UNIT-II

KANI'S METHOD

"Gasper kani", a german engineer, developed another distribution procedure based on slope deflection equations.

→ This method is very useful for the analysis of multistorey frames.

→ The greatest advantage of this method is, even if a mistake is committed in distribution in one of the cycles, it converges finally to the correct answer.

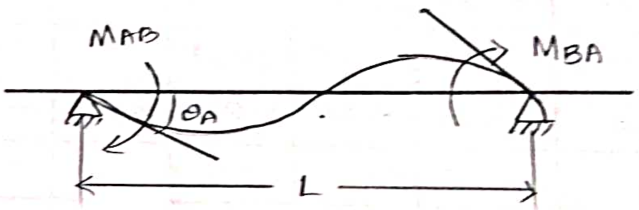
→ Even today, many practising engineers who are not familiar with computer methods, use kani's method for the analysis of 3 to 4 storey building frames.

→ This method is first explained for structures with fixed ends.

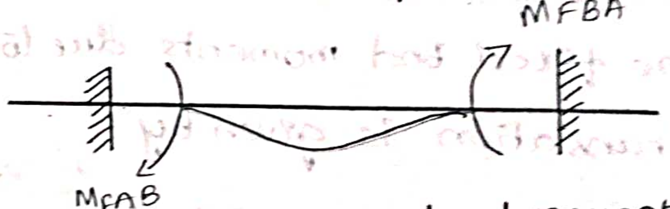
→ Then, the modifications to handle simply supported and overhanging ends are discussed.

→ Analysis of symmetric frames making use of the symmetry is also explained, after which an analysis of general frames is taken up.

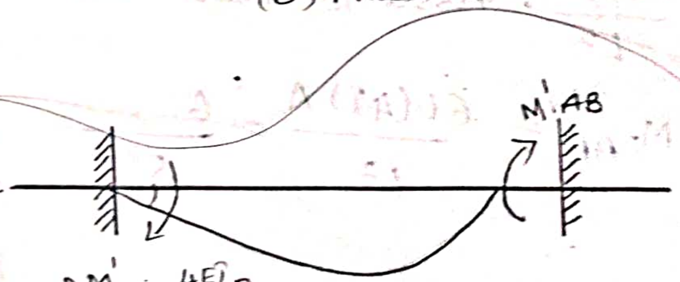
Analysis of continuous beams including settlement of supports



(a) A typical member



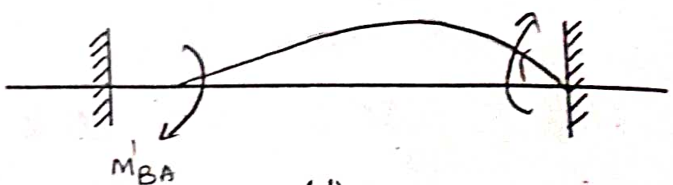
(b) Fixed end moments



$2M'_{AB} = \frac{4EI}{L}\theta_A$

(c) Moment due to rotation of end 'A'

$2M'_{BA} = \frac{4EI}{L}\theta_B$



(d)

Moment due to rotation of end B.

→ Members AB, shown in figure (a), is an intermediate member of a beam/frame, which has no relative displacements at the ends (i.e., ends A and B are at the same level).

→ Let M_{AB} and M_{BA} be the final end moments.

→ M_{AB} may consist of:

(i) Fixed end moments $(\theta_A = \theta_B = 0)$ (figure (b))

(ii) Moment due to rotation of end A only (figure (c))

(iii) Moment due to rotation of end B only (figure (d))

→ Let the moment developed at A due to rotation θ_A only be $2M'_{AB}$.

→ Naturally, it is equal to $\frac{4EI}{L}\theta_A$.

→ Hence, moment developed at B = M'_{AB}.

→ Similarly, the moments developed at ends A and B due to rotation θ_B only are M'_{BA} and

$2M'_{BA} = \left[\frac{4EI}{L}\theta_B\right]$ respectively.

$\therefore\ M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA}$ —— (1)

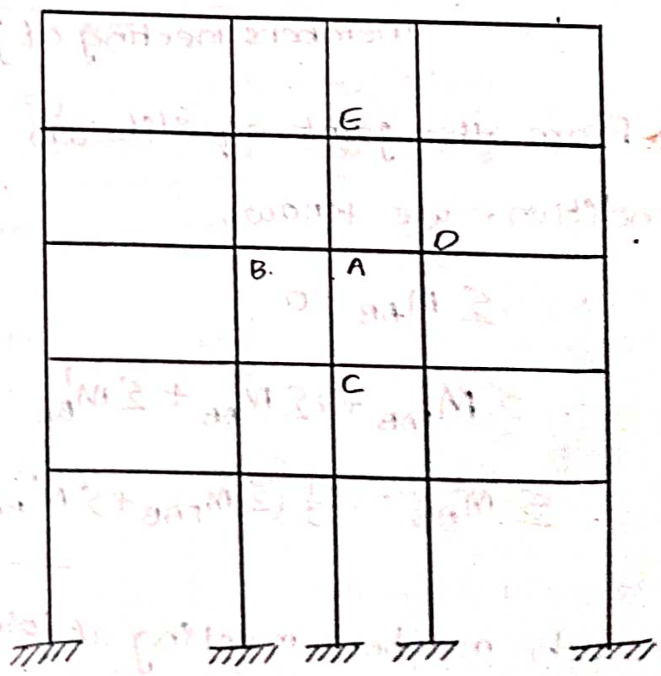
$M_{BA} = M_{FBA} + M'_{AB} + 2M'_{BA}$ —— (2)

→ The moments M'_{AB} and M'_{BA} are called rotation contributions.

→ In general, eq^n (1) may be stated as

Final moment = Fixed end moment + 2 [Rotational contribution of near end] + [Rotation contribution of far end]. —— (3)

→ Now, consider the moments at joint A in the frame shown in figure.



A typical frame.

According to eq^n (3)

$M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA}$

$M_{AC} = M_{FAC} + 2M'_{AC} + M'_{CA}$

$M_{AD} = M_{FAD} + 2M'_{AD} + M'_{DA}$

$M_{AC} = M_{FAC} + 2M'_{AC} + M'_{CA}$

$\Sigma M_{AB} = \Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA}$

where ΣM_{AB} = sum of near end moments in all the members meeting at joint A.

$\Sigma M'_{FAB}$ = sum of fixed end moments in all the members at joint A.

$\Sigma M'_{AB}$ = sum in near end rotation contributions of all the members meeting at joint A.

$\Sigma M'_{BA}$ = sum of far end rotation contributions of all the members meeting at joint A'.

→ From the joint equilibrium condition, we know,

$$\Sigma M_{AB} = 0$$

$$\Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA} = 0$$

$$\Sigma M'_{AB} = -\frac{1}{2}(\Sigma M_{FAB} + \Sigma M'_{BA}) \quad \text{— (4)}$$

For each member meeting at joint A,

$$2M'_{AB} = \left(\frac{4EI}{L}\right)\theta_A = K_{AB}\theta_A$$

$$M'_{AB} = \frac{1}{2}K_{AB}\theta_A$$

$$\Sigma M'_{AB} = \frac{1}{2}\Sigma K_{AB}\theta_A$$

$$= \frac{1}{2}\theta_A \Sigma K_{AB},$$

Since, θ_A is the same for all the members meeting at joint A

$$\frac{M'_{AB}}{\Sigma M'_{AB}} = \frac{K_{AB}}{\Sigma K_{AB}}$$

$$M'_{AB} = \left[\frac{K_{AB}}{\Sigma K_{AB}}\right]\Sigma M'_{AB} \quad \text{— (5)}$$

Substituting eq^n (4) & (5),

$$M'_{AB} = -\frac{1}{2}\left[\frac{K_{AB}}{\Sigma K_{AB}}\right](\Sigma M_{FAB} + \Sigma M'_{BA}) \quad \text{— (6)}$$

→ The expression $\boxed{-\frac{1}{2}\left[\frac{K_{AB}}{\Sigma K_{AB}}\right]}$ is called the Rotation factor(RF) for member AB at joint A.

→ From the eq^n, Kani developed the rotation contribution method.

→ In any given problem, the fixed end moments at all joints can be found.

→ Hence, at any joint, ΣM_{FAB}, can be found.

→ To calculate the rotation contributions from eq^n (6), we require the far end contributions which are not known.

→ Assuming them to be zero, calculate the near end contribution using eq^n. (6).

→ Likewise, we calculate the near end rotation contribution for the next joint taking the far end contribution, if available, or, otherwise, assuming it to be zero.

→ In this manner, we calculate the rotation contributions at all joints, which completes the first cycle.

→ Now, the rotation contributions at the far ends are also available.

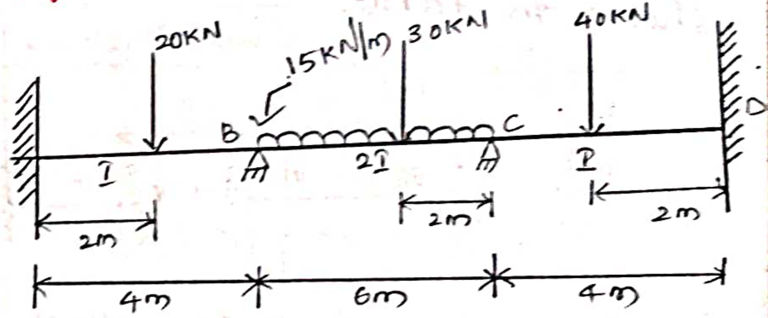
→ Using eq^n (6) again, we calculate the near end contributions at all the joints to complete the second cycle.

→ Repeat the procedure until a change in the rotation contributions of two successive iterations is negligible. Then, calculate the final moments using eq^n (3)

Application of Kani's Method to continuous Beams with fixed ends.

The above procedure may be applied to continous beams with fixed ends noting that the rotation contribution at the end is zero, since the end being fixed, its rotation is zero.

Problem no (1):- Analyse the continuous beam shown in figure 3.3 by Kani's method.



Continous beam.

Sol: Step-1:- Fixed End Moments:-

$$M_{FAB} = \frac{-Wl}{8} \Rightarrow \frac{-20\times4}{8} \Rightarrow -10KNm$$

$$M_{FBA} = \frac{+Wl}{8} \Rightarrow \frac{20\times4}{8} \Rightarrow +10KNm$$

$$M_{FBC} = \frac{-wl^2}{12} \Rightarrow \frac{-15\times6^2}{12} - \frac{30\times4\times2^2}{6^2}$$

$$\Rightarrow -58.33KNm.$$

$$M_{FCB} = \frac{+wl^2}{12} \Rightarrow \frac{+15\times6^2}{12} + \frac{30\times4^2\times2}{6^2}$$

$$\Rightarrow 71.67KNm.$$

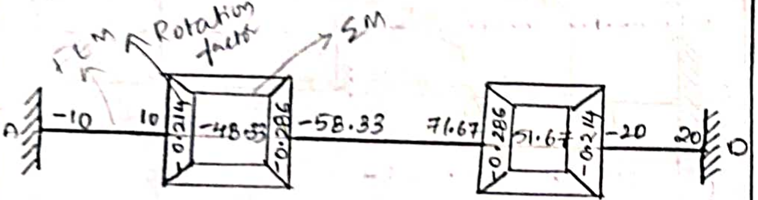
$$M_{FCD} = \frac{-40\times4}{8} \Rightarrow -20KNm$$

$$M_{FDC} = 20KNm$$

Step 2: Rotation factor (RF) = $\boxed{-\frac{1}{2}\left(\frac{K_{AB}}{\Sigma K}\right)}$

Joint	Members	K	ΣK	RF
B	BA	$\frac{4EI}{4} = EI$	2.33EI	-0.214
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		-0.286
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	2.33EI	-0.286
	CD	$\frac{4EI}{4} = EI$		-0.214

Step 3: Rotation contribution are calculated in the tabular form shown in table.



10.34	13.84	-18.73	-14.01
14.35	19.18	-20.26	-15.16
14.68	19.62	-20.39	-15.26
14.70	19.65	-20.40	-15.26

@ joint B $\Sigma M_{FEM} = 10 - 58.33 = -48.33$
@ joint C $\Sigma FEM = 71.67 - 20 = 51.67$

Iteration No. 1:

① M'_{BA} = Rotation factor × [ΣM @ joint B + Σ Rotation contribution from far end]

$M'_{BA} = -0.214 \times [-48.33+0]$

$\boxed{M'_{BA} = 10.34}$

② $M'_{BC} = -0.286 \times [-48.33+0]$

$\boxed{M'_{BC} = 13.82}$

③ $M'_{CB} = -0.286 \times [51.67+13.82]$

$\boxed{M'_{CB} = -18.73}$

④ $M'_{CD} = -0.214 \times [51.67+13.82]$

$\boxed{M'_{CD} = -14.01}$

All joints which rotate have been considered.

→ Hence, a cycle is completed.

Iteration no. 2:

The 2nd cycle again starts from joint B. The far end contribution of member BC is -18.73

Σ far end contribution = -18.73

$M'_{BA} = -0.214 \times [-48.33 - 18.73]$

$\boxed{M'_{BA} = 14.35}$

$M'_{BC} = -0.286 \times [-48.33 - 18.73]$

$\boxed{M'_{BC} = 19.18}$

$M'_{CB} = -0.286 \times [51.67 + 19.18]$

$\boxed{M'_{CB} = -20.26}$

$M'_{CD} = -0.214 \times [51.67 + 19.18]$

$\boxed{M'_{CD} = -15.16}$

→ Similarly, further cycles are carried out.

→ The difference between rotation contribution of cycle 4 and cycle 3 are negligible.

→ Hence, the distribution

procedure is stopped.

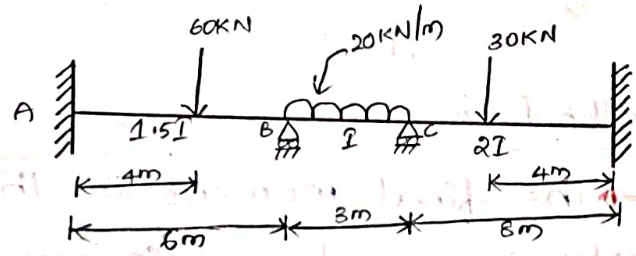
Final moment = Fixed end moment

+2 (Near end rotation cont^{n})

+ Far end contribution.

Joints	A	B		C		D
FEM	−10	10	−58.33	71.67	−20	20
Near end Contribution	2×0=0	2×14.7 = 29.4	2×19.65 = 39.3	2×20.40 = 40.8	2×(−15.16) = −30.32	2×0 = 0
Far end contribution	14.70	0	−20.40	19.65	0	−15.26
Final Moments.	4.70	39.4	−39.4	50.52	−50.52	4.74

problem no. ②: Analyse the continuous beam shown in figure by kani's method.



continuous beam.

Sol: Fixed End Moments:-

$-\frac{wab^2}{L^2}$ $\quad M_{FAB} = \frac{-60\times4\times2^2}{6^2} = \boxed{-26.67\,KNm}$

$\frac{wa^2b}{L}$ $\quad M_{FBA} = \frac{60\times4^2\times2}{6^2} = \boxed{53.33\,KNm}$

$-\frac{wl^2}{12}$ $\quad M_{FBC} = -\frac{20\times3^2}{12} = \boxed{-15\,KNm}$

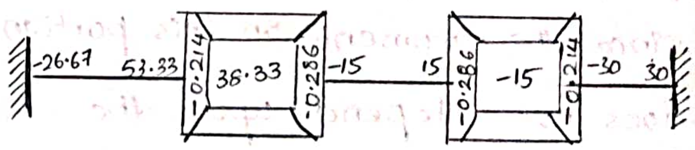
$+\frac{wl^2}{12}$ $\quad M_{FCB} = \frac{+20\times3^2}{12} = \boxed{15\,KNm}$

$-\frac{wl}{8}$ $\quad M_{FCD} = \frac{-30\times8}{8} = \boxed{-30\,KNm}$

$+\frac{wl}{8}$ $\quad M_{FDC} = \frac{30\times8}{8} = \boxed{30\,KNm}$

Rotation factor (RF) $= -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

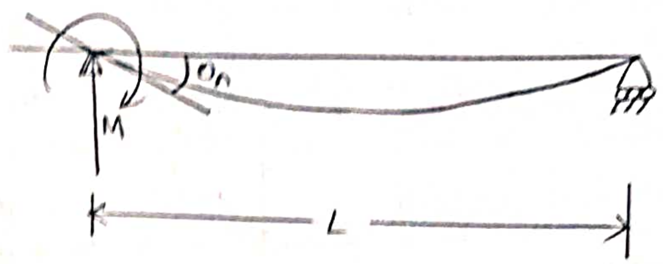
Joints	Members	K	ΣK	R.F
B	BA	$\frac{4E(1.5I)}{6}$ ⇒ EI	2.33EI	−0.214
	BC	$\frac{4EI}{3}$		−0.286
C	CB	$\frac{4EI}{3}$	2.33EI	−0.286
	CD	$\frac{4E(2I)}{8}$ = EI		−0.214



−8.20	−10.96	7.43	5.56
−9.79	−13.09	8.03	6.01
−9.92	−13.26	8.08	6.04
$\boxed{-9.93}$	$\boxed{-13.27}$	$\boxed{8.09}$	$\boxed{6.05}$

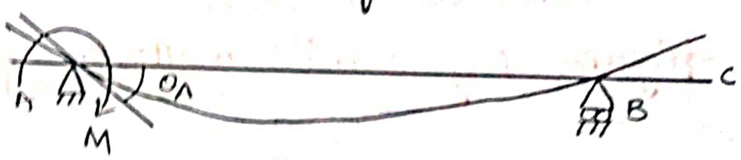
Joint	A	B		C		D
FEM	-26.61	28.88	-15	15	-30	30
Near end contribution	2×0 = 0	-2×9.93 = -19.86	-2×13.27 = -26.54	2×8.09 = 16.18	2×6.05 = 12.10	2×0 = 0
Far end contribution	-9.93	0	8.09	-18.27	0	-6.05
Final Moments	-36.60	38.74	-38.74	17.91	-17.91	36.05

Application to Continuous Beams with simply supported and overhanging Ends:



(a)

Rotation factors.



(b)

Method - I:-

→ If the end of the continuous beam is simply supported (or) has overhang, the last support also rotates.

→ Hence, rotation contribution of that joint also should be found.

→ Stiffness of overhanging portion may be taken as zero, since the moment in this portion does not depend upon the loading in the other portion.

→ Then, rotation factor for interior member at such joint is -0.5.

Method - II:-

→ The final moment at the simply supported end is zero and at the end support in the overhanging beam is equal to the end moment in the cantilever.

→ Referring to above figure, we know that rotation at the inner end A due to moment 'M' applied is

$$\boxed{\theta_A = \frac{ML}{3EI}}$$

∴ Moment required for unit rotation is $\boxed{\frac{3EI}{L}}$.

→ Thus, stiffness of member AB is $\boxed{\frac{3EI}{L}}$

→ If stiffness of member AB is modified as $\frac{3EI}{L}$, no moment gets transferred from end A to end B due to the rotation of end A.

→ In the first step, fixed end moments are calculated at end B also, as if it is a fixed end.

→ To achieve the first final moment at end B (zero in case of s.s. end & at cantilever end moment in overhanging beam), balancing moment shall be applied at joint B.

→ Then, half of this balancing moment goes to inner end A.

→ Thus, fixed end moment of inner end A is modified as

$M_{FAB} = M_{FAB} + 0.5 \times$ Balancing moment at B

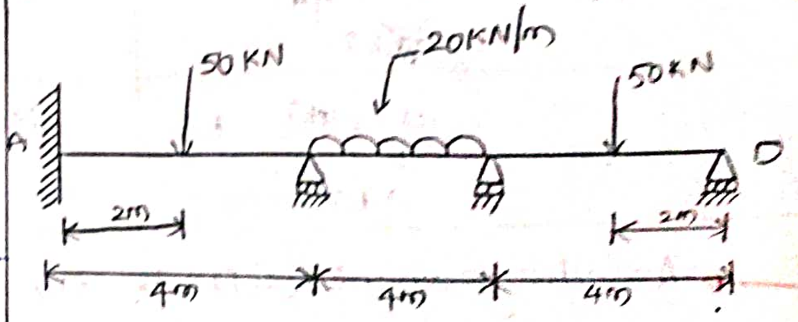
$= M_{FAB} - 0.5 \times$ unbalanced moment at B.

→ Method - I is a direct application of Kani's concept of rotation contribution, while method - II is a modification using the concept of moment distribution.

→ In method - II, the number of joints to be considered for calculating the rotation contribution is reduced.

→ Hence, calculation effort is reduced.

Problem no ③ :- Analyse the continuous beam shown in figure by Kani's method. Flexural rigidity is constant throughout.



Continuous Beam.

Fixed End Moments:

$M_{FAB} = -\frac{50\times4}{8} = \boxed{-25\,KNm}$

$M_{FBA} = +\frac{50\times4}{8} = \boxed{25\,KNm}$

$M_{FBC} = -\frac{20\times4^2}{12} = \boxed{-26.67\,KNm}$

$M_{FCB} = +\frac{20\times4^2}{12} = \boxed{26.67\,KNm}$

$M_{FCD} = -\frac{50\times4}{8} = \boxed{-25\,KNm}$

$M_{FDC} = +\frac{50\times4}{8} = \boxed{25\,KNm}$

Method - I:

Rotation factor (RF) $= -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Joint	Members	K	ΣK	RF
B	BA	$\frac{4EI}{L} \Rightarrow \frac{4EI}{4} \Rightarrow EI$	$EI+EI = 2EI$	-0.25
	BC	$\frac{4EI}{L} = \frac{4EI}{4} \Rightarrow EI$		-0.25
C	CB	$\frac{4EI}{L} = \frac{4EI}{4} \Rightarrow EI$	$EI+EI$ (0.75) $= 1.75EI$	-0.286
	CD	$\frac{3EI}{L} = \frac{3EI}{4} = 0.75EI$		-0.214
D	DC	$\frac{4EI}{L} = \frac{4EI}{4} \Rightarrow EI$	EI	-0.5

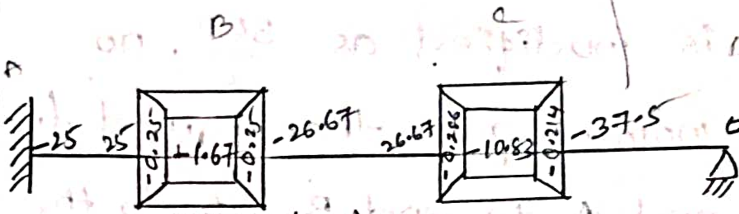
→ As the moment @D is zero
∴ Modification can be done in last span.

$M'_{FCD} = M_{FCD} - 0.5 \times$ unbalanced moment @ D

$= -25 - 0.5(25)$

$\boxed{M'_{FCD} = -37.5\,KNm}$

∴ $\boxed{M_{FDC} = 0\,KNm.}$ modified



(summation)

$-26.67 + 25 \Rightarrow -1.67$

$-37.5 + 26 \Rightarrow -10.83$

Iteration - 1:

$M'_{BA} = \text{Rotation factor} \times \left[\Sigma M^{FE} @ \text{joint B} + \Sigma \text{Rotation contribution from far end}\right]$

$= -0.25 \times [-1.67 + 0]$

$\boxed{M'_{BA} = 0.4175}$

$M'_{BC} = -0.25 \times [-1.67 + 0]$

$\boxed{M'_{BC} = 0.4175}$

$M'_{CB} = -0.286 \times [-10.83 + 0.4175]$

$\boxed{M'_{CB} = 2.97}$

$M'_{CD} = -0.214 \times [-10.83 + 0.4175]$

$\boxed{M'_{CD} = 2.22}$

Iteration - 2:

Rotation contribution from far end

$M'_{BA} = -0.25 \times [-1.67 + 2.97]$

$\boxed{M'_{BA} = -0.325}$

$M'_{BC} = -0.25 \times [-1.67 + 2.97]$

$\boxed{M'_{BC} = -0.325}$

$M'_{CB} = -0.286 \times [-10.83 - 0.325]$

$\boxed{M'_{CB} = 3.190}$

$M'_{CD} = -0.214 \times [-10.83 - 0.325]$

$\boxed{M'_{CD} = 2.38}$

Iteration - 3:-

$M'_{BA} = -0.25 \times [-1.67 + 3.190]$

$\boxed{M'_{BA} = -0.380}$

$M'_{BC} = -0.25 + [-1.67 + 3.190]$

$\boxed{M'_{BC} = -0.380}$

$M'_{CB} = -0.286 \times [-10.83 - 0.38]$

$\boxed{M'_{CB} = 3.20}$

$M'_{CD} = -0.2\cancel{86}14 \times [-10.83 - 0.38]$

$\boxed{M'_{CD} = 2.39}$

These iterations are to be done until we get the similar values between two rows.

→ Another iteration we required.

Iteration - 4:-

$M'_{BA} = -0.25 \times [-1.67 + 3.20]$

$\boxed{M'_{BA} = -0.382}$

$M'_{BC} = -0.25 \times [-1.67 + 3.20]$

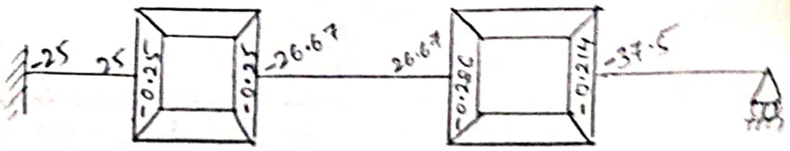
$\boxed{M'_{BC} = -0.382}$

$M'_{CB} = -0.286 \times [-10.83 - 0.38]$

$\boxed{M'_{CB} = 3.20}$

$M'_{CD} = -0.214 \times [-10.83 - 0.38]$

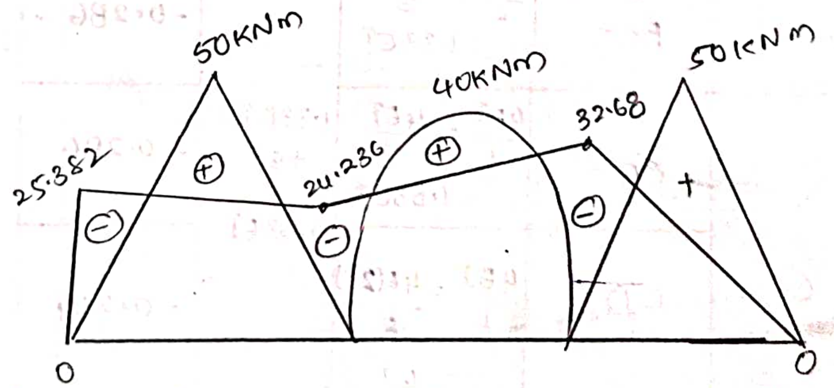
$\boxed{M'_{CD} = 2.39}$



0.4175	0.4175	2.97	2.22
-0.325	-0.325	3.190	2.38
-0.380	-0.380	3.20	2.39
$\boxed{-0.382}$	$\boxed{-0.382}$	$\boxed{3.20}$	$\boxed{2.39}$

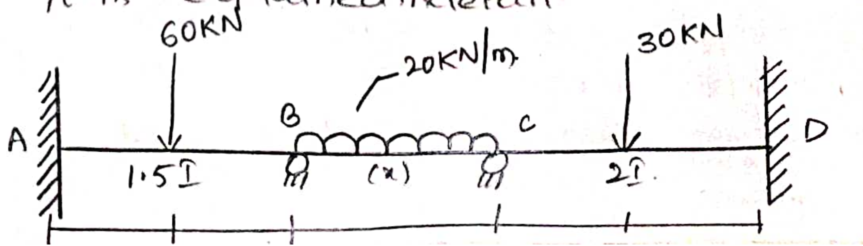
Joints	A	B		C		D
FEM	-25	+25	-26.67	+26.67	~~-37.5~~	0
Near end contribution	2×0 =0	2X-0.382 =-0.764	2X-0.382 =-0.764	2X3.20 =6.4	2X2.39 4.78	—
Farend contribution	-0.382	0	3.20	-0.382	0	—
Final Moment	25.382	24.236	-24.236	32.68	-32.7	—

BMD with original loading & final moments.



problem no 4:- Analyse the continuous beam by kani's method

problem 2 & 4 are same but here it is explained in detail.



Fixed End Moments:-

$$M_{FAB} = \frac{-Wab^2}{L^2} = \frac{-60\times4\times2^2}{6} = -26.67\,kNm$$

$$M_{FBA} = \frac{Wa^2b}{L^2} = \frac{60\times4^2\times2}{6} = +53.33\,kNm$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-20\times3^2}{12} = -15\,kNm$$

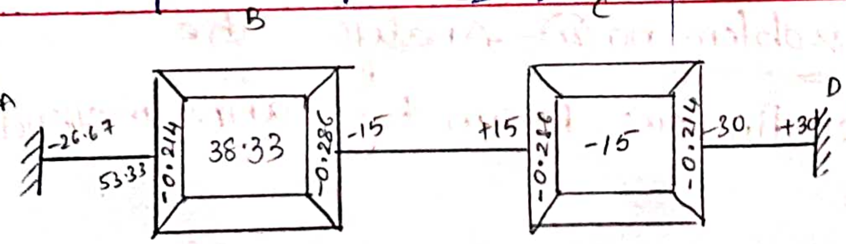
$$M_{FCB} = \frac{WL^2}{12} = \frac{20\times3^2}{12} = 15\,kNm.$$

$$M_{FCD} = \frac{-WL}{8} = \frac{-30\times8}{8} \Rightarrow -30\,kNm$$

$$M_{FDC} = \frac{+WL}{8} = \frac{+30\times8}{8} \Rightarrow 30\,kNm.$$

Rotation Factors:-

Joint	Members	K	ΣK	$RF = -\frac{1}{2}\frac{K}{\Sigma K}$
B	BA	$\frac{4EI}{L} = \frac{4EI.5}{6} = EI$	$EI + 1.33EI = 2.33EI$	-0.214
	BC	$\frac{4EI}{L} = \frac{4EI}{3} = 1.33EI$		-0.286
C	CB	$\frac{4EI}{L} = \frac{4EI}{3} = 1.33EI$	$1.33EI + EI = 2.33EI$	-0.286
	CD	$\frac{4EI}{L} = \frac{4E(2I)}{8} = EI$		-0.214



-8.20	-10.96	7.42	5.55
-9.79	-13.08	8.03	6.00
-9.92	-13.25	8.07	6.04
$\boxed{-9.93}$	$\boxed{-13.27}$	$\boxed{8.08}$	$\boxed{6.04}$

Iteration -1:-

$= RF \times (\Sigma FEM @ \text{Joint} + \text{far end rotation contribution})$

$M'_{BA} = -0.214 \times [38.33 + 0]$

$\boxed{M'_{BA} = -8.20}$

$M'_{BC} = -0.286 \times [38.33 + 0]$

$\boxed{M'_{BC} = -10.96}$

$M'_{CB} = -0.286 \times [-15 + -10.96]$

$\boxed{M'_{CB} = 7.42}$

$M'_{CD} = -0.214 \times [-15 - 10.96]$

$\boxed{M'_{CD} = 5.56}$

Iteration -2:-

$M'_{BA} = -0.214 \times [38.33 + 7.42]$

$\boxed{M'_{BA} = -9.79}$

$M'_{BC} = -0.286 \times [38.33 + 7.42]$

$\boxed{M'_{BC} = -13.08}$

$M'_{CB} = -0.286 \times [-15 - 13.08]$

$\boxed{M'_{CB} = 8.03}$

$M'_{CD} = -0.214 \times [-15 - 13.08]$

$\boxed{M'_{CD} = 6.00}$

Iteration -3:-

$M'_{BA} = -0.214 \times [38.33 + 8.03]$

$\boxed{M'_{BA} = -9.92}$

$M'_{BC} = -0.286 \times [38.33 + 8.03]$

$\boxed{M'_{BC} = -13.25}$

$M'_{CB} = -0.286 \times [-15 - 13.25]$

$$\boxed{M'_{CB} = 8.07}$$

$$M'_{CD} = -0.214 \times [-15 - 13.25]$$

$$\boxed{M'_{CD} = 6.04}$$

Iteration-4:-

$$M'_{BA} = -0.214 \times [38.33 + 8.07]$$

$$\boxed{M'_{BA} = -9.92}$$

$$M'_{BC} = -0.286 \times [38.33 + 8.07]$$

$$\boxed{M'_{BC} = -13.27}$$

$$M'_{CB} = -0.286 \times [-15 - 13.27]$$

$$\boxed{M'_{CB} = 8.08}$$

$$M'_{CD} = -0.214 \times [-15 - 13.27]$$

$$\boxed{M'_{CD} = 6.04}$$

Iteration-5:-

$$M'_{BA} = -0.214 \times [38.33 + 8.08]$$

$$\boxed{M'_{BA} = -9.93}$$

$$M'_{BC} = -0.286 \times [38.33 + 8.08]$$

$$\boxed{M'_{BC} = -13.27}$$

$$M'_{CB} = -0.286 \times [\cancel{8} -15 - 13.27]$$

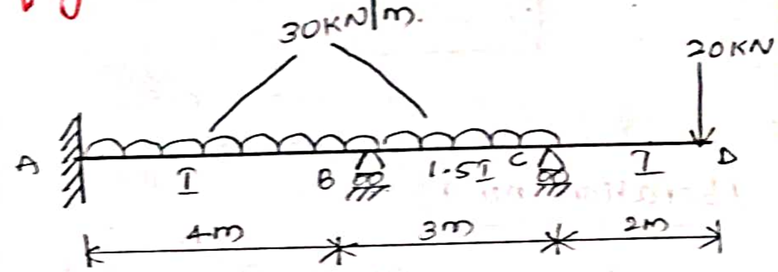
$$\boxed{M'_{CB} = 8.08}$$

$$M'_{CD} = -0.214 \times [-15 - 13.27]$$

$$\boxed{M'_{CD} = 6.04}$$

Joints	A	B		C		D
FEM	-26.67	53.33	-15	15	-30	30
Near end contribution	2×0	2× 9.92 = -19.86	2× 13.27 = -26.54	2×8.09 = 16.18	2×6.05 = 12.10	2×0 = 0
Far end contribution	-9.93	0	8.09	-13.27	0	-6.05
Final Moments	-36.60	33.74	-33.74	17.91	-17.91	36.05

Problem no. 5:- Analyse the continuous beam shown in figure.



Sol: **Fixed End Moments:-**

$$M_{FAB} = -\frac{WL^2}{12} = \frac{-30\times4^2}{12} = -40\,KNm$$

$$M_{FBA} = +\frac{WL^2}{12} = \frac{+30\times4^2}{12} = 40\,KNm$$

$$M_{FBC} = -\frac{WL^2}{12} = \frac{-30\times3^2}{12} = -22.5\,KNm$$

$$M_{FCB} = +\frac{WL^2}{12} = \frac{+30\times3^2}{12} = 22.5\,KNm$$

$$M_{FCD} = -\frac{WL}{\cancel{12}} \Rightarrow -20\times2 \Rightarrow -40\,KNm$$

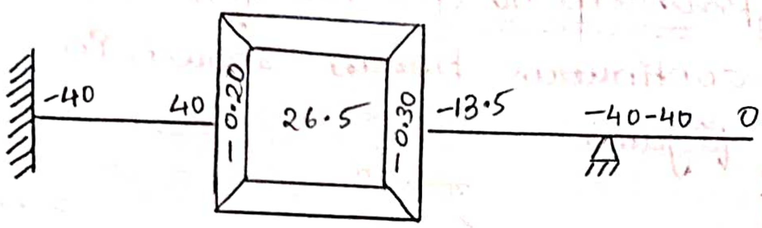
modification to take care of rotation of support C.

Modified $M_{FBC} = M_{FBC} - 0.5 \times$ unbalanced moment at C

$$M_{FBC} = -22.5 - 0.5(22.5 - 40)$$
$$= -13.5\,KNm$$

Rotation factor (RF) = $-\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Joint	Members	K	ΣK	RF
B	BA	$\frac{4EI}{4} = EI$	$EI + 1.5EI \Rightarrow 2.5EI$	$\Rightarrow -\frac{1}{2} \times \frac{EI}{2.5EI} = -0.20$
	BC	$\frac{3E(1.5I)}{3} \Rightarrow 1.5EI$		$\Rightarrow -\frac{1}{2} \times \frac{1.5EI}{2.5EI} = -0.30$



@ joint B = 40 − 13.5 = 26.5

−5.3 −7.95

Iteration no. 1:-

$M'_{BA} = -0.20 \times [26.5 + 0]$

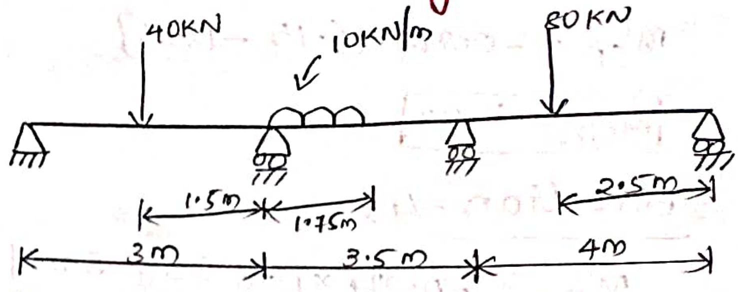
$= -5.3$

$M'_{CB} = -0.30 \times [26.5 + 0]$

$= -7.95$

Joints	A	B		C		D
FEM	−40	40	−13.5	40	−40	0
Near end contribution	2×0 = 0	2×(−5.3) = −10.6	2×(−7.95) = −15.9	0	0	0
Far end contribution	−5.3	0	0	0	0	0
Final moments	−45.3	+29.40	−29.4	40	−40	0

Problem no. ⑥ :- Analyse the continuous beam shown in figure. by Kani's method. EI is constant throughout.



sol: **Fixed End Moments:-**

$$M_{FAB} = \frac{-WL}{8} \Rightarrow \frac{-40\times3}{8} = -15\,KNm$$

$$M_{FBA} = \frac{+WL}{8} \Rightarrow \frac{40\times3}{8} = 15\,KNm$$

$$M_{FBC} = -\int_0^{1.75} \frac{10dx \times x(3.5-x)^2}{(3.5)^2}$$

$$= \frac{-10}{(3.5)^2}\int_0^{1.75}\left[(3.5)^2x - 7x^2 + x^3\right]dx$$

$$= \frac{-10}{(3.5)^2}\left[(3.5)^2\left(\frac{x^2}{2}\right) - 7\left(\frac{x^3}{3}\right) + \frac{x^4}{4}\right]_0^{1.75}$$

$$= -7.02\,KNm$$

$$M_{FCB} = \int_0^{1.75} \frac{10dx \times x^2(3.5-x)}{(3.5)^2}$$

$$= \frac{10}{(3.5)^2}\int_0^{1.75}(3.5x^2 - x^3)dx$$

$$= \frac{10}{(3.5)^2}\left[3.5\left(\frac{x^3}{3}\right) - \frac{x^4}{4}\right]_0^{1.75}$$

$$= 3.19\,KNm$$

$$M_{FCD} = -\frac{Wab^2}{L^2} = \frac{-80\times1.5\times2.5^2}{4^2}$$

$$M_{FCD} = -46.88\,KNm$$

$$M_{FDC} = -\frac{Wa^2b}{L^2} = \frac{-80\times1.5^2\times2.5}{4^2}$$

$$= 28.13$$

modification to account for rotation of A and D.

$M_{FAB} = 0$

$M_{FBA} = 15 - 0.5(-15)$

$= 22.5 \, kNm$

$M_{FCD} = -46.88 - 0.5(28.13)$

$= -60.95 \, kNm.$

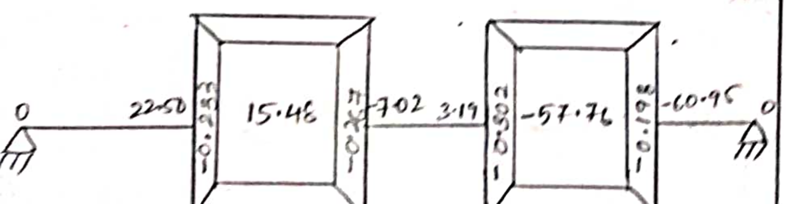
$M_{FDC} = 0$

Rotation factor $(RF) = -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Joint	Members	K	ΣK	$RF = \frac{1}{2}\left(\frac{K}{\Sigma K}\right)$
B	BA	$\frac{3EI}{3} \Rightarrow EI$	2.143EI	-0.233
	BC	$\frac{4EI}{3.5} = 1.143EI$		-0.267
C	CB	1.143EI	1.893EI	-0.302
	CD	$\frac{3}{4}EI = 0.75EI$		-0.198

@ Joint B ⇒ 22.50 − 7.02 = 15.48

@ Joint C ⇒ 3.19 − 60.95 = −57.76



-3.60	-4.13	18.69	12.25
-7.96	-9.12	20.19	13.24
-8.31	-9.52	20.31	13.32
$\boxed{-8.38}$	$\boxed{-9.55}$	$\boxed{20.32}$	$\boxed{13.32}$

Iteration no. 1;

$M'_{BA} = -0.233 \times [15.48 + 0]$

$\boxed{M'_{BA} = -3.60}$

$M'_{BC} = -0.267 \times [15.48 + 0]$

$\boxed{M'_{BC} = -4.13}$

$M'_{CB} = -0.302 \times [-57.76 - 4.13]$

$\boxed{M'_{CB} = +18.69}$

$M'_{CD} = -0.198 \times [-57.76 - 4.13]$

$\boxed{M'_{CD} = 12.25}$

Iteration no. 2 :

$M'_{BA} = -0.233 \times [15.48 + 18.69]$

$\boxed{M'_{BA} = -7.96}$

$M'_{BC} = -0.267 \times [15.48 + 18.69]$

$\boxed{M'_{BC} = -9.12}$

$M'_{CB} = -0.302 \times [-57.76 - 9.12]$

$\boxed{M'_{CB} = 20.19}$

$M'_{CD} = -0.198 \times [-57.76 - 9.12]$

$\boxed{M'_{CD} = 13.24}$

Iteration no. 3 :-

$M'_{BA} = -0.233 \times [15.48 + 20.19]$

$\boxed{M'_{BA} = -8.31}$

$M'_{BC} = -0.267 \times [15.48 + \cancel{18.6}\ 20.19]$

$\boxed{M'_{BA} = -9.52}$

$M'_{CB} = -0.302 \times [-57.76 - 9.52]$

$\boxed{M'_{CB} = 20.31}$

$M'_{CD} = -0.198 \times [-57.76 - 9.52]$

$\boxed{M'_{CB} = 13.32}$

$\cancel{M'_{CB} = 1}$

Iteration no (4):

$M'_{BA} = -0.233 \times [15.48 + 20.31]$

$\boxed{M'_{BA} = -8.33}$

$M'_{BC} = -0.267 \times [15.48 + 20.31]$

$\boxed{M'_{BC} = -9.55}$

$M'_{CB} = -0.302 \times [-57.76 - 9.55]$

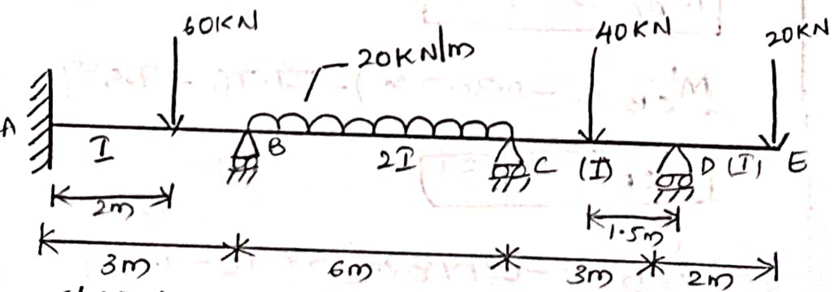
$\boxed{M'_{CB} = 20.32}$

$M'_{CD} = -0.198 \times [-57.76 - 9.55]$

$\boxed{M'_{CD} = 13.32}$

Joints	A	B		C		D
FEM	0	22.5	-7.02	3.19	-60.95	0
Near end contribution	2×0 =0	2×(-8.34) =-16.68	2×(9.56) =-19.12	2×20.33 =40.66	2×13.33 =26.66	2×0 =0
Far end contribution	0	0	20.33	-9.56	0	0
Final moments	0	5.82	-5.82	34.29	-34.29	0

Problem no. (7): Analyse the continuous beam shown in figure by Kani's method.



Sol: Step-1: **Fixed End Moments:**

$M_{FAB} = -\frac{Wab^2}{L^2} \Rightarrow \frac{-60\times2\times1^2}{3^2} = -13.33\,KNm$

$M_{FBA} = \frac{Wa^2b}{L^2} \Rightarrow \frac{60\times2^2\times1}{3^2} = 26.66\,KNm$

$M_{FBC} = -\frac{wL^2}{12} = -\frac{20\times6^2}{12} = -60\,KNm$

$M_{FCB} = +\frac{wL^2}{12} = \frac{20\times6^2}{12} = 60\,KNm$

$M_{FCD} = -\frac{WL}{8} = \frac{-40\times3}{8} = -15\,KNm$

$M_{FDC} = +\frac{WL}{8} = \frac{40\times3}{8} = 15\,KNm.$

$M_{FDE} = -2\times20 = -40\,KNm$

Modification in FEM to account for rotation of D

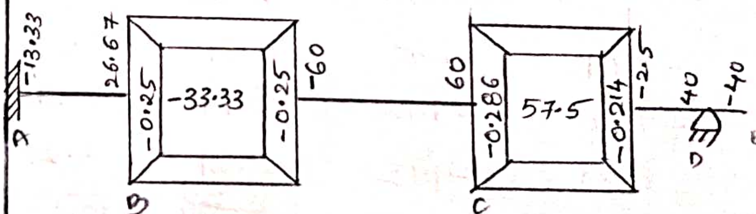
$M_{FDC} = -40\,KNm$ (to balance joint D)

$M_{FCD} = -15 - 0.5(15-40)$

$= -2.5\,KNm.$

Step-2: Rotational factor (RF) $= -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$

Joint	Members	K	ΣK	RF
B	BA	$\frac{4EI}{3}$	$\frac{8}{3}EI$	-0.25
	BC	$\frac{4E(2I)}{6} \Rightarrow \frac{4}{3}EI$		-0.25
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	2.33EI	-0.286
	CD	$\frac{3EI}{3} \Rightarrow EI$		-0.214



8.33	8.33	-18.82	-14.08
13.03	13.03	-20.17	-15.09
13.37	13.37	-20.26	-15.16
$\boxed{13.39}$	$\boxed{13.39}$	$\boxed{-20.27}$	$\boxed{-15.17}$

Iteration no. 1:-

$M'_{BA} = -0.25 \times [-33.33 + 0]$

$\boxed{M'_{BA} = 8.33}$

$M'_{BC} = -0.25 \times [-33.33 + 0]$

$\boxed{M'_{BC} = 8.33}$

$M'_{CB} = -0.286 \times [57.5 + 8.33]$

$\boxed{M'_{CB} = -18.82}$

$M'_{CD} = -0.214 \times [57.5 + 8.33]$

$\boxed{M'_{CD} = -14.08}$

Iteration no. 2:-

$M'_{BA} = -0.25 \times [-33.33 - 18.82]$

$\boxed{M'_{BA} = 13.03}$

$M'_{BC} = -0.25 \times [-33.33 - 18.82]$

$\boxed{M'_{BC} = 13.03}$

$M'_{CB} = -0.286 \times [57.5 + 13.03]$

$\boxed{M'_{CB} = -20.17}$

$M'_{CD} = -0.214 \times [57.5 + 13.03]$

$\boxed{M'_{CD} = -15.09}$

Iteration no. 3:-

$M'_{BA} = -0.25 \times [-33.33 - 20.17]$

$\boxed{M'_{BA} = 13.37}$

$M'_{BC} = -0.25 \times [-33.33 - 20.17]$

$\boxed{M'_{BC} = 13.37}$

$M'_{CB} = -0.286 \times [57.5 + 13.37]$

$\boxed{M'_{CB} = -20.26}$

$M'_{CD} = -0.214 \times [57.5 + 13.37]$

$\boxed{M'_{CD} = -15.16}$

Iteration no. 4:-

$M'_{BA} = -0.25 \times [-33.33 + (-20.26)]$

$\boxed{M'_{BA} = 13.39}$

$M'_{BC} = -0.25 \times [-33.33 - 20.26]$

$\boxed{M'_{BC} = 13.39}$

$M'_{CB} = -0.286 \times [57.5 + 13.39]$

$\boxed{M'_{CB} = -20.27}$

$M'_{CD} = -0.214 \times [57.5 + 13.39]$

$\boxed{M'_{CD} = -15.17}$

Final Moment Calculations:-

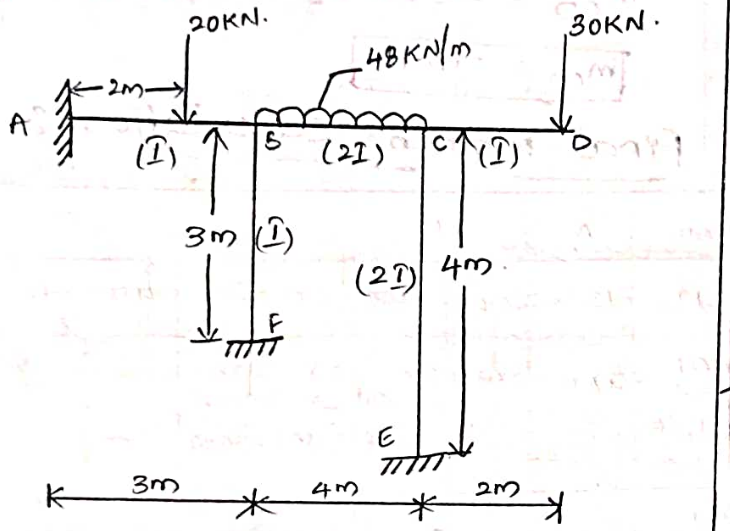
Joint	A	B		C		D		E
FEM	-13.33	26.66	-60	60	-2.5	40	-40	—
Near end contribution	2×0 =0	2×13.39 =26.78	2×(13.39) =26.78	2×-(20.27) =-40.54	2×-(15.17) =-30.34	0 —	— —	— —
Far end contribution	13.39	0	-20.27	13.39	0	—	—	—
Final Moments	0.06	53.44	-53.49	32.65	-32.84	40	-40	—

Analysis of frames without Lateral sway:-

→ If there is no sway in the frame, the analysis procedure is exactly the same as that for continuous beams, except that there may be more than two members meeting at the joint in the frames.

→ While calculating rotation factors and rotation contributions all the members meeting at the joints should be considered.

problem no. ⑧:- Analyse the rigid frame shown in figure by Kani's method.



Sol:- **Fixed End Moments:-**

$$M_{FAB} = \frac{-Wab^2}{L^2} = \frac{-20\times2\times1^2}{3^2} = -4.44\,KNm$$

$$M_{FBA} = \frac{Wa^2b}{L^2} = \frac{20\times2^2\times1}{3^2} = 8.88\,KNm$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-48\times4^2}{12} = -64\,KNm$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{48\times4^2}{12} = 64\,KNm$$

$$M_{FCD} = -30\times2 = -60\,KNm$$

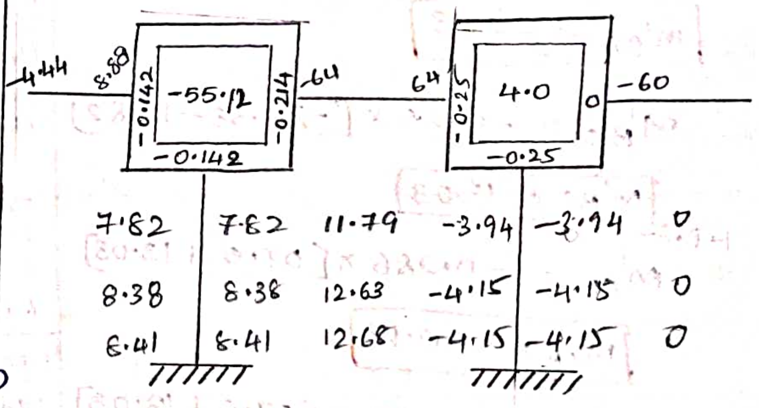
$$M_{FBF} = M_{FFB} = M_{FCE} = M_{FEC} = 0$$

Rotation factor:-

$$R.F = -\frac{1}{2}\left(\frac{K}{\Sigma K}\right)$$

Joint	Member	K	ΣK	RF
B	BA	$\frac{4EI}{L} = \frac{4EI}{3}$	$\frac{4EI}{3} + \frac{4EI}{3} + 2EI = 4.66EI$	-0.142
	BF	$\frac{4EI}{L} = \frac{4EI}{3}$		-0.142
	BC	$\frac{4E(2I)}{L} \Rightarrow \frac{4E(2I)}{4} \Rightarrow 2EI$		-0.214
C	CB	$2EI$	$2EI+2EI \Rightarrow 4EI$	-0.25
	CD	0		0
	CE	$\frac{4E(2I)}{4} \Rightarrow 2EI$		-0.25

Kani's Representation:-



Final Moments:-

Iteration no. ①:-

~~Final moments = FEM + 2(Nearend rotⁿ contⁿ) + (farend contⁿ)~~

~~$M_{FAB} = -4.44 + 2(0) + 8$~~

$$M'_{BA} = -0.142\times[-55.12+0] = 7.82$$

$$M'_{BF} = -0.142\times[-55.12+0] = 7.82$$

$$M'_{BC} = -0.214\times[-55.12+0] = 11.79$$

$M'_{CB} = -0.25 \times [4+11.79]$
$= 3.94$

$M'_{CD} = 0$

$M'_{CE} = -0.25 \times [4+11.79]$
$= -3.94$

Iteration no ②:-

$M'_{BA} = -0.142 \times [-55.12 + (-3.94)]$
$= 8.38$

$M'_{BF} = -0.142 \times [-55.12 - 3.94]$
$= 8.38$

$M'_{BC} = -0.214 \times [-55.12 - 3.94]$
$= 12.63$

$M'_{CB} = -0.25 \times [4 + 12.63]$
$= -4.15$

$M'_{CE} = -0.25 \times [4 + 12.63]$
$= -4.15$

$M'_{CD} = 0$

Iteration no ③:-

$M'_{BA} = -0.142 \times [-55.12 - 4.15]$
$= 8.41$

$M'_{BF} = -0.142 \times [-55.12 - 4.15]$
$= 8.41$

$M'_{BC} = -0.214 \times [-55.12 - 4.15]$
$= 12.68$

$M'_{CB} = -0.25 \times [4 + 12.68]$
$= -4.15$

$M'_{CE} = -0.25 \times [4 + 12.68]$
$= -4.15$

$M'_{CD} = 0$

Final Moments:-

Final moments = FEM + 2(Near end contributions + (far end contributions)

$M_{FAB} = -4.44 + 2(0) + 8.41$
$= 3.97 \, KNm$

$M_{BA} = 8.88 + 2(8.41) + 0$
$= 25.7 \, KNm$

$M_{BF} = 0 + 2 \times 8.41 + 0$
$= 16.82 \, KNm$

$M_{BC} = -64 + 2 \times 12.68 - 4.15$
$= -42.79 \, KNm$

$M_{CB} = 64 + 2(-4.15) + 12.68$
$= 68.38 \, KNm$

$M_{CD} = -60 \, KNm$

$M_{CE} = 0 + 2(-4.15) + 0$
$= -8.3 \, KNm$

$M_{EC} = 0 + 2 \times 0 - 4.15$
$= -4.15 \, KNm$

$M_{FB} = 0 + 2 + 0 + 8.41$
$= 8.41 \, KNm.$

CABLES AND SUSPENSION BRIDGES

- Cables provide an efficient means for supporting loads from bridges and roof systems.
- They are commonly used as cable car systems at sky slopes around the world, guys for derricks, radio towers etc.
- The steel cables manufacted from high-strength steel wire may provide the lowest cost-to-strength ratio of any common structural members.
- They are easily handled and positioned, even for long spans.
- The structural efficiency of the cable is due to its funicular shape taken by it while resisting the loads.
- A structure is said to behave in a funicular manner when it carries the loads by only internal tension (or) compression force.
- When the shape of the structure corresponds with the funicular curve for the loading present, no significant undesirable bending is present in the structure and efficient use of materials attained.
- Since a freely deforming cable subjected to the loads takes the shape of funicular curve, a profile assumed finally naturally, it develops only tensile forces.
- Inverting this structural form yields a new structure called arch which is analogous to the cable structure except that the forces developed are in a

state of compression rather than pure tension.

→ These two structures are collectively referred to as a group of funicular structures.

→ A cable of constant cross section supporting only its own dead weight deforms in to a catenary, while a cable carrying the load that is uniformly distributed along the horizontal projection of the cable, as in the case of suspension bridge, deforms in to a parabola.

→ Cables carrying concentrated point loads will deform in to a series of straight-line segments.

→ The magnitude of the forces developed in an arch (or) a cable depends on the relative height (or) depth of funicular shape in relation to its length as well as the magnitude and location of applied loads.

→ The greater the rise of an arch (or) the sag of a cable, the smaller is the internal forces developed in the structure, and vice versa.

→ The reactive forces developed at the arch (or) cable supports also depend on these parameters.

→ End reactions have both vertical and horizontal components which must be resisted by foundations (or) some other elements such as tie rods (or) struts.

Suspended Cable:-

→ suspended cable systems can span large distances.

→ However, simply suspended cable systems are particularly sensitive to vibratory wind effects, which have in the past caused the failure of many cable structures.

- Special precautions must be taken to prevent wind-induced instability.
- For analysis a cable is generally conceived as a continuous series of discrete elements joined together by pinned connections i.e., it is analogous to a chain.
- Since reactive force at each end has components one each in the vertical and horizontal directions.
- Thus, a total of four unknown components are present but only three independent equations of statics are available for their determination.
- The fourth condition is provided by the fact that the cable does not carry any bending moment i.e., the sum of all rotational effects produced by external and internal forces at a location on the cable must be zero.
- In a cable the external shear at a section is balanced by an internal resisting shear that is provided by the vertical components of the internal axial force (tension) in the cable, thus the cable must have a vertical projection in order to support the loads.
- The greatest the vertical projection, the smaller will be the cable tension.
- The overall external bending moment at a section is balanced by an internal resisting moment that is provided by the couple formed between the horizontal component of the tension in the cable at the same section and the horizontal component of

support reaction.

→ The resultant tension at any point is obtained from

$$\boxed{F_t = \sqrt{H^2+V^2}}$$

where, H, V are respectively,

→ the horizontal and vertical components of tensile force in the cable at that point.

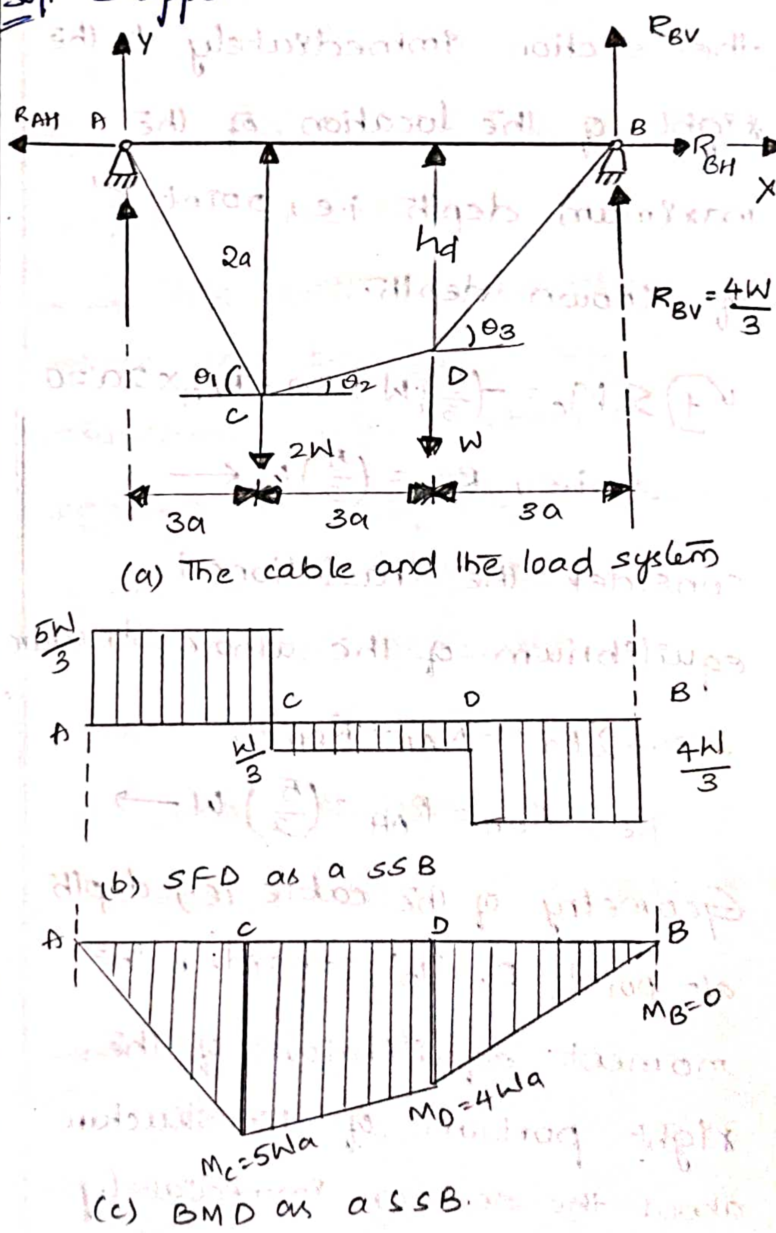
→ It is evident that the tension varies along the length of the cable.

→ In case of vertical loads only the value of H will be constant throughout the cable and the maximum tension will occur at the point where the vertical force is the maximum.

Cable Subjected to Concentrated Loads:-

Problem: Determine the deflected shape, reactions at the supports, internal forces (or) tensions in the various segments and the total length of the cable structure carrying two concentrated loads at one-third span points as shown in figure. The maximum depth under first load is given.

Sol: Support Reactions R_A & R_D.



(a) The cable and the load system

(b) SFD as a SSB

(c) BMD as a SSB.

Vertical reactions: Consider the moment equilibrium of the whole structure about A (or) D.

$\circlearrowleft + \sum M_A = -2W \times 3a - W \times 6a \times R_{DV} \times 9a = 0$

i.e., $R_{DV} = \left(\frac{4}{3}\right) W \uparrow$

$\circlearrowleft + \sum M_D = -R_{AV} \times 9a + 2W \times 6a + W \times 3a = 0$

i.e., $R_{AV} = (5/3) W \uparrow$

Horizontal reactions:- Consider the moment eq^m of the left portion of the structure about the section immediately to the right of the location of the maximum depth i.e., point 'C' of known depth.

$\circlearrowleft + \sum M_C = -\left(\frac{5}{3}\right) W \times 3a + R_{AH} \times 2a = 0$

∴ i.e., $R_{AH} = \left(\frac{5}{2}\right) W \leftarrow$

consider the traslational equilibrium of the whole structure.

$\rightarrow + \sum F_x = -R_{AH} + R_{DH} = 0$

i.e., $R_{DH} = R_{AH} = \left(\frac{5}{2}\right) W \rightarrow$

Geometry of the cable ie., depth at point D, h_D consider the moment equilibrium of the right portion of the structure about the section immediately to the left of point D.

$\circlearrowleft + \sum M_D = \left(\frac{4}{3}\right) W \times 3a - \left(\frac{5}{2}\right) W \times h_D = 0$

i.e., $h_D = \left(\frac{8}{5}\right) a$.

Inclination of segment CD

$$\theta_{CD} = \tan^{-1}\left[\frac{2a - 1.6a}{3a}\right]$$

$$= 7.59^\circ$$

Forces in the cable The forces in the individual segments are determined by using the method of joints.

Joint A The force in cable segment AC is the resultant of vertical and horizontal components.

Thus

$$F_{AC} = \sqrt{R_{AH}^2 + R_{AV}^2}$$

$$= \sqrt{\left(\frac{5W}{2}\right)^2 + \left(\frac{5W}{3}\right)^2}$$

$$= 3.005W \quad \text{(Tension)}$$

Joint C:

Equilibrium in horizontal direction, $\sum F_x = 0 \rightarrow +$

− Horizontal component of force in the segment AC + Horizontal component of force in the segment CD = 0

$$-\left(\frac{5}{2}\right) W + F_{CD} (\cos\theta) = 0$$

(or) $F_{CD} = \left(\frac{5}{2}\right) W (\sec\theta) = 2.522W$

(Tension)

The member BC is in tension as assumed, since the sign is positive.

The next step in method of joints is to proceed to an adjacent joint D.

However, it is convenient to consider joint B.

Joint B The force in the segment DB of the cable is the resultant of its vertical and horizontal components.

Thus

$$F_{DB} = \sqrt{R_{BH}^2 + R_{BV}^2}$$

$$= \sqrt{\left(\frac{5W}{2}\right)^2 + \left(\frac{4W}{2}\right)^2}$$

$$= 2.833W \quad \text{(Tension)}$$

It should be noted that if the loads are vertical the horizontal component of the force in the cable is always equal to the horizontal reaction because of translatory equilibrium in the horizontal direction.

Alternatively, the analysis can be carried out on the basis of load transfer mechanism ie, the external shear is balanced by the vertical component of the force in the cable.

The external bending moment is balanced by the couple formed between the horizontal component of the force in the cable and the horizontal component of support reaction.

This method uses shear force and bending moment diagrams for the externally applied load system and method of sections to calculate the vertical and horizontal components of cable forces.

Horizontal reaction :- Consider moment transfer criteria at point C.

Internal resisting couple = external bending moment

(or) $R_{AH} \times (2a) = 5Wa$ (from moment diagram)

ie, $R_{AH} = \left(\frac{5}{2}\right) W$

Depth at point D, h_D Use moment transfer criteria at point D.

$$\left(\frac{5}{2}\right)W \times h_D = 4Wa \text{ (from moment diagram)}$$

i.e., $h_D = \left(\frac{8}{5}\right)a$

Forces in the cable Vertical component of the force in the cable segment = External SF.

Segment AC $\theta_{AC} = \tan^{-1}\left(\frac{2a}{3a}\right)$

$$\theta_{AC} = 33.69^\circ$$

$$F_{AC}(\sin\theta_{AC}) = \left(\frac{5}{3}\right)W$$

i.e., $F_{AC} = +3.005W$ (Tension)

Segment CD $\theta_{CD} = 7.59^\circ$

$$F_{CD}(\sin\theta_{CD}) = \frac{1}{3}W$$

i.e, $F_{CD} = +2.524W$ (Tension)

Segment DB

$$\theta_{DB} = \tan^{-1}\left(\frac{1.6a}{3a}\right)$$

$$= 28.07^\circ$$

$$F_{DB}(\sin\theta_{DB}) = \left(\frac{4}{3}\right)W$$

i.e, $F_{DB} = +2.834W$ (Tension)

→ It should be noted that the reactions and internal forces depend on the height of structure.

Total length of cable = Sum of lengths of segments AC, CD and DB.

$$S = 3a \times (\sec\theta_{AC} + \sec\theta_{CD} + \sec\theta_{DB})$$

$$= 3a \times (1.2018 + 1.0088 + 1.1333)$$

$$= 10.032a.$$

General case:

→ A more general cable which has supports at different levels can be analyzed in much the same way as the case discussed above.

→ consider the case of a cable A–B of span L having its support B at height 'h' above the support A as shown in figure.

→ To determine support reactions of the cable, consider the total moment equilibrium about point A:

$$\circlearrowleft + \quad \Sigma M_A = -[W_1x_1 + W_2x_2 + \cdots] - R_{BH} \times h + R_{BV} \times L = 0$$

$$R_{BV} = \frac{\Sigma M_{A,ext}}{L} + \frac{R_{BH} \times h}{L}$$

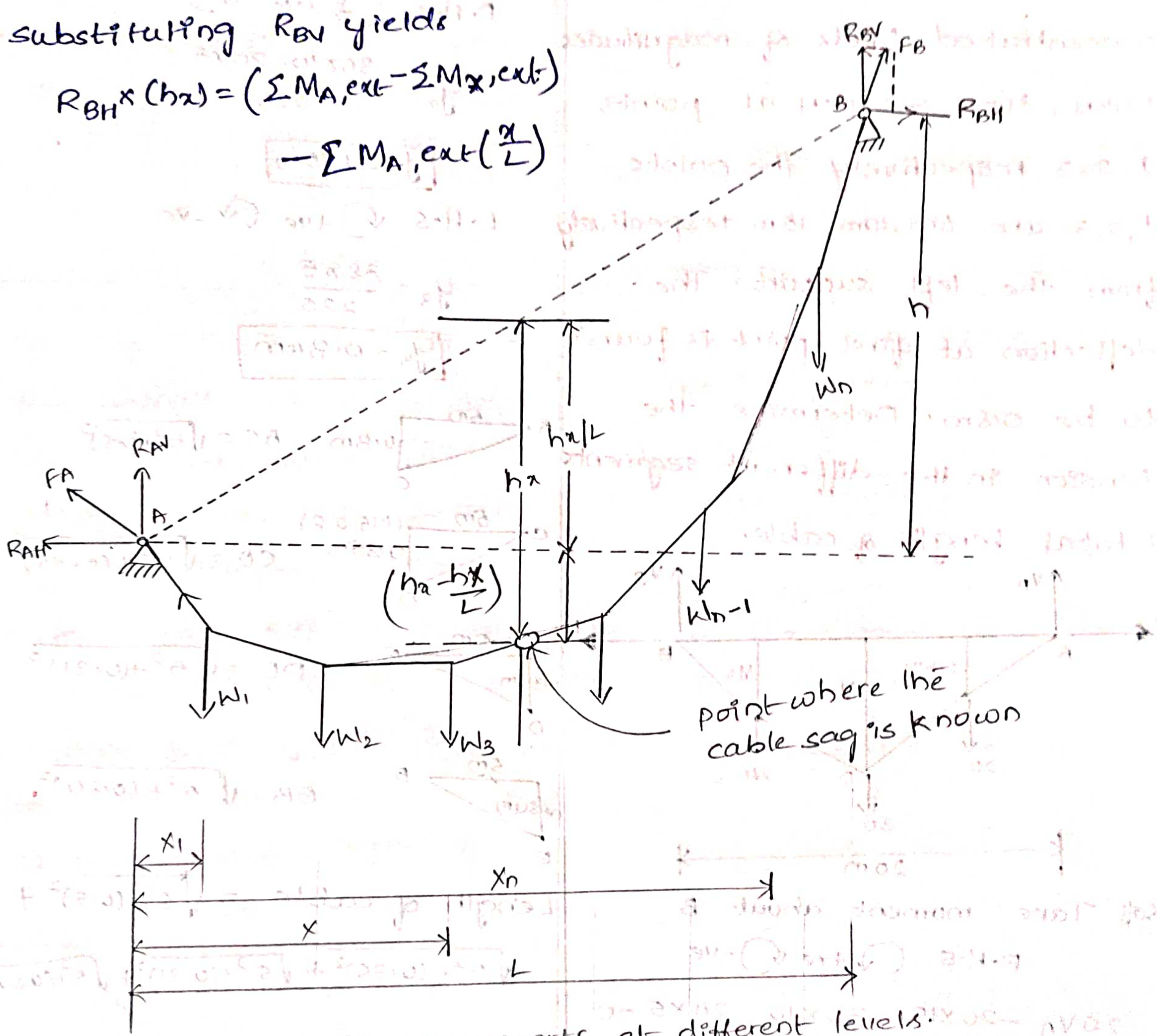
$$\Sigma M_{A,ext} = W_1x_1 + W_2x_2 + \cdots$$

→ To determine R_{BH} consider the moment equilibrium of the right portion about a point (say, x) on the cable where cable sag is known (or) specified.

$$\circlearrowleft + \; \Sigma M_x = -\Sigma M_{x,ext} - R_{BH} \times \left[h - \frac{hx}{L} + h_x\right] + R_{BV} \times (L - x) = 0$$

substituting R_{BV} yields

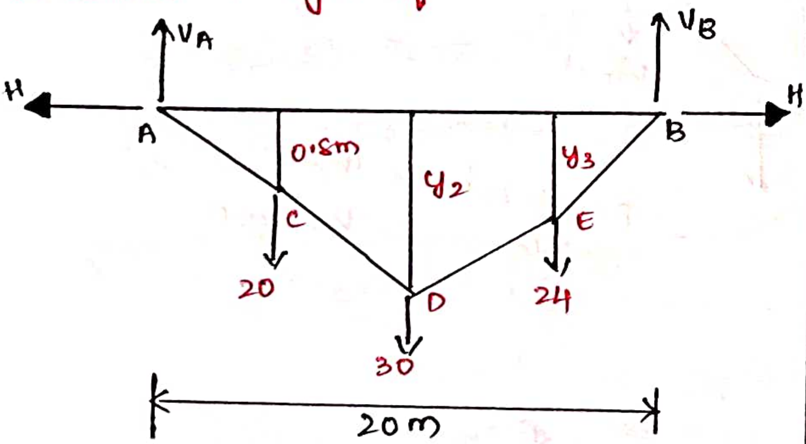
$$R_{BH} \times (h_x) = (\Sigma M_{A,ext} - \Sigma M_{x,ext}) - \Sigma M_{A,ext}\left(\frac{x}{L}\right)$$



cable with supports at different levels.

- Where h_x is the cable sag measured from the line joining two end-points of the cable.
- If h_x is known at a point, R_{BH} can be evaluated.
- Once cable reactions are known, the unknown cable sags, & cable forces can be found by equilibrium as described previously.

Problem no ①:- A light cable is supported at two points 20m apart which are at the same level. The cable carries three concentrated loads of magnitudes 20KN, 30KN & 24KN at points 1, 2, 3 respectively. The points 1, 2, 3 are 5m, 10m, 15m respectively from the left support. The deflection at first point is found to be 0.8m. Determine the tension in the different segments & total length of cable.



Sol: Take moment about B

R.H.S ↻ +ve ↺ -ve

$$20V_A - 20\times15 - 30\times10 - 24\times5 = 0$$

$$V_A = \frac{720}{20} \Rightarrow \boxed{36KN = V_A}$$

$$\Sigma V = 0$$

$$36 + V_B - 20 - 30 - 24 = 0$$

$$\boxed{V_B = 38KN}$$

$$y = \frac{\text{Beam moment}}{H}$$

$$0.8 = \frac{36\times5}{H}$$

$$H = \frac{180}{0.8}$$

$$\boxed{H = 225KN}$$

R.H.S ↻ +ve ↺ -ve

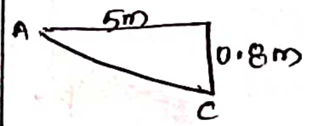
$$y_2 = \frac{36\times10 - 20\times5}{225}$$

$$\boxed{y_2 = 1.15m}$$

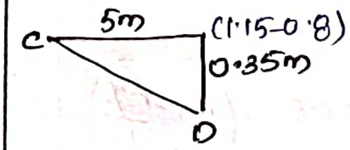
L.H.S ↺ +ve ↻ -ve

$$y_3 = \frac{38\times5}{225}$$

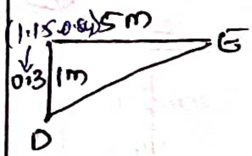
$$\boxed{y_3 = 0.84m}$$



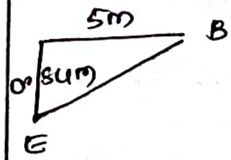
$$AC = \sqrt{5^2+(0.8)^2}$$



$$CD = \sqrt{5^2+(0.35)^2}$$



$$DE = \sqrt{5^2+(0.31)^2}$$



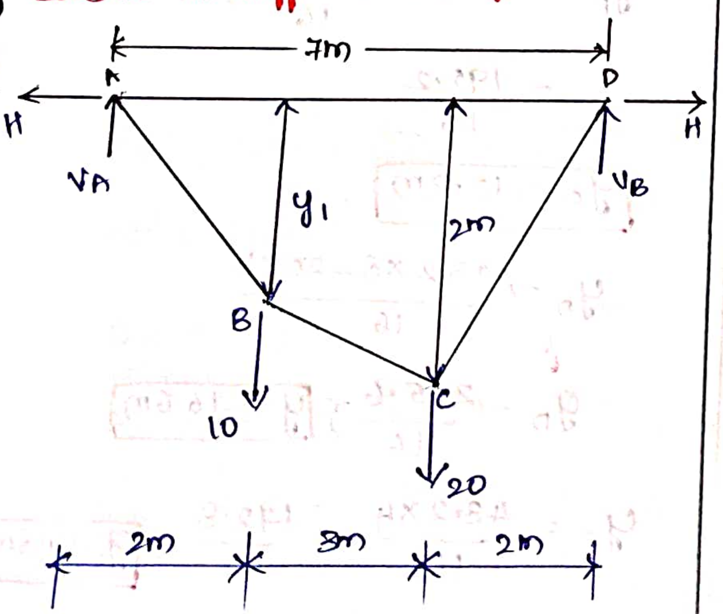
$$EB = \sqrt{5^2+(0.84)^2}$$

$$\text{Length of cable} = \sqrt{5^2+(0.8)^2} + \sqrt{5^2+(0.35)^2} + \sqrt{5^2+(0.31)^2} + \sqrt{5^2+(0.84)^2}$$

$$= 20.$$

Segment	Horizontal component (KN)	Vertical component (KN)	Resultant Tension (KN)
AC	225	36	$\sqrt{(225)^2+36^2} = 227.86$
CD	225	$(36-20) = 16$	$\sqrt{(225)^2+16^2} = 225.57$
DE	225	$(36-20-30) = -14$	$\sqrt{(225)^2+(-14)^2} = 225.43$
EB	225	$(36-20-30-24) = -38$	$\sqrt{(225)^2+(-38)^2} = 228.18$

Problem no (2):- A cable AD span of 7metre is supporting two concentrated loads 10KN and 20KN at points B & C. which are 2m & 5m from left support. Support A & D are in the same level. Dip of point C is 2m. calculate the support reactions & dip of point B. Also calculate the tension in the cable in different segments.



Sol: Take moment about D

R.H.S ↻ +ve ↺ -ve.

$7V_A - 10\times5 - 20\times2 = 0$

$V_A = \frac{90}{7}$

$\boxed{V_A = 12.86KN}$

$\Sigma V = 0$

$12.86 + V_B - 10 - 20 = 0$

$V_B = 30 - 12.86$

$\boxed{V_B = 17.14KN}$

$y = \frac{\text{Beam moment}}{H}$

$2 = \frac{12.86\times5 - 10\times3}{H}$

$H = \frac{34.3}{2}$

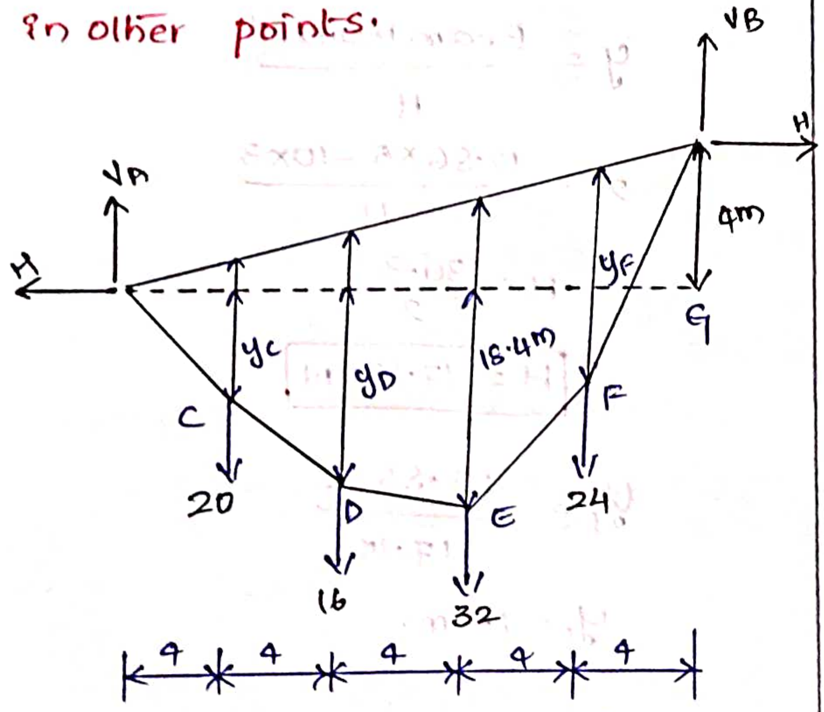
$\boxed{H = 17.15KN}$

$y_1 = \frac{12.86\times2}{17.15}$

$y_1 = 1.5m.$

Segment	Horizontal component (KN)	Vertical component (KN)	Resultant Tension (KN)
AB	17.15	12.86	$\sqrt{(17.15)^2+(12.86)^2}$ $=21.44$
BC	17.15	$(12.86-10)$ $=2.86$	$\sqrt{(17.15)^2+(2.86)^2}$ $=17.39$
CD	17.15	$(12.86-10-20)$ $=-17.14$	$\sqrt{(17.15)^2+(-17.14)^2}$ $=24.45$

Problem no. ③: A suspension cable is suspended between two supports at a distance of 20m & loaded as shown in figure. Calculate reactions, tension in each segment of the cable and length of the cable, if level difference between A & B is 4m & dip under 32KN load is 18.4m. Calculate the dips in other points.



Sol: Take moment about G

R.H.S (↻ +ve ↺ -ve)

$$20R_A - 20\times16 - 16\times12 - 32\times8 - 24\times4 = 0$$

$$R_A = \frac{864}{20}$$

$$\boxed{R_A = 43.2KN}$$

$$y = \frac{\text{Beam moment}}{H}$$

R.H.S (↻ +ve ↺ -ve)

$$18.4 = \frac{43.2\times12 - 20\times8 - 16\times4}{H}$$

$$H = \frac{294.4}{18.4}$$

$$\boxed{H = 16KN}$$

R.H.S. (↻ +ve ↺ -ve)

$$y_F = \frac{43.2\times16 - 20\times12 - 16\times8 - 32\times4}{16}$$

$$= \frac{195.2}{16}$$

$$\boxed{y_F = 12.2m}$$

$$y_D = \frac{43.2\times8 - 20\times4}{16}$$

$$y_D = \frac{265.6}{16} ; \boxed{y = 16.6m}$$

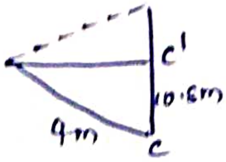
$$y_c = \frac{43.2\times4}{16} = \frac{172.8}{16} ; \boxed{y_c = 10.8m}$$

Take moment about B

$20V_A - 20\times16 - 16\times12 - 32\times8 - 24\times4 + 16\times4 = 0$

$V_A = \frac{800}{20}$

$\boxed{V_A = 40KN}$



$CC' = 10.8 - \frac{4}{20}\times4$

$= 10m.$

Take moment about C

R.H.S (↻)+ve (↺)-ve

$4V_A - 16\times10 = 0$

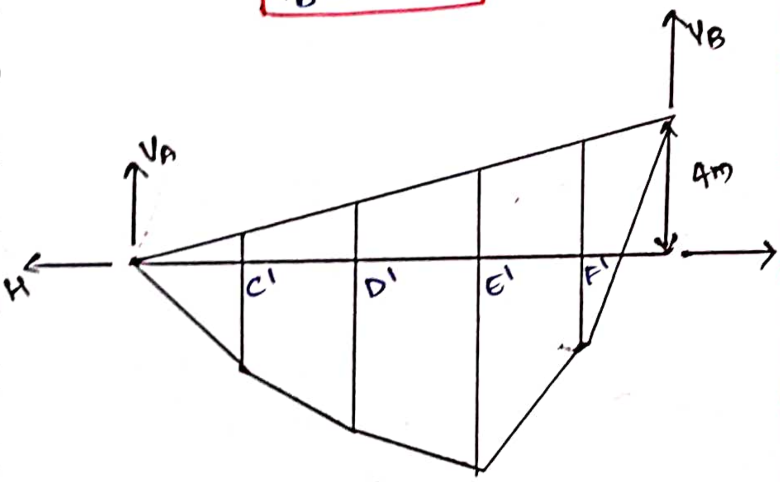
$V_A = \frac{160}{4}$

$\boxed{V_A = 40KN}$

$\Sigma V = 0$

$40 + V_B - 20 - 16 - 32 - 24 = 0$

$\boxed{V_B = 52KN}$



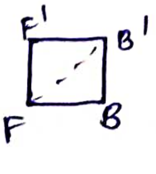
$CC' = 10.8 - \frac{4}{20}\times4 = 10m$

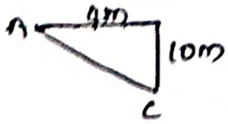
$DD' = 16.6 - \frac{4}{20}\times8 = 15m$

$EE' = 18.4 - \frac{4}{20}\times12 = 16m$

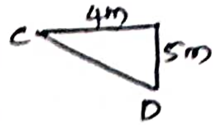
$FF' = 12.2 - \frac{4}{20}\times16 = 9m$

$BB' = 4+9 = 13m.$

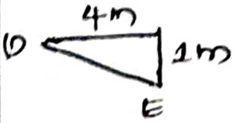




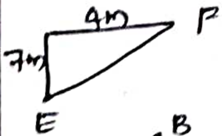
$AC = \sqrt{4^2+10^2}$



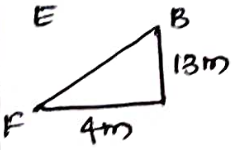
$CD = \sqrt{4^2+5^2}$



$DE = \sqrt{4^2+1^2}$



$EF = \sqrt{4^2+7^2}$



$FB = \sqrt{4^2+13^2}$

Length of cable $= \sqrt{4^2+10^2} + \sqrt{4^2+5^2} + \sqrt{4^2+1^2} + \sqrt{4^2+7^2} + \sqrt{4^2+13^2}$

$= 42.96m.$

Segment	Horizontal component (KN)	Vertical component (KN)	Resultant Tension (KN)
AC	16	40	$\sqrt{16^2+40^2} = 43.08$
CD	16	$(40-20) = 20$	$\sqrt{16^2+20^2} = 25.61$
DE	16	$40-20-16 = 4$	$\sqrt{16^2+4^2} = 16.49$
EF	16	$40-20-16-32 = -28$	$\sqrt{16^2+(-28)^2} = 32.25$
FB.	16	$40-20-16-32-24 = -52$	$\sqrt{16^2+(-52)^2} = 54.40$