = 7.5$

$-\frac{7.5}{2} \Rightarrow -3.75$

$\frac{11.25}{2} \Rightarrow 5.62$; $\frac{33.75}{2} = 16.87$

$5.62 \times 0.4 \Rightarrow 2.248 \simeq 2.25$

$5.62 \times 0.6 \Rightarrow 3.37$

$16.87 \times 0.6 \Rightarrow 10.12$

$16.87 \times 0.4 \Rightarrow 6.748 \simeq 6.75$

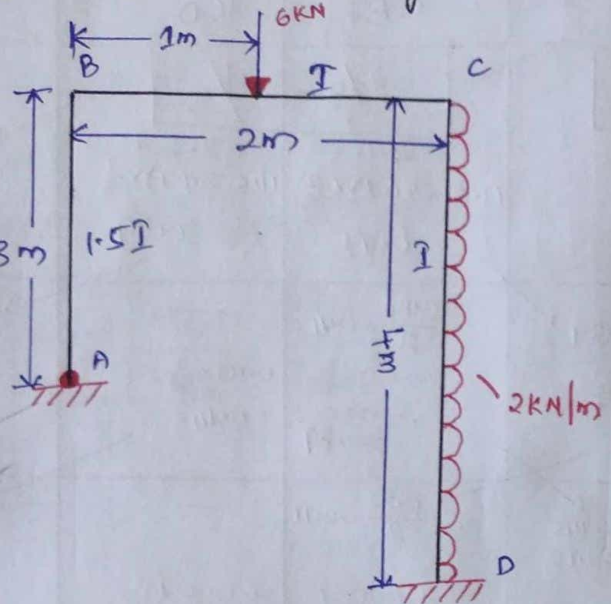
$2.25/2 \Rightarrow 1.125$ | $6.75/2 \Rightarrow 3.375$

$10.12/2 \Rightarrow 5.06$

$3.37/2 \Rightarrow 1.68$

Problems on Analysis of portal frame with side sway.

(1)



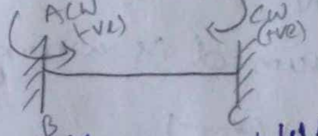
Step-1: Fixed End Moments

» we know FEM means we will assume all these supports have as fixed. so the end moments corresponding to the external loadings, here we have external loadings on BC & CD and we don't have on AB.

$\therefore \; M_{FAB} = M_{FBA} = 0$

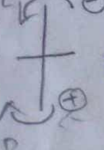
$M_{FBC} = \frac{-Wab^2}{l^2} = \frac{-wl}{8}$

$= \frac{-6\times2}{8} = -1.5\,KNm$



$M_{FCB} = \frac{+Wba^2}{l^2} = \frac{+wl}{8}$

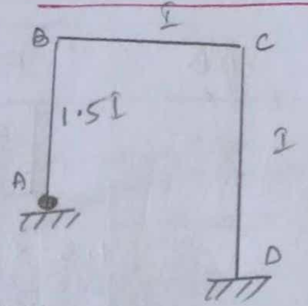
$= \frac{6\times2}{8} = +1.5\,KNm$



$M_{FCD} = \frac{-wl^2}{12} = \frac{-2\times4^2}{12} = -2.67\,KN\text{-}m$

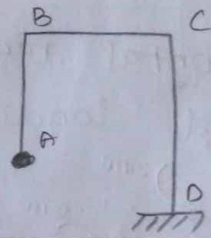
$M_{FDC} = \frac{wl^2}{12} = \frac{2\times4^2}{12} = 2.67\,KN\text{-}m$

Step 2: Distribution Factors



Joints	Members	Relative stiffness (K)	Sum ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\cdot\frac{I}{L} = \frac{3}{4}\times\frac{1.5I}{3} = \frac{1.5I}{4}$	$\frac{1.5I}{4}+\frac{I}{2} = \frac{7I}{8}$	$\frac{1.5I/4}{7I/8} = \frac{3}{7}$
	BC	$\frac{I}{L} = \frac{I}{2}$		$\frac{4}{7}$
C	CB	$\frac{I}{L} = \frac{I}{2}$	$\frac{3I}{4}$	2/3
	CD	$\frac{I}{4} = \frac{I}{L}$		1/3

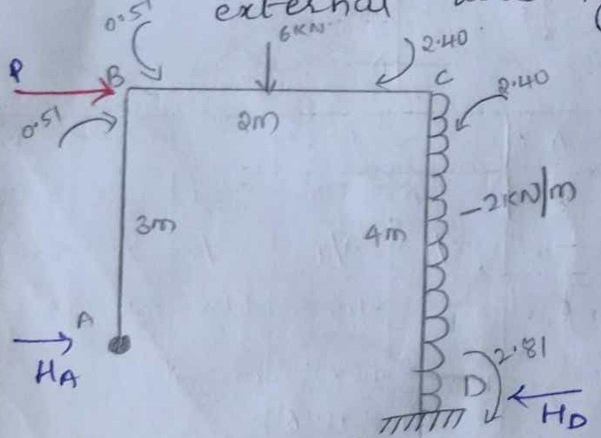
STEP-3: Moment distribution Table



Joints	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	–	3/7	4/7	2/3	1/3	–
FEM	0	0	-1.5	+1.5	-2.67	2.67
Balance	–	$1.5\times\frac{3}{7}$ = 0.64	$+1.5\times\frac{4}{7}$ = +0.86	$(1.5-2.67)\times\frac{2}{3}$ = +0.78	$(1.5-2.67)\times\frac{1}{3}$ = +0.39	–
Carry over	–	–	$\frac{0.78}{2}$ = 0.39	$\frac{0.86}{2}$ = 0.43		

Joints	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	—	$\boxed{3/7}$	$\boxed{4/7}$	$\boxed{2/3}$	$\boxed{1/3}$	—
FEM	—	—	-1.5	$+1.5$	-2.67	$+2.67$
Balance	—	$1.5 \times \frac{3}{7} = 0.67$	$+1.5 \times \frac{4}{7} = 0.86$	$(1.5-2.67) \times \frac{2}{3} = 0.78$	$(1.5-2.67) \times \frac{1}{3} = 0.39$	—
carry over	—	—	$\frac{0.78}{2} = 0.39$	$\frac{0.86}{2} = 0.43$	—	$\frac{0.39}{2} = 0.195 \simeq 0.20$
Balance	—	$(0+0.39) \times \frac{3}{7} = -0.167 \simeq -0.17$	$(0+0.39) \times \frac{4}{7} = -0.22$	$(0.43) \times \frac{2}{3} = -0.286 \simeq -0.29$	$0.43 \times \frac{1}{3} = 0.143$	—
Carry over	—	—	$\frac{0.29}{2} = 0.145 \simeq 0.15$	$\frac{0.22}{2} = -0.11$	—	$\frac{0.143}{2} = -0.07$
Balance		$+0.06$	$+0.09$	$+0.07$	$+0.04$	—
carryover	—	—	$+0.04$	$+0.05$	—	0.02
Balance xCO	—	0.02	-0.02	-0.03	-0.02 as end is fixed half of it will CO	-0.01
Final Moments	—	$+0.51$	-0.51	$+2.40$	-2.40	$+2.81$
	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}

Step-4: Horizontal reactions due to Moments due to external loadings.



$\Sigma M_B = 0$; left of B.

$-H_A \times 3 + 0.51 = 0$

$H_A = 0.17\,KN\ (\rightarrow)$ → as we got +ve our assumed dir^n is right.

$\Sigma M_C = 0$, right of B

$H_D \times 4 + 2.81 + (2 \times 4) \times 2 - 2.40 = 0$

$H_D = -4.10\,KN$

as we got -ve the assumed is wrong.

$H_D = 4.10\,KN\ (\leftarrow)$

We have to assume a horizontal force that can be applied at B to hold your frame against the translation so, an imaginary load 'p' is taken @ B.

(Difference b/w all the forces)

$p = -0.17 - 4.10 - 2 \times 4 = -3.73\,KN.$ (←)

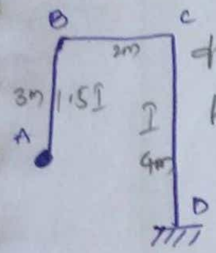
(left) (Net hold force)

@ B, $p \Rightarrow 3.73\,KN\ (\rightarrow)$

Step 5: Side sway

Sway force $S = 3.73\,KN\,(\leftarrow)$ @ C

⇒ We are giving sway force @ C because as we are already assumed horizontal force @ B



fixed / hinge $\left\{ \dfrac{M_{CD}}{M_{BA}} = \dfrac{2I_1/L_1^2}{I_2/L_2^2} \right.$ — one end is hinge & other is fixed.

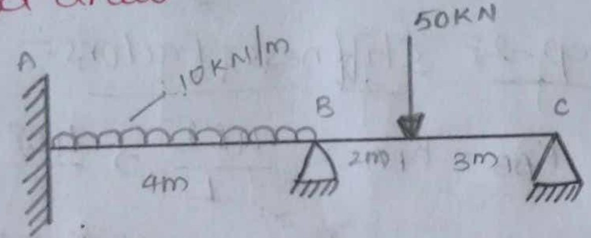
$= \dfrac{2I/16\ (4^2)}{1.5I/9\ (3)^2}$

$= \dfrac{6}{8}$

Let $M_{CD} = M_{DC} = +6KN$

$M_{BA} = M_{AB} = 8KN$

Problem no. 10: Analyse the frame given in figure by moment distribution method and draw the B.M.D & S.F.D.



Soln

Step-1: Fixed End Moment:

$MF_{AB} = \dfrac{-WL^2}{12} \Rightarrow \dfrac{-10\times4^2}{12}$

$\boxed{MF_{AB} = -13.33\,KNm}$

$MF_{BA} = \dfrac{WL^2}{12} \Rightarrow \dfrac{10\times4^2}{12}$

$\boxed{MF_{BA} = +13.33\,KNm}$

$MF_{BC} = \dfrac{-Wab^2}{L^2} = \dfrac{-50\times2\times3^2}{5^2} = -36\,KNm$

$\boxed{MF_{BC} = -36\,KNm}$

$MF_{CB} = \dfrac{Wa^2b}{L^2} = \dfrac{50\times2^2\times3}{5^2}$

$\boxed{MF_{CB} = 24\,KNm}$

Step-2: Stiffness.

$K_{AB} = K_{BA} = \dfrac{4EI}{L} = \dfrac{4EI}{4} \Rightarrow EI$

$K_{BC} = K_{CB} = \dfrac{3EI}{L} \Rightarrow \dfrac{3EI}{5} \Rightarrow 0.6E$

Step-3: Moment Distribution Factor:

Joint B

$D^F_{BA} = \dfrac{K_{BA}}{(K_{BA}+K_{BC})} = 0.63$

$D^F_{BC} = \dfrac{K_{BC}}{(K_{BA}+K_{BC})} = 0.37$

Step-4: Moment distribution.

Member.	AB	B		CB
		BA	BC	
D.F	0	0.673	0.337	0
FEM	-13.33	+13.33	-36	+24
Balancing	0	0	0	-24
carryover Factor	0	0	-12	0
carry over moment	-13.33	+13.33	-48	0
Balancing	0	21.84	12.83	0
carryover factor	10.92	0	0	0
Moments Final	-2.4	35.17	-35.17	0

Step-5: Reactions

Span AB:

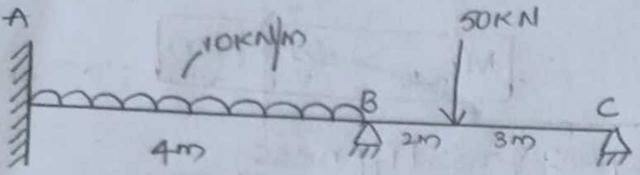
$R_A = 11.81\,KN$

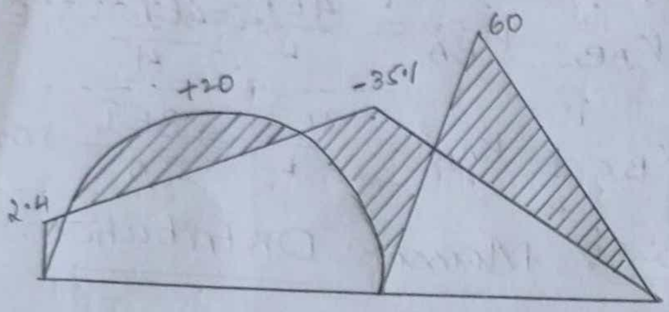
$R_{B_1} = 28.19\,KN$

Span BC:

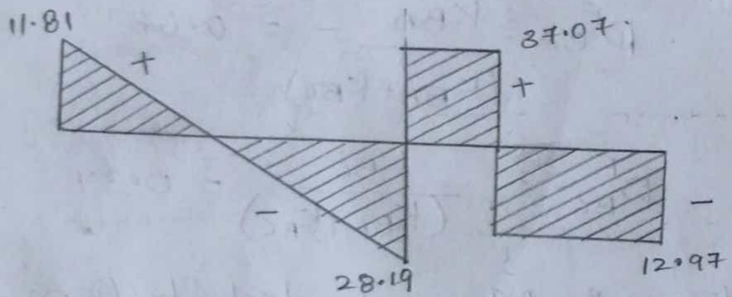
$R_{B_2} = 37.03\,KN$

$R_C = 12.97\,KN$



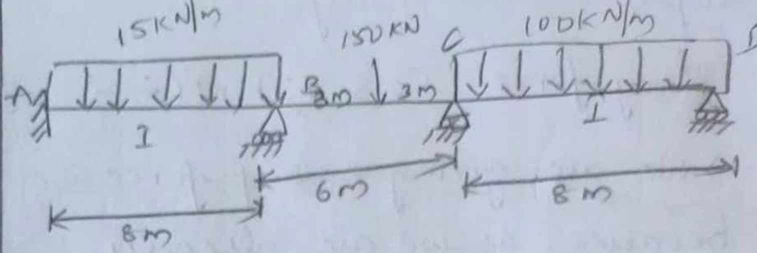


Combined BMD



SFD

problem (11) :- The continuous beam ABCD, subjected to the given loads, as shown in figure below. Assume that only rotation of joints occurs at B, C and D, and that no support displacements occur at B, C and D. Due to the applied loads in span AB, BC and CD, rotations occur at B, C and D using MDM. (Moment Distribution method)



step-1 :- Fixed End Moments:-

$$M_{AB} = -M_{BA} = \frac{-wl^2}{12}$$

$$= \frac{-15\times 8^2}{12}$$

$$= -80\,KNm$$

$$M_{BC} = -M_{CB} = \frac{-wl}{8}$$

$$= \frac{-150\times 6}{8}$$

$$= -112.5\,KNm$$

$$M_{CD} = -M_{DC} = \frac{-wl^2}{12}$$

$$= \frac{-10\times 8^2}{12}$$

$$= -53.33\,KNm$$

Step-2 :- Stiffness Factors:-

$$K_{AB} = K_{BA} = \frac{4EI}{L} \Rightarrow \frac{4EI}{8}$$

$$= 0.5EI$$

$$K_{BC} = K_{CB} = \frac{4EI}{L} = \frac{4EI}{6}$$

$$= 0.667EI$$

$$K_{CD} = \left[\frac{\cancel{4}^{3}EI}{8}\right] = \frac{\cancel{4}^{3}}{8}EI$$

$$= \cancel{0.5EI}\ 0.375$$

$$K_{DC} = \frac{\cancel{4}^{3}EI}{8} \Rightarrow 0.375$$

$$= \cancel{0.5EI}$$

Step-3: Distribution factor :-

$$DF_{AB} = \frac{K_{BA}}{K_{BA} + K_{wall}}$$

$$= \frac{0.5EI}{0.5 + \infty \text{ (wall stiffness)}} = 0$$

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}}$$

$$= \frac{0.5EI}{0.5EI + 0.667EI} = 0.4284$$

$$DF_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{0.667EI}{0.5EI + 0.667EI}$$

$$= 0.5716$$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}}$$

$$= \frac{0.667EI}{0.667EI + \cancel{0.500EI}\ 0.375} = \cancel{0.5716}\ 0.64$$

$$DF_{CD} = \frac{K_{CD}}{K_{CB} + K_{CD}}$$

$$= \frac{\cancel{0.500EI}\ 0.375}{0.667EI + \cancel{0.500EI}\ 0.375} = \cancel{0.4284}\ 0.359 \approx 0.36$$

$$DF_{DC} = \frac{K_{DC}}{K_{DC}} = 1.00$$

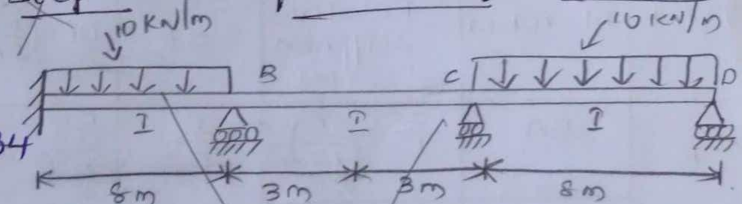
$(112.5-80) \times 0.4284 \Rightarrow 13.923$
$(112.5-80) \times 0.5716 \Rightarrow 18.577$
$(112.5-53.33) \times 0.64 \Rightarrow 37.868$
$(112.5-53.33) \times 0.36 \Rightarrow 21.30$
$(18.93-0) \times 0.4284 \Rightarrow 8.109$

$13.923 \div 2 \Rightarrow 6.961$

Step-4: Moment Distribution :-

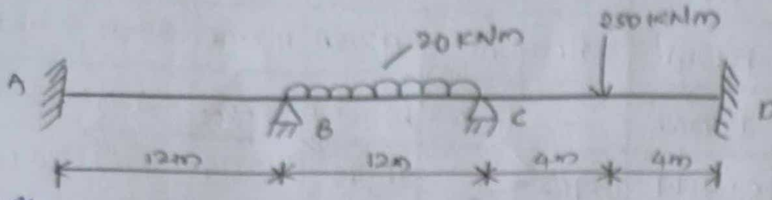
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.4284	0.5716	0.64	0.36	1
FEM	-80	80	-112.50	112.50	-53.33	53.33
1st Dist	—	13.923	18.577	-37.87	-21.3	-53.33
C.O.M	6.962	—	18.93	9.289	-26.67	-10.65
2nd dist	—	8.111	10.823	11.122	6.256	10.65
C.O.M	4.056	—	5.561	5.412	5.325	3.128
3rd dist	—	-2.382	-3.179	-6.872	-3.865	-3.128
C.O.M	-1.191	—	-3.436	-1.59	-1.564	-1.933
4th Dist	—	1.472	1.964	2.019	1.135	1.933
C.O.M	0.736	—	1.01	0.982	0.967	0.568
5th Dist	—	-0.433	-0.577	-1.247	-0.702	-0.568
C.O.M	—	—	—	—	—	—
Moment Final	-69.44	100.69	-100.7	-93.74	93.75	0

Step-5: Computation of shear Forces



Simply supported reaction	60	60	75	75	40	40
End reaction due to left hand FEM	8.726	8.726	16.665	-16.67	12.079	-12.08
End reaction due to right hand FEM	-12.5	12.498	-16.1	16.102	0	0
summed -up moments	56.228	63.772	75.563	74.437	53.077	27.923

Problem no. 13:- Analyse the beam as shown in figure by moment distribution method and draw the BMD. Assume EI is constant.



Sol:-

Step-1:- Fixed End Moments:-

$M_{F_{AB}} = 0$

$M_{F_{BA}} = 0$

$M_{F_{BC}} = -\dfrac{wL^2}{12} = \dfrac{-20 \times 12^2}{12} = -240\,KNm$

$M_{F_{CB}} = \dfrac{wL^2}{12} = \dfrac{20 \times 12^2}{12} = 240\,KNm$

$M_{F_{CD}} = -\dfrac{WL}{8} = \dfrac{-250 \times 8}{8} = -250\,KNm$

$M_{F_{DC}} = \dfrac{WL}{8} = \dfrac{250 \times 8}{8} = 250\,KNm$

Step-2:- Distribution factor:-

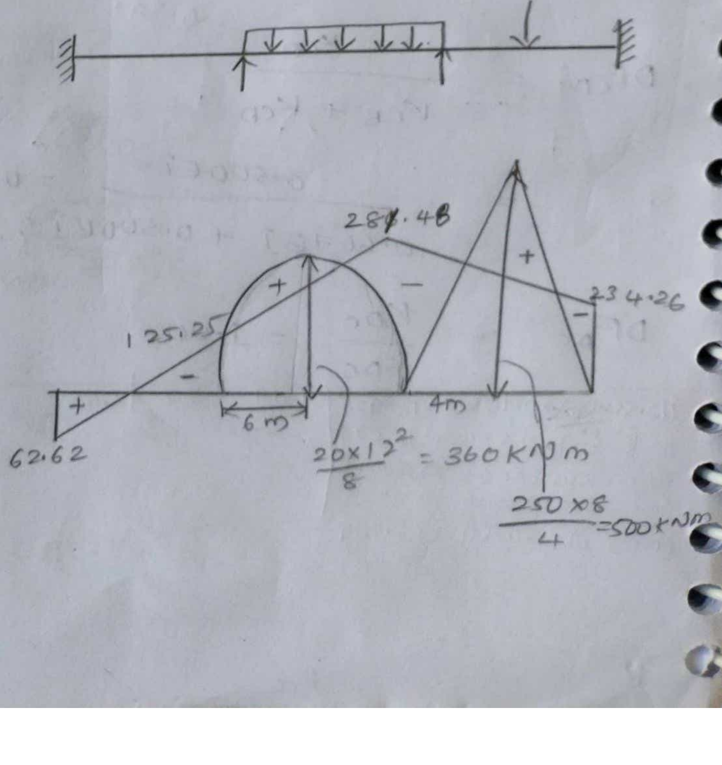
Joint	Member	Relative stiffness (K)	ΣK	$DF = \left(\frac{K}{\Sigma K}\right)$
B	BA	$\frac{I}{L} = \left(\frac{I}{12}\right)$	$\frac{I}{6}$	$\frac{I/12}{I/6} = 0.50$
	BC	$\frac{I}{L} = \left(\frac{I}{12}\right)$		$\frac{I/12}{I/6} = 0.50$
C	CB	$\frac{I}{L} = \left(\frac{I}{12}\right)$	$\frac{5I}{24}$	$\frac{I/12}{5I/24} = 0.40$
	CD	$\frac{I}{L} = \left(\frac{I}{8}\right)$		$\frac{I/8}{5I/24} = 0.60$

$(250 - 240) \times 0.4 = 4$

$(250 - 240) \times 0.6 = 6$

Step-3:- Moment distribution M.

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	
D.F	0	0.5	0.5	0.4	0.6	0
FEM	0	0	-240	+240	-250	+250
Balance	–	+120	+120	4	6	–
C.O	60	–	2	60	–	3
Balance	–	-1	-1	-24	-36	–
C.O	-0.5	–	-12	-0.5	–	-18
Balance	–	+6	+6	0.2	0.3	–
C.O	3	–	0.1	3	–	0.15
Balance	–	0.05	-0.05	-1.2	-1.8	–
C.O	-0.03	–	-0.6	-0.03	–	0.9
Balance	–	0.3	+0.3	0.01	0.02	–
C.O	0.15	–	–	–	–	0.01
Final Moments	62.62	125.25	-125.25	281.48	-281.48	234.26



Analysis of inclined frames - SFD, BMD, Elastic curve.

→ Inclined members are used, though less frequently, in pitched roofs, in high trestles (an open braced framework), in framed girders for bridges.

→ Mostly inclined members are used in buildings for elagance in appearance.

→ Rigid jointed structures involving inclined members in their construction are of two types.

→ First one is the single (or) multi bay rigid portal frame.

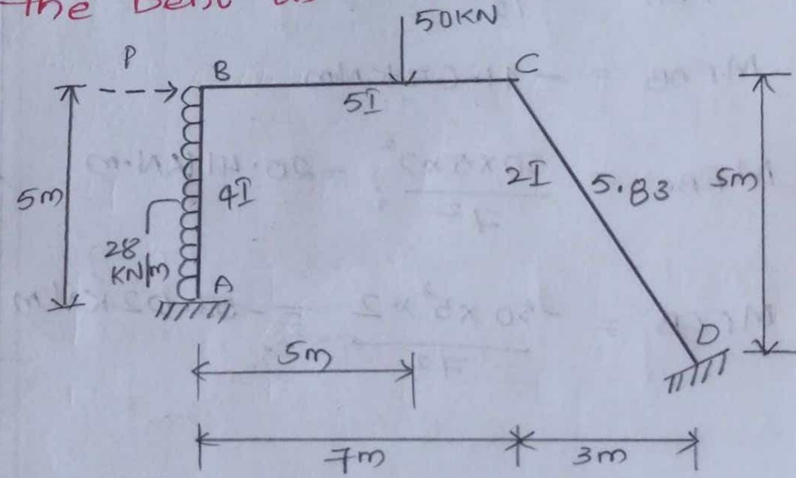
→ These types are used for construction of factories & it not only presents a clean & elegant appearance but also provides unrestricted internal space by bracing members.

→ The second type is the open - frame cantilever (or) girder. This is used a trestles in vertical orientation (or) as a bridge in horizontal position.

→ In these types of applications, the inclined members supports the roof in portal frame constuction, also help to resist lateral forces better in open frame.

→ In such application, the inclined members is both functional & pleasing.

Problem no (13): Find the end moments in the members of the bent as shown below.



Sol: The length of the member

$$CD = \sqrt{5^2+3^2}$$
$$= 5.83m.$$

The loading & geometrical configuration of the frame is unsymmetrical, it undergoes sway moment.

Moment distribution under vertical loadings :-

Relative stiffness:-

Member	K	ΣK	$DF = \left(\frac{K}{\Sigma K}\right)$
AB	$4I/L$	$\frac{4I}{5} = 0.8I$	$\frac{0.8I}{0.8I+0.714I} = 0.528\ (DF_{BA})$
BC	$\frac{5I}{L}$	$\frac{5I}{7} = 0.714I$	$\frac{0.74I}{0.714I+0.343I} = 0.675\ (DF_{CB})$
CD	$\frac{2I}{L}$	$\frac{2I}{5.83} = \frac{0.343}{I}$	$\frac{0.343I}{0.741I+0.343I} = 0.35\ (DF_{CD})$

$D.F_{AB} = 0$; $DF_{BA} = 0.528$; $DF_{BC} = 0.472$

$DF_{CB} = 0.675$; $DF_{CD} = 0.325$

Fixed-End Moments:-

$$M_{FBA} = \frac{20 \times 5^2}{12} = 41.67\,KNm$$

$$M_{FAB} = -41.67\,KNm$$

$$M_{FBC} = \frac{50 \times 5 \times 2^2}{7^2} = 20.41\,KN.m$$

$$M_{FCB} = \frac{-50 \times 5^2 \times 2}{7^2} = -51.02\,KNm$$

	A	B		C		D
DF	0	0.528	0.472	0.675	0.325	0
FEM	+41.67	-41.67	20.42	-51.02	0	0
Bal	—	11.23	10.03	34.44	16.58	—
C.O	5.615	—	17.22	5.015	—	8.29
Balance	—	-9.09	-8.13	-3.385	-1.63	—
C.O	-4.545	—	-1.693	-4.065	—	0.815
Balance	—	0.894	0.8	2.744	1.321	—
C.O	0.447	0	1.372	0.4	0	0.661
Balance	—	-0.724	-0.648	-0.27	-0.13	—
C.O	-0.362	—	-0.135	-0.34	—	0.65
B	—	0.0713	0.064	0.2187	0.1053	—
	42.825	-39.29	39.29	-16.246	16.246	8.201