UNIT – III

APPROXIMATE METHODS OF ANALYSIS

Introduction:-

In the case of multistorey frames, the degree of indeterminacy is very high and hence solution by consistent deformation, slope deflection, moment distribution (or) column analogy methods is always almost ruled out.

→ Kani's method, however may be employed, but it needs more computational efforts.

→ For quick solutions, design engineers use the following approximate methods of analysis

1. Substitute frame method for vertical loads.
2. Any one of the following methods for horizontal loads
   - (a) portal method.
   - (b) Cantilever method.
   - (c) Factor method.

→ Multistorey frames are subjected to horizontal forces due to wind and seismic forces.

→ The horizontal forces are assumed to act at joints and any one.

Portal method:-

Assumptions:-

1. Point of contraflexure occurs at the middle of all the members of the frame.
2. Horizontal shear taken by each interior column is double of that taken by external columns.

→ With the above two assumptions, the frame becomes a determine and hence analysis is easy.

→ The method was presented by "Albert Smith in the journal of

"Western society of Engineers" in 1915.

Cantilever method:-

Assumptions:-

1. There is a point of contraflexure at the centre of each member.
2. The intensity of axial stress in each column of a storey is proportional to the horizontal distance of that column from the centre of gravity of all the columns of the storey under consideration.

→ The analysis is carried out storey by storey.

→ Consider the top storey of the frame show in figure.

→ Let a_1, a_2, a_3 & a_4 be the areas of columns AE, BF, CG and DH respectively.

→ Let H_1, H_2, H_3 & H_4 be the horizontal shears in the columns and V_1, V_2, V_3 & V_4 be the respective vertical forces.

$$\frac{\left(\frac{V_1}{a_1}\right)}{x_1} = \frac{\left(\frac{V_2}{a_2}\right)}{x_2} = \frac{\left(\frac{V_3}{a_3}\right)}{x_3} = \frac{\left(\frac{V_4}{a_4}\right)}{x_4} \quad \text{—(1)}$$

According to assumptions made in this method, where x_1, x_2, x_3 & x_4 are the distances of columns AE, BF, CG and DH respectively from the centre of gravity of the column.

→ Taking moment about the point of contraflexure of the first column, we get

$$P_1\left(\frac{h}{2}\right) = V_2 L_1 - V_3(L_1+L_2) - V_4(L_1+L_2+L_3) \quad \text{—— (2)}$$

where 'h' is the height of the columns and L_1, L_2, L_3 are distances between the columns AE, BF, CG & DH, respectively.

→ From eq^n (1) & (2) vertical forces V_1, V_2, V_3, V_4 in the columns can be found.

→ Considering the moment equilibrium condition about the centre of beams, horizontal forces H_1, H_2, H_3, H_4 can be found.

→ The other forces and moments can be found

figure 6.11 (pg. no. 197 Bhavikatti)

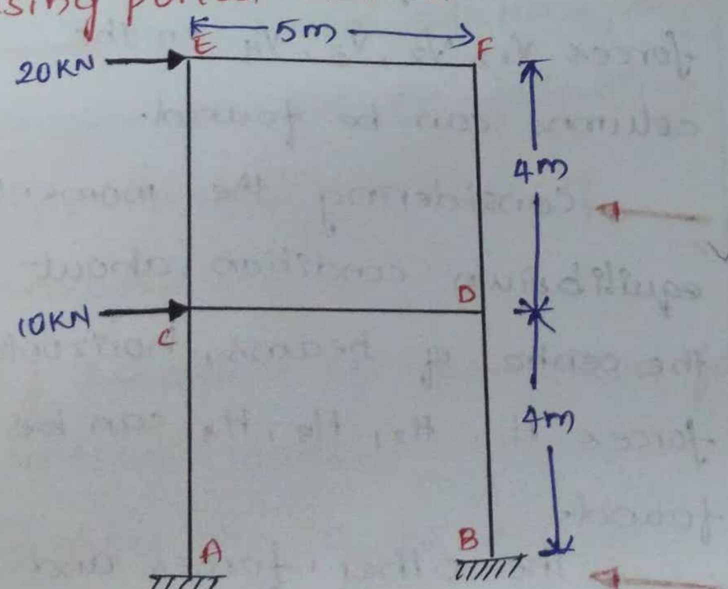
using equations of statics.

Factor method :-

→ Prof. J. B. Wilber published this method in his article, 'A New method for analysing stress due to lateral loads on building frames'.

→ It is an approximate slope deflection method presented in steps using simple instructions by which the desired results can be obtained without the knowledge of principle of elasticity upon which it is based.

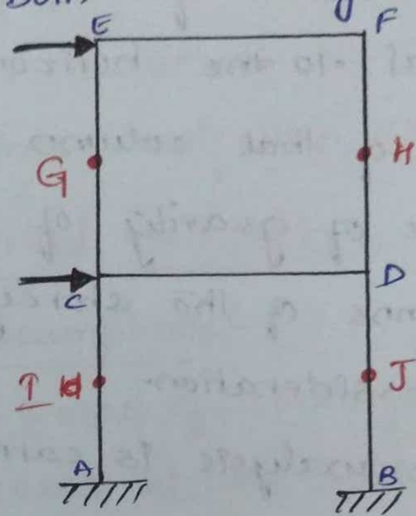
Problem no (1) :- Analyse the frame using portal method.



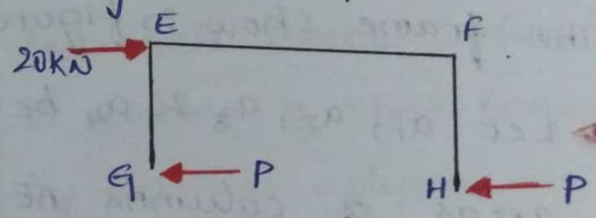
Soln As per our assumptions in portal frame we dont find any interior frame in the given frame.

→ In this case both of the columns will take the same shear force.

→ Now will make centre in both the storey's



→ Will make the upper storey in two parts.



'P' - shear force

→ Both columns take the same shear force 'P'

$$\Sigma H = 0$$

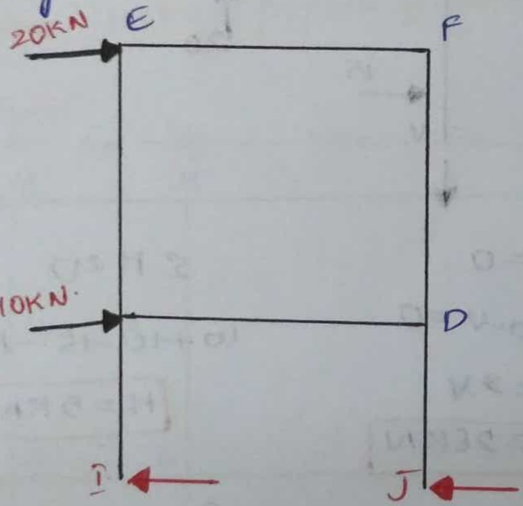
$$-P - P + 20 = 0$$

$$2P = 20$$

$$P = \frac{20}{2} \quad ; \quad \boxed{P = 10\,KN}$$

→ we have calculated the shear forces at just above the centre of the columns, below the centre of the columns they will be acting in the opposite direction.

→ Now let us take the centre for bottom storey but we need to consider the storey of upward to the bottom storey.



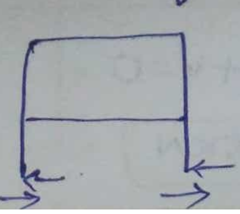
$\Sigma H = 0$

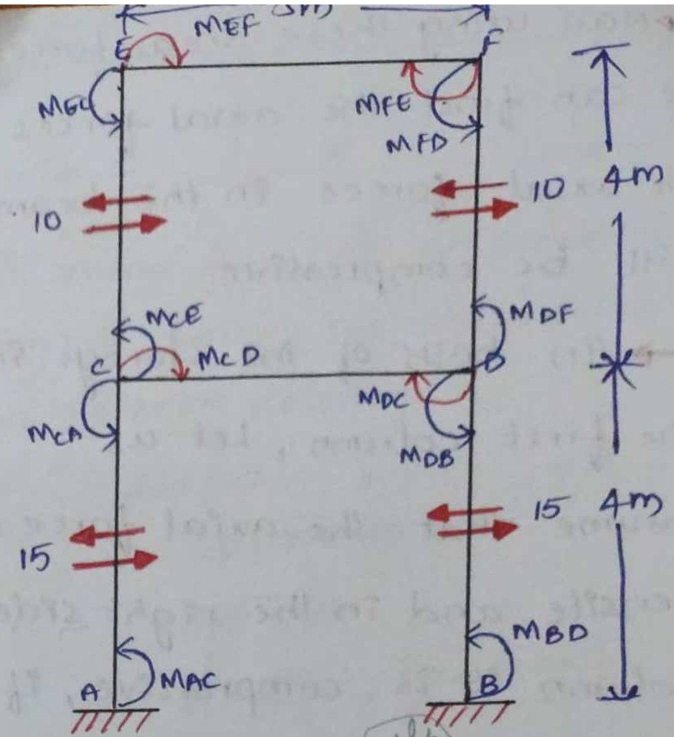
$-P - P + 20 + 10 = 0$

$2P = 30$

$P = \frac{30}{2}$; $\boxed{P = 15KN}$

Same as above opposite directing shear forces.





$M_{EC} = M_{CE} = \frac{10 \times 4}{2} = 20KNm$ (wl/2)

$M_{FD} = M_{DF} = \frac{10 \times 4}{2} = 20KNm$

$M_{CA} = M_{AC} = \frac{15 \times 4}{2} = 30KNm$

$M_{DB} = M_{BD} = \frac{15 \times 4}{2} = 30KNm$

Beams

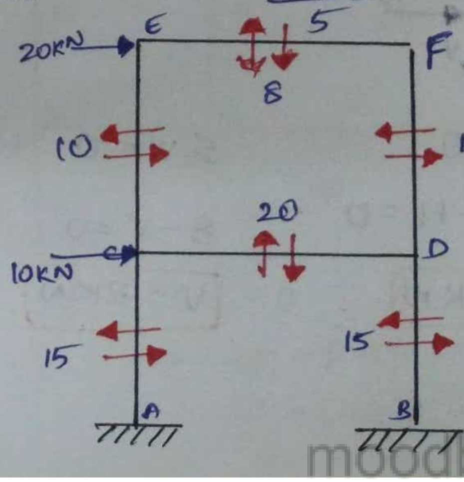
$M_{EF} = M_{FE} = 20KNm$

$M_{CD} = M_{DC} = 30 + 20 = 50KNm$

Shear force in beam.

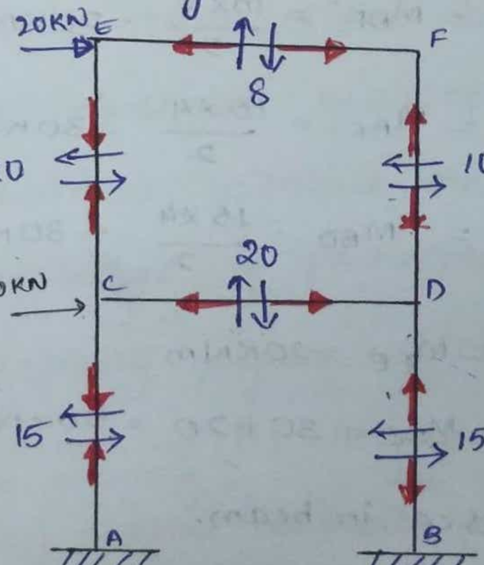
① M_{EF} & M_{FE} $\Rightarrow \frac{20+20}{5} = \frac{40}{5} = 8$

M_{CD} & M_{DC} $\Rightarrow \frac{50+50}{5} = \frac{100}{5} = 20$

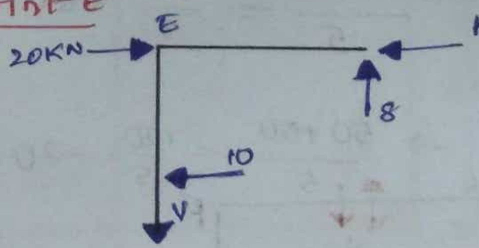


→ Now using these shear forces we can find the axial forces the axial forces in the beams will be compressive.

→ In both of the storey's in the first column, Let us assume that the axial force is tensile and in the right side column it is compressive, if we get any -ve value we can change the direction.



@ Joint E



$\Sigma H = 0$

$20 - 10 - H = 0$

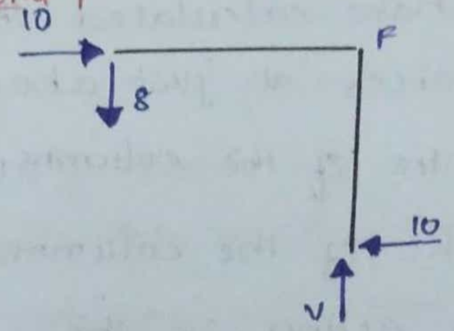
$\boxed{H = 10KN}$

$\Sigma V = 0$

$8 - V = 0$

$\boxed{V = 8KN}$

@ Joint F

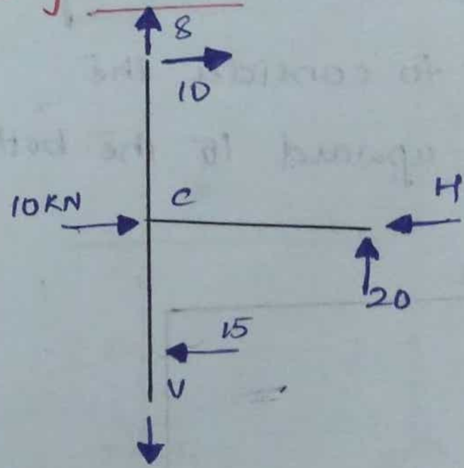


$\Sigma V = 0$

$-8 + V = 0$

$\boxed{V = 8KN}$

@ Joint 'C'



$\Sigma V = 0$

$8 + 20 - V = 0$

$\cancel{28 = 8V}$

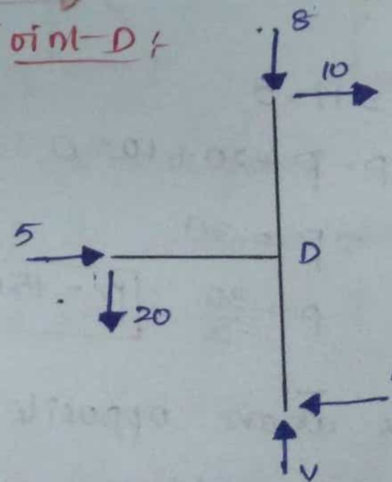
$\boxed{V = 28KN}$

$\Sigma H = 0$

$10 + 10 - 15 - H = 0$

$\boxed{H = 5KN}$

@ Joint D

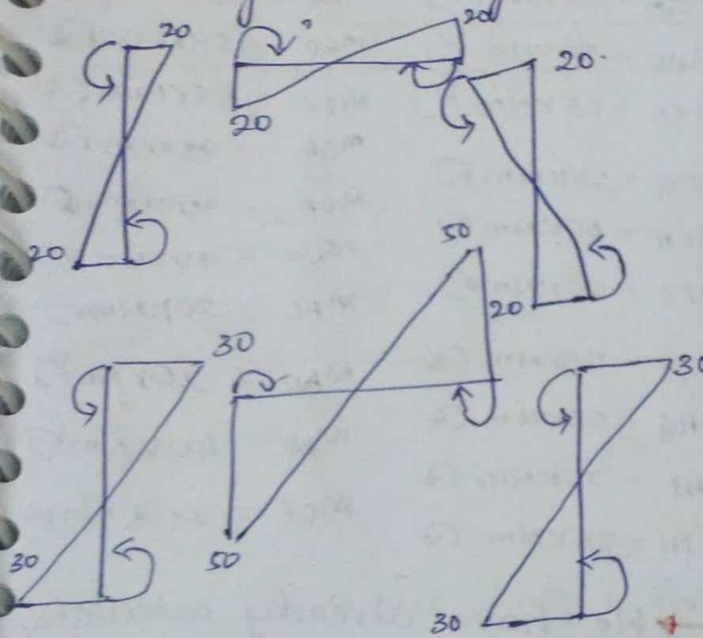


$\Sigma V = 0$

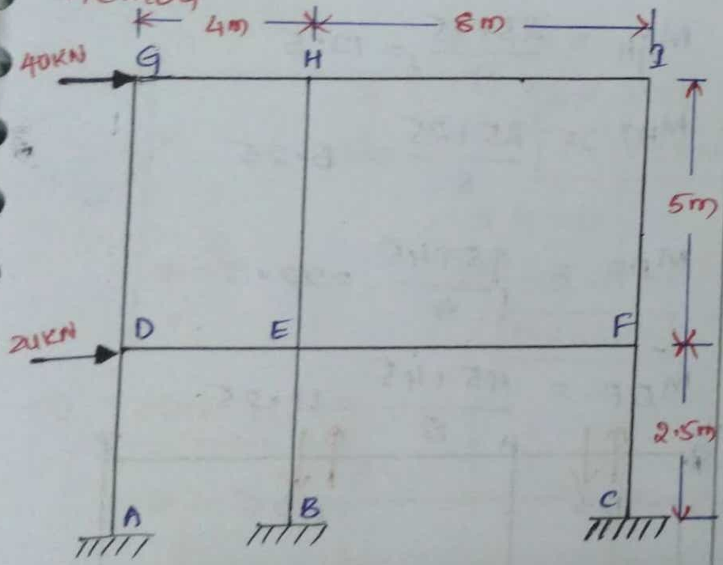
$-20 - 8 + V = 0$

$\boxed{V = 28KN}$

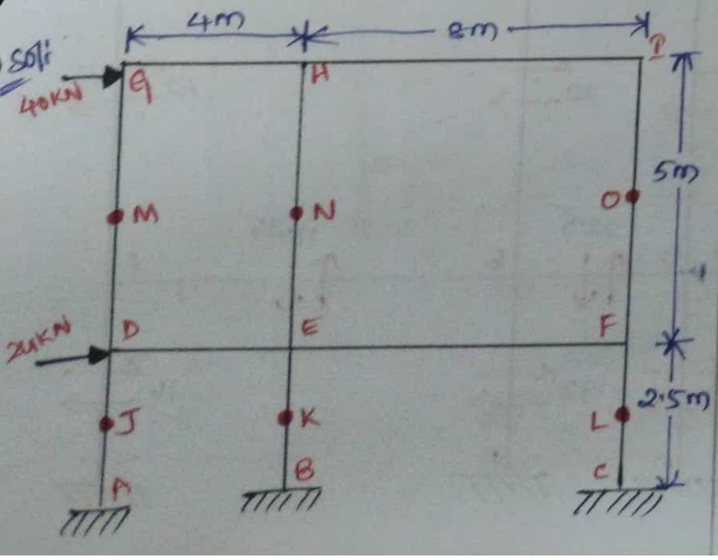
Bending moment diagram

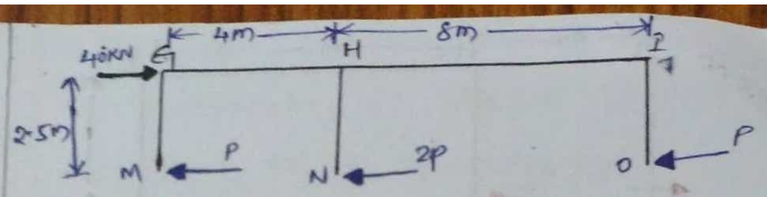


problem no. ② :- Analyse the frame using portal frame method.



Soln





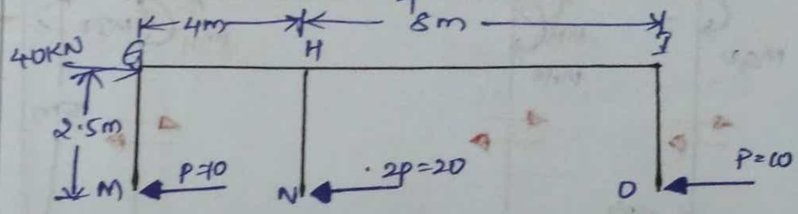
→ The shear force in an interior column is twice the shear force in an exterior column.

$$\Sigma H = 0$$

$$-P - 2P - P + 40 = 0$$

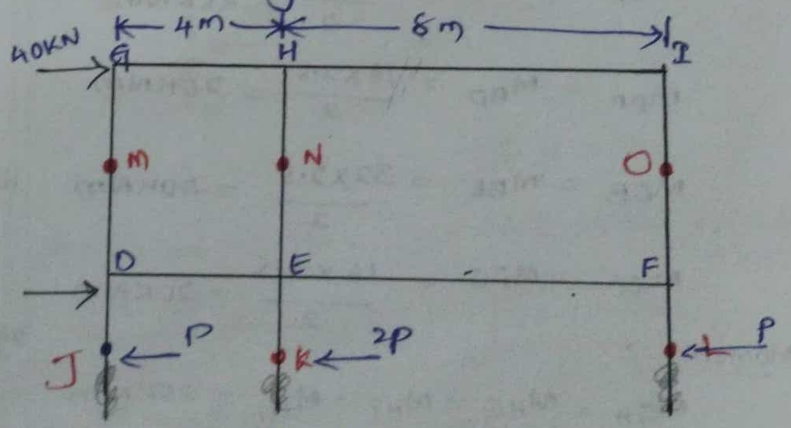
$$4P = 40$$

$$P = \frac{40}{4} \; ; \; \boxed{P = 10\,KN}$$



→ We have calculated the shear forces just above centre of the columns GD, HE & IF below the centre of the columns there will be opposite direction

→ Now we will calculate the bottom storey.

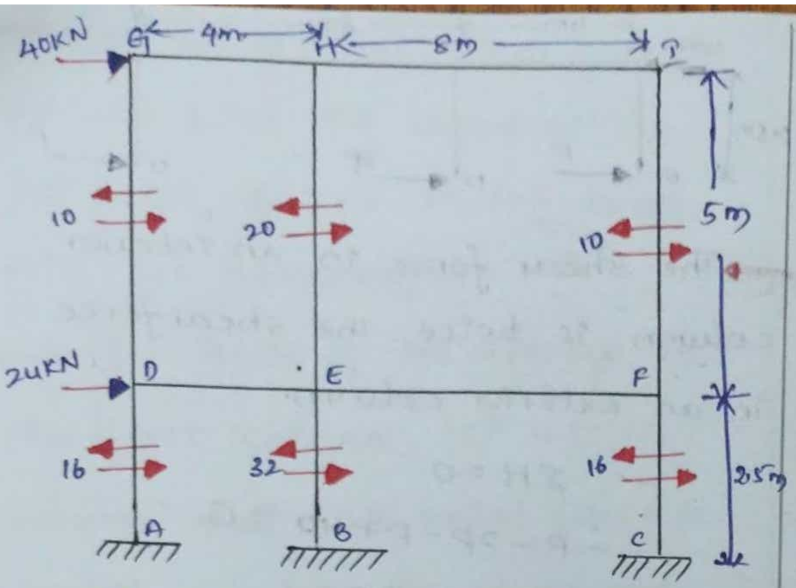


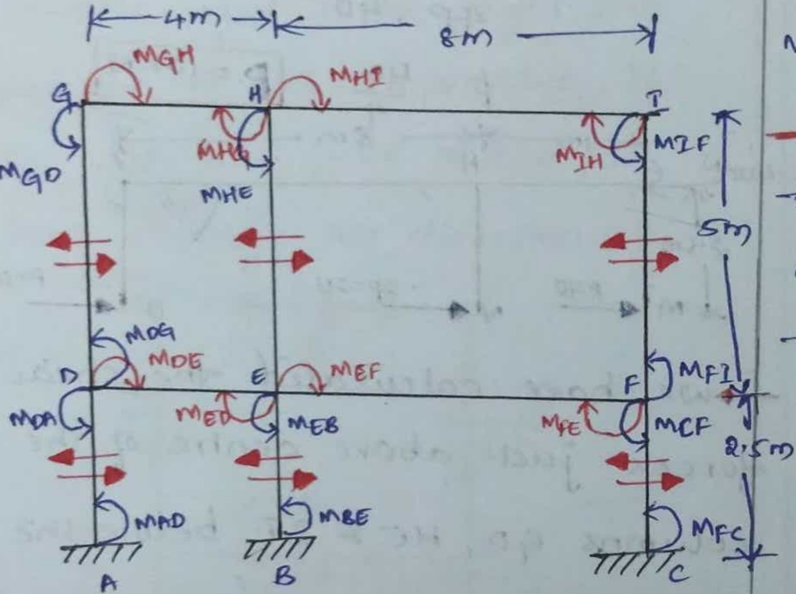
$$\Sigma H = 0$$

$$-P - 2P - P + 40 + 24 = 0$$

$$4P = 64$$

$$P = 64/4 \; ; \; \boxed{P = 16\,KN}$$





$M_{GD} = M_{DG} = \frac{10 \times 5}{2} = 25\,KNm$

$M_{HE} = M_{EH} = \frac{20 \times 5}{2} = 50\,KNm$

$M_{IF} = M_{FI} = \frac{10 \times 5}{2} = 25\,KNm$

$M_{DA} = M_{AD} = \frac{16 \times 2.5}{2} = 20\,KNm$

$M_{EB} = M_{BE} = \frac{32 \times 2.5}{2} = 40\,KNm$

$M_{CF} = M_{FC} = \frac{16 \times 2.5}{2} = 20\,KNm$

Moments

$M_{GH} = M_{HG} = M_{HI} = M_{IH} = 25\,KNm$

$M_{DE} = 20 + 25 = 45\,KNm$

$M_{ED} = M_{EF} = M_{FE} = 45\,KNm$

$M_{GD} = 25\,KNm$

$M_{HE} = 50\,KNm$

$M_{IF} = 25\,KNm$

$M_{DG} = 25\,KNm$

$M_{EH} = 50\,KNm$

$M_{FI} = 25\,KNm$

$M_{GH} = 25\,KNm$

$M_{HG} = 25\,KNm$

$M_{HI} = 25\,KNm$

$M_{IH} = 25\,KNm$

$M_{DE} = 45\,KNm$

$M_{ED} = 45\,KNm$

$M_{EF} = 45\,KNm$

$M_{FE} = 45\,KNm$

$M_{DA} = 20\,KNm$

$M_{EB} = 40\,KNm$

$M_{FC} = 20\,KNm$

$M_{AD} = 20\,KNm$

M_{BE} $40\,KNm$

$M_{CF} = 20\,KNm$

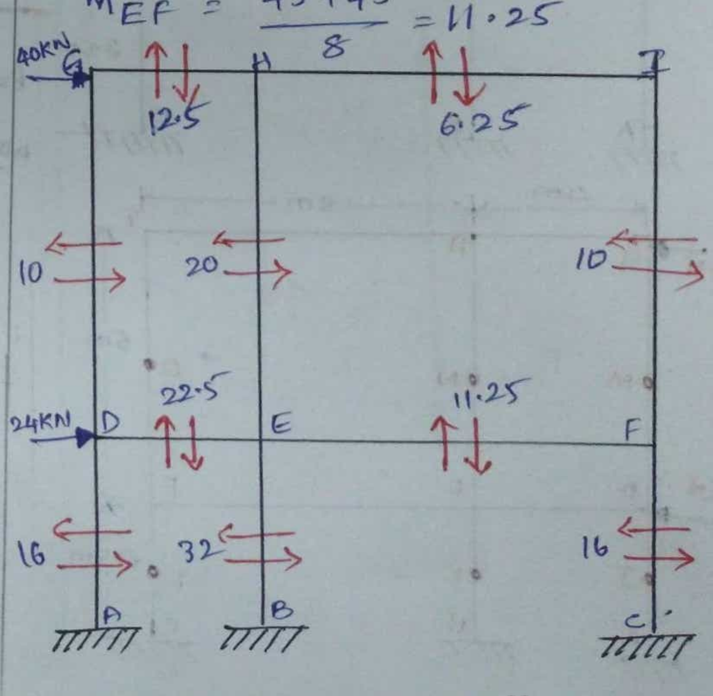
→ We have already calculated the shear forces in the columns now let us find them in beams.

$M_{GH} = \frac{25+25}{4} = 12.5$

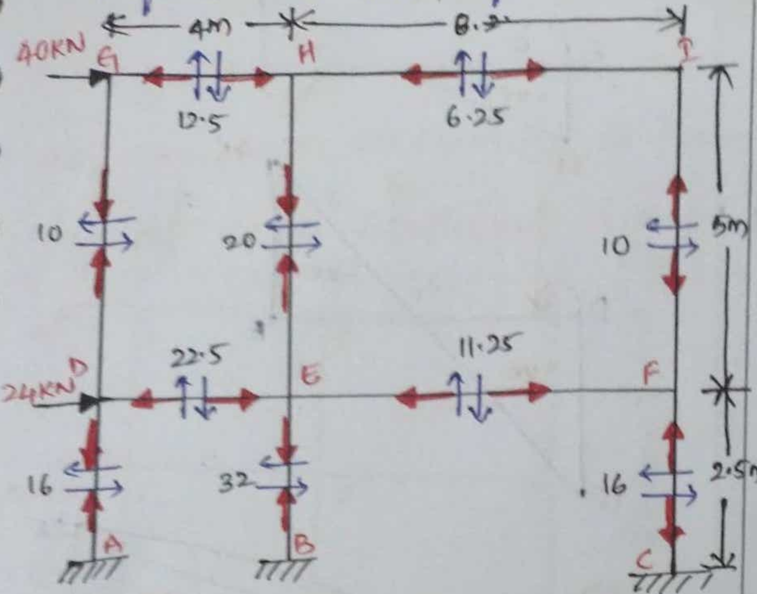
$M_{HI} = \frac{25+25}{8} = 6.25$

$M_{DE} = \frac{45+45}{4} = 22.5$

$M_{EF} = \frac{45+45}{8} = 11.25$

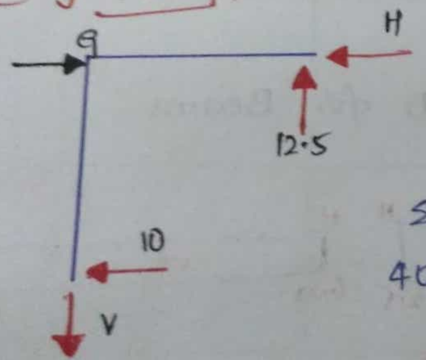


To find the axial forces.



→ When we select half of the members shear force of each member towards joint is considered.

@ Joint G :-



$\Sigma V = 0$

$12.5 - V = 0$

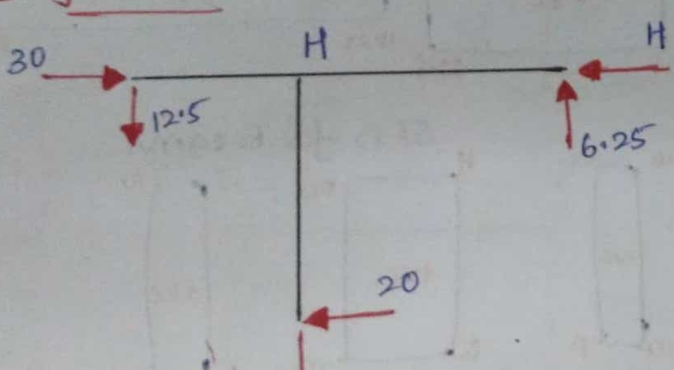
$\boxed{V = 12.5KN}$

$\Sigma H = 0$

$40 - 10 - H = 0$

$\boxed{H = 30KN}$

@ Joint H :-



$\Sigma H = 0$

$30 - 20 - H = 0$

$\boxed{H = 10KN}$

$\Sigma V = 0$

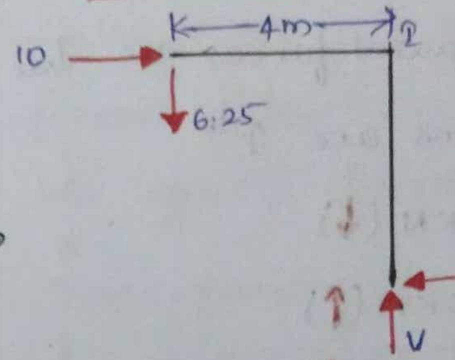
$-12.5 + 6.25 - V = 0$

$\boxed{V = -6.25KN(\uparrow)}$

'V' dir^n ✗

i.e., our assumed 'V' direction is incorrect, so the 'V' is not tensile, it is compressive.

@ Joint I :-

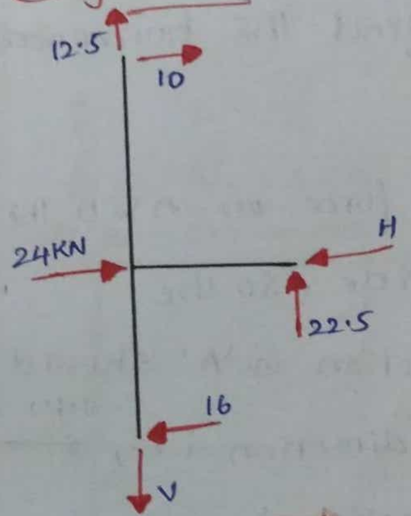


$\Sigma V = 0$

$-6.25 + V = 0$

$\boxed{V = 6.25KN}$

@ Joint D :-



$\Sigma H = 0$

$10 - 16 + 24 - H = 0$

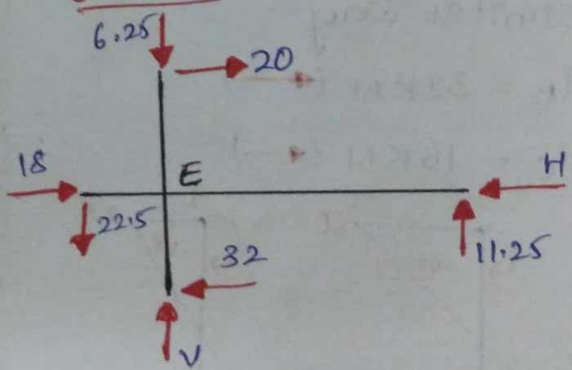
$\boxed{H = 18KN}$

$\Sigma V = 0$

$12.5 + 22.5 - V = 0$

$\boxed{V = 35KN}$

@ Joint E :-



$\Sigma H = 0$

$18 + 20 - 32 - H = 0$

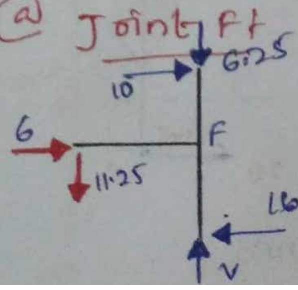
$\boxed{H = 6KN}$

$\Sigma V = 0$

$-22.5 - 6.25 + 11.25 + V = 0$

$\boxed{V = 17.5KN}$

@ Joint F :-



$\Sigma V = 0$

$-11.25 - 6.25 + V = 0$

$\boxed{V = 17.5KN}$

→ Now Lets find reactions @ fixed ends.

→ At the end 'A' the axial force is ↑ds so the reaction will be ↓ds

'B' & 'C' the axial forces are ↓ds so the reactions are ↑ds

$R_A = 35KN\ (\downarrow)$

$R_B = 17.5KN\ (\uparrow)$

$R_C = 17.5KN\ (\uparrow)$

→ Now will find the horizontal reactions

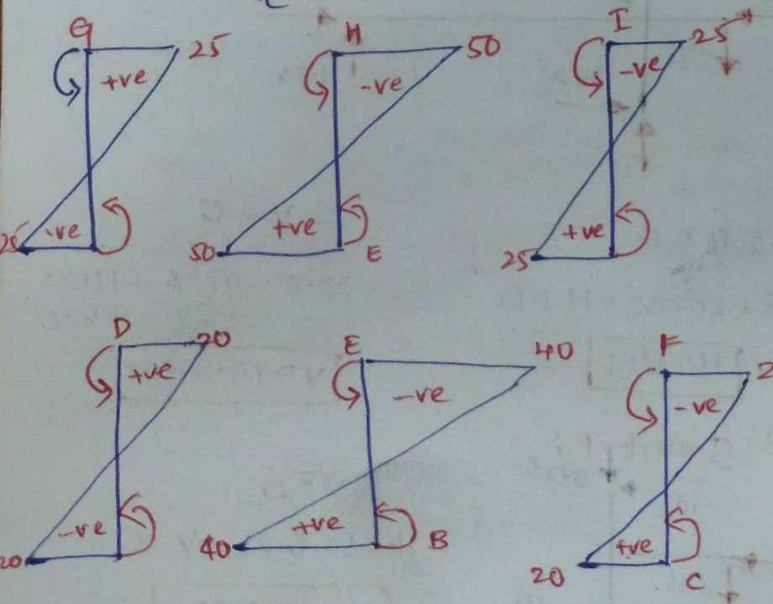
→ As the shear force on A & D the force is right side, so the horizontal reaction @ 'A' should be in opposite direction i.e., ← (left)

$H_A = 16KN\ (\leftarrow)$

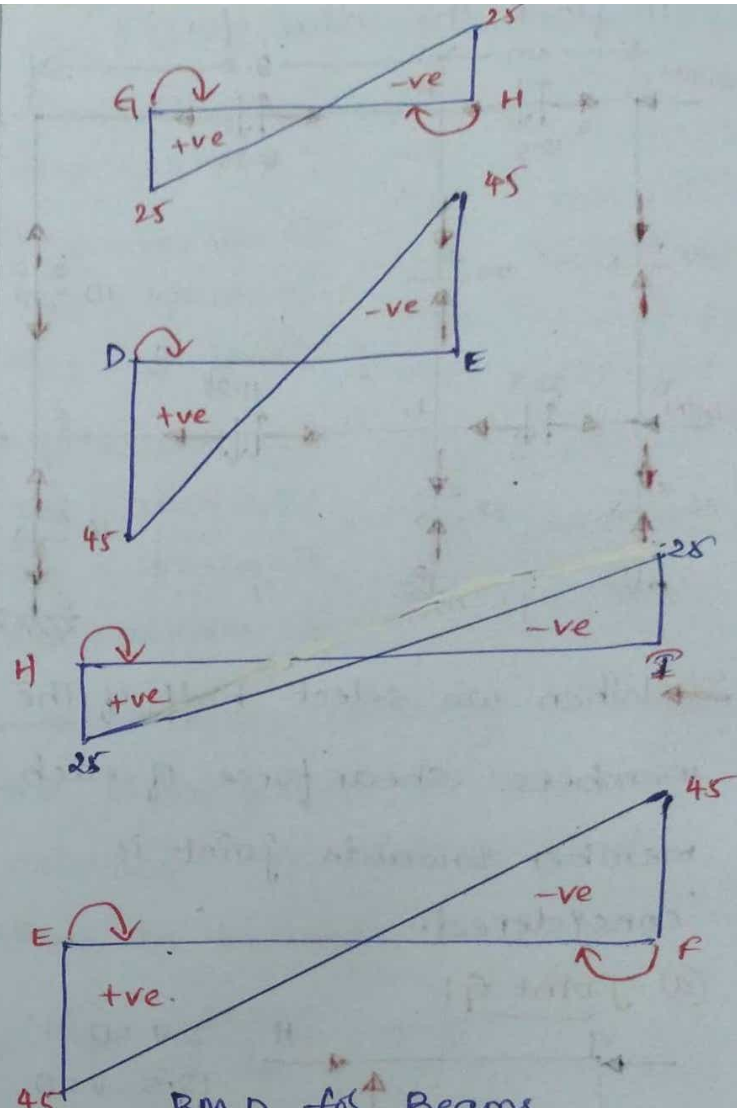
in the similar way

$H_B = 32KN\ (\leftarrow)$

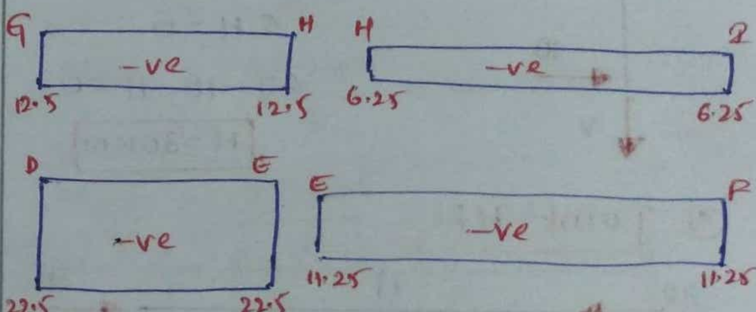
$H_C = 16KN\ (\leftarrow)$



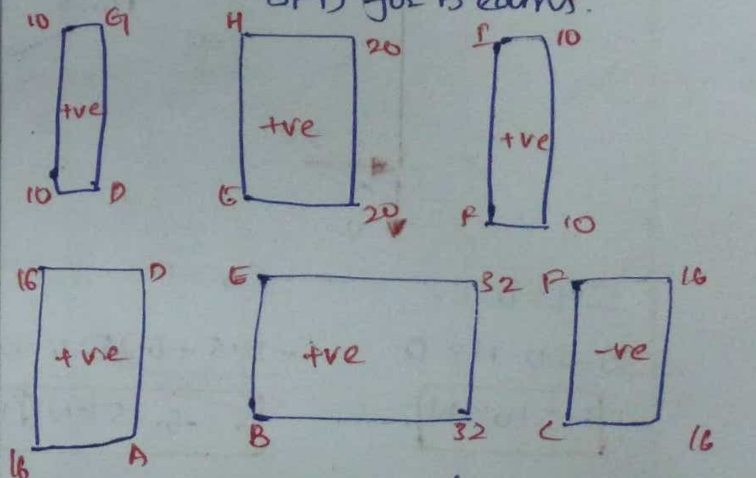
BMD for columns



BMD for Beams.

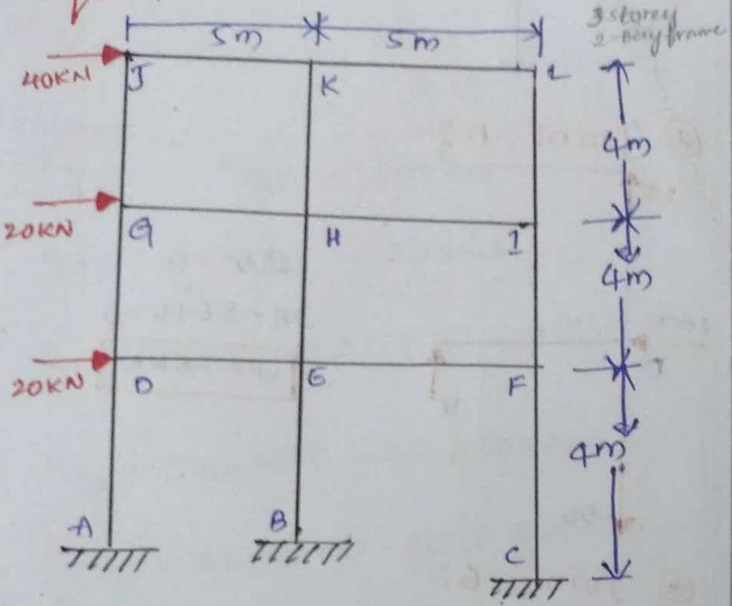


SFD for Beams.

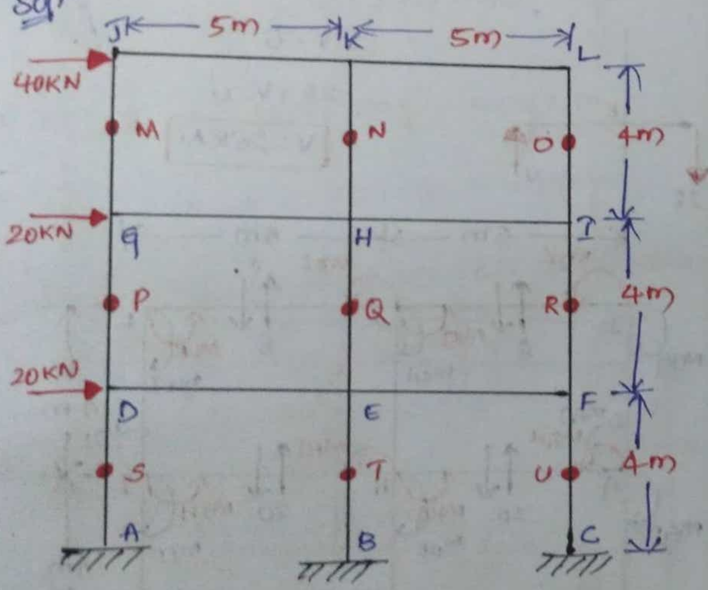


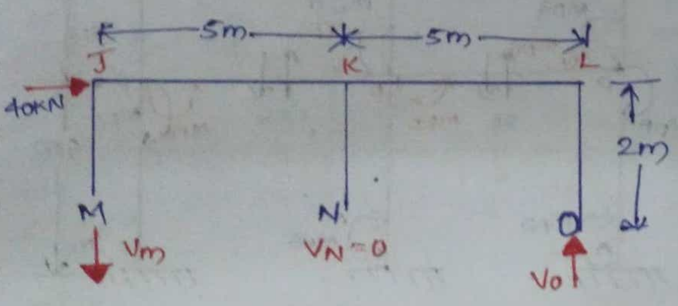
SFD for columns.

Problem no ③ :- Analyse the frame using Cantilever method. The columns are assumed to have equal cross sectional areas.



Sol:-





→ Now, Let us find the centre of gravity '$\bar{x}$' & this frame is vertically symmetrical so centre of gravity lies in centre so $\bar{x} = 5m$

→ Since centre of gravity is on member KN, $V_N = 0$

→ V_m & V_0 are the axial forces value will be same but opposite direction. $V_m \uparrow$, $V_0 \downarrow$

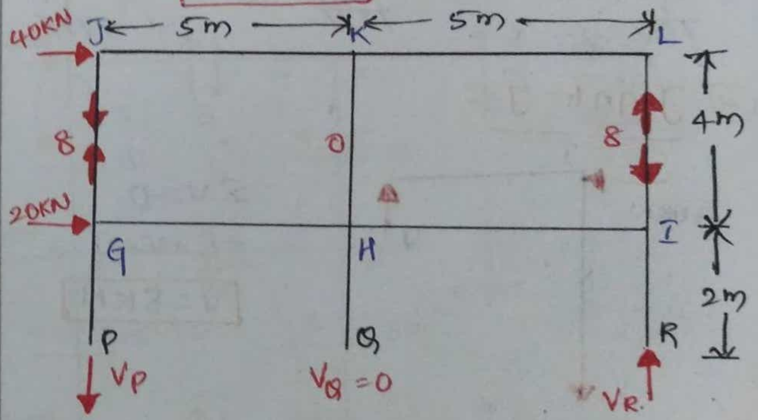
Take moment about 'M'

L.H.S ↻ -ve ↺ +ve.

$$10V_0 - 40 \times 2 = 0$$

$$V_0 = \frac{80}{10}$$

$\boxed{V_0 = 8KN}$ (↓)

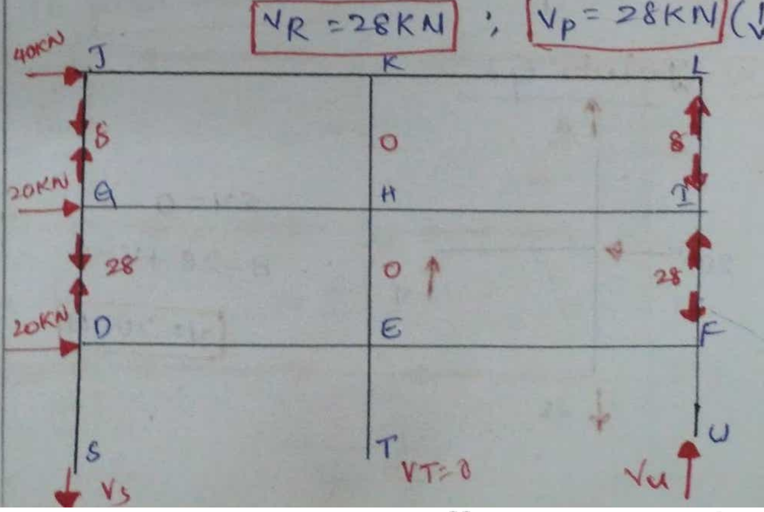


Take moment about P L.H.S ↻ -ve ↺ +ve

$$10V_R - 40 \times 6 - 20 \times 2 = 0$$

$$V_R = \frac{280}{10}$$

$\boxed{V_R = 28KN}$; $\boxed{V_P = 28KN}$ (↓)

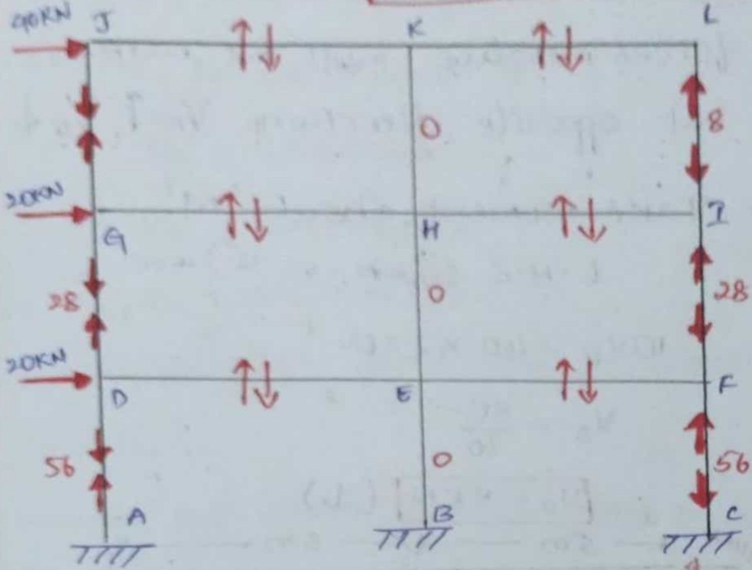


Take moment about 'S'

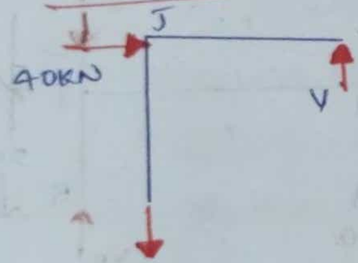
$10V_u - 40\times10 - 20\times6 - 20\times2 = 0$

$V_u = \frac{560}{10}$; $\boxed{V_u = 56\,KN}$

$\boxed{V_s = 56\,KN}$ (↓)



@ Joint J:-

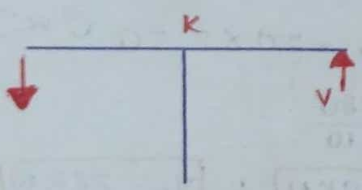


$\Sigma V = 0$

$-8 + V = 0$

$\boxed{V = 8\,KN}$

@ Joint K:-

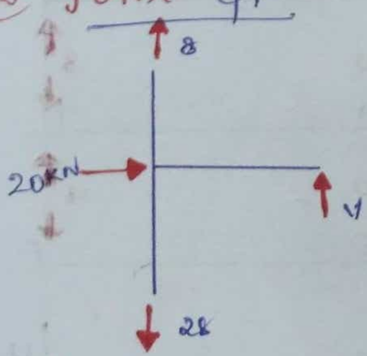


$\Sigma V = 0$

$-8 + V = 0$

$\boxed{V = 8\,KN}$

@ Joint G:-

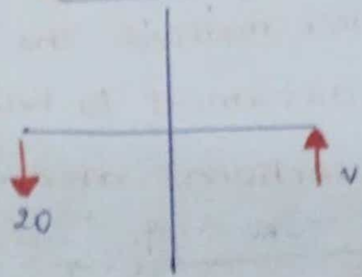


$\Sigma V = 0$

$8 - 28 + V = 0$

$\boxed{V = 20\,KN}$

@ Joint H:-

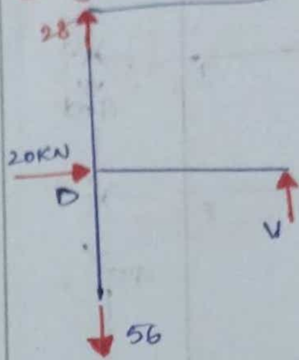


$\Sigma V = 0$

$-20 + V = 0$

$\boxed{V = 20\,KN}$

@ Joint D:-

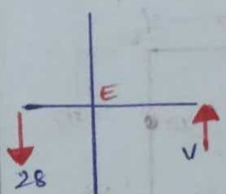


$\Sigma V = 0$

$28 - 56 + V = 0$

$\boxed{V = 28\,KN}$

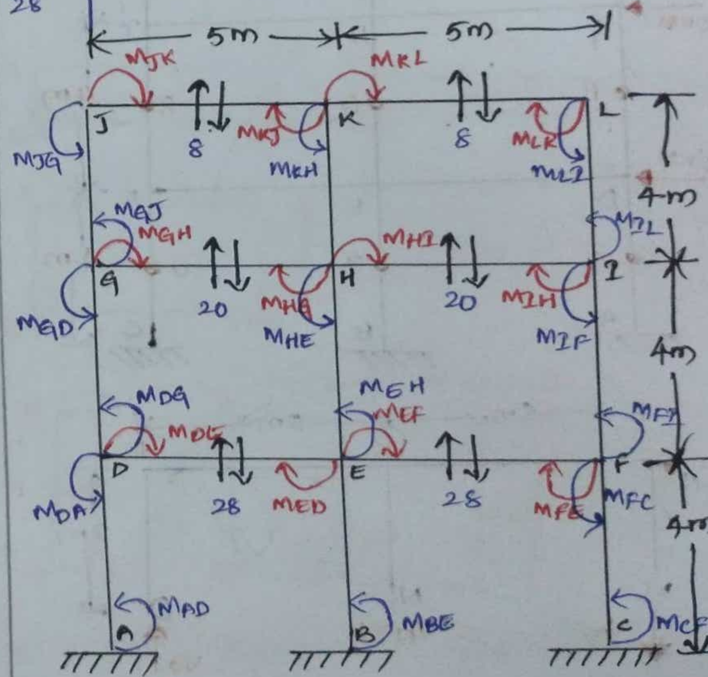
@ Joint E:-



$\Sigma V = 0$

$-28 + V = 0$

$\boxed{V = 28\,KN}$



$M_{JK} = M_{KJ} = \frac{8\times5}{2} = 20\,KNm$

$M_{KL} = M_{LK} = \frac{8\times5}{2} = 20\,KNm$

$M_{GH} = M_{HG} = \frac{20 \times 5}{2} = 50 KNm$

$M_{HI} = M_{IH} = \frac{20 \times 5}{2} = 50 KNm$

$M_{DE} = M_{ED} = \frac{28 \times 5}{2} = 70 KNm$

$M_{EF} = M_{FE} = \frac{28 \times 5}{2} = 70 KNm$

Columns

$M_{JG} = M_{GJ} = 20 KNm$

$M_{LI} = M_{IL} = 20 KNm$

$M_{KH} = M_{HK} = 20 + 20 = 40 KNm$

$M_{GD} = M_{DG} = 50 - 20 = 30 KNm$

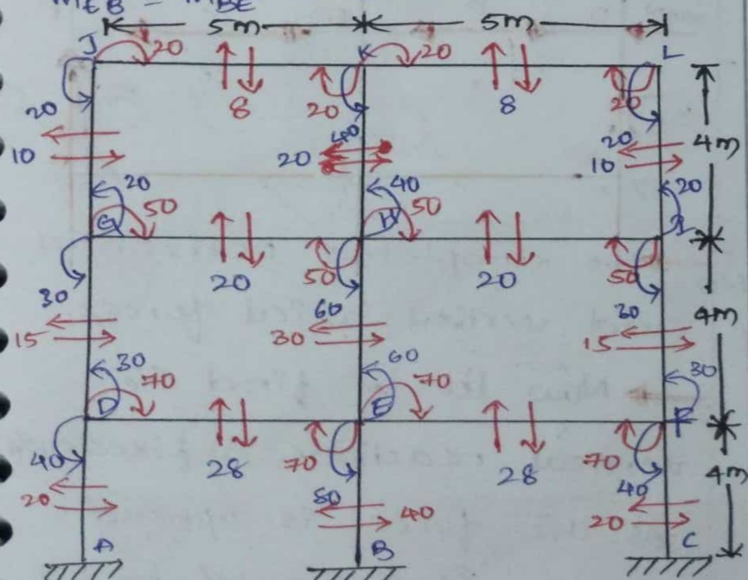
$M_{IF} = M_{FI} = 50 - 20 = 30 KNm$

$M_{HE} = M_{EH} = 50 + 50 - 40 = 60 KNm$

$M_{DA} = M_{AD} = 70 - 30 = 40 KNm$

$M_{FC} = M_{CF} = 70 - 30 = 40 KNm$

$M_{EB} = M_{BE} = 70 + 70 - 60 = 80 KNm$



→ Now Let us find the shear forces in columns & Member.

$M_{GJ} \Rightarrow \frac{20 + 20}{4} = 10$

$M_{KH} \Rightarrow \frac{40 + 40}{4} = 20$

$M_{IL} \Rightarrow \frac{20 + 20}{4} = 10$

$M_{GD} = \frac{30 + 30}{4} = 15$

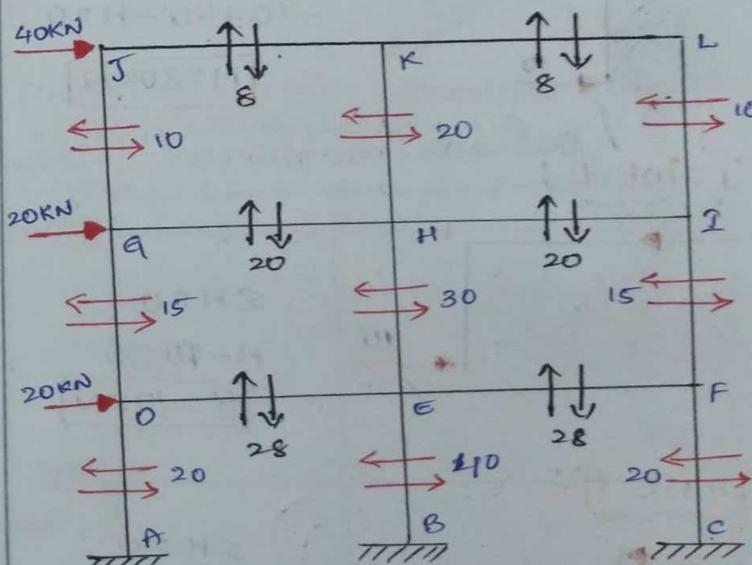
$M_{HE} = \frac{60 + 60}{4} \Rightarrow 30$

$M_{IF} = \frac{30 + 30}{4} \Rightarrow 15$

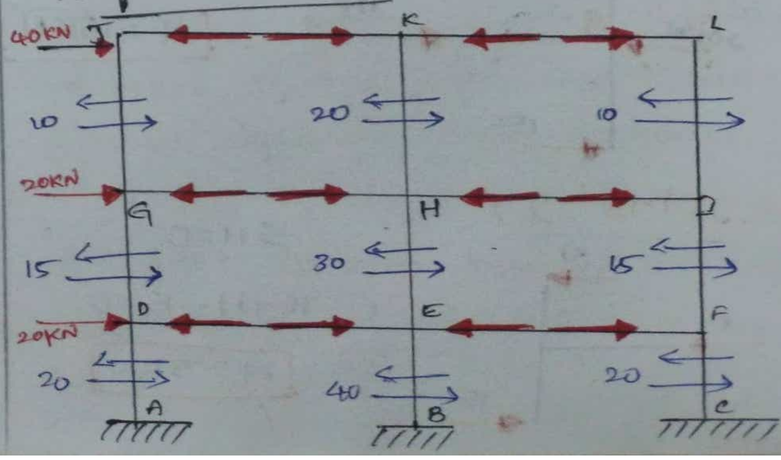
$M_{DA} = \frac{40 + 40}{4} \Rightarrow 20$

$M_{EB} = \frac{80 + 80}{4} \Rightarrow 40$

$M_{FC} = \frac{40 + 40}{4} \Rightarrow 20$



To find axial forces:



Now let us find the horizontal reactions @ fixed ends.

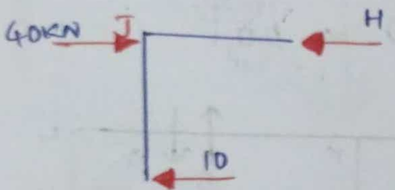
$H_A = 20\,KN$ (←)

$H_B = 40\,KN$ (←)

$H_C = 20\,KN$ (←)

We have calculated axial forces in the columns now let us find axial force in the beams.

@ Joint J:



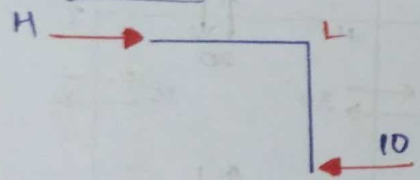
$\Sigma H = 0$

$-10 + 40 - H = 0$

$\boxed{H = 30\,KN}$

(To find axial force b/w K & L we can take joint 'K' & 'L'. 'L' will be easy)

@ Joint L:

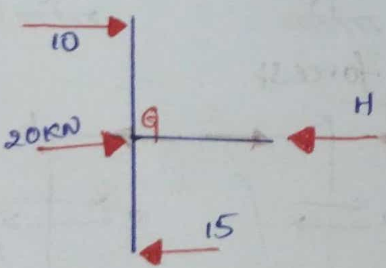


$\Sigma H = 0$

$H - 10 = 0$

$\boxed{H = 10\,KN}$

@ Joint G:

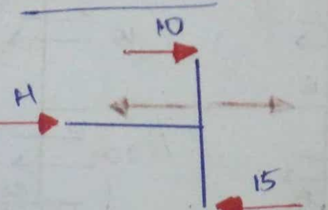


$\Sigma H = 0$

$10 + 20 - 15 - H = 0$

$\boxed{H = 15\,KN}$

@ Joint I:

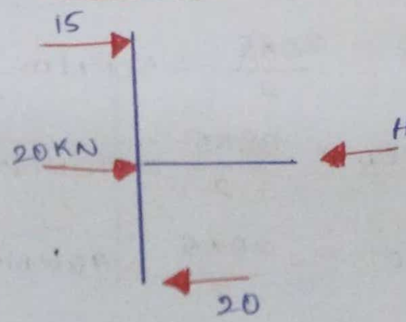


$\Sigma H = 0$

$10 + H - 15 = 0$

$\boxed{H = 5\,KN}$

@ Joint D:

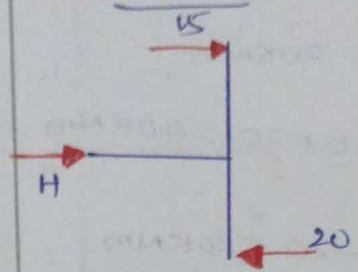


$\Sigma H = 0$

$15 + 20 - 20 - H = 0$

$\boxed{H = 15\,KN}$

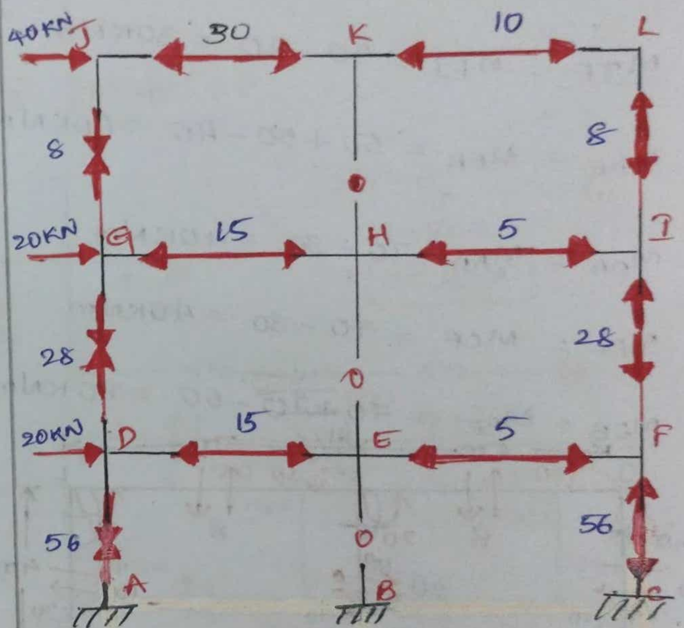
@ Joint F:



$\Sigma H = 0$

$H + 15 - 20 = 0$

$\boxed{H = 5\,KN}$



We completed horizontal and vertical axial forces.

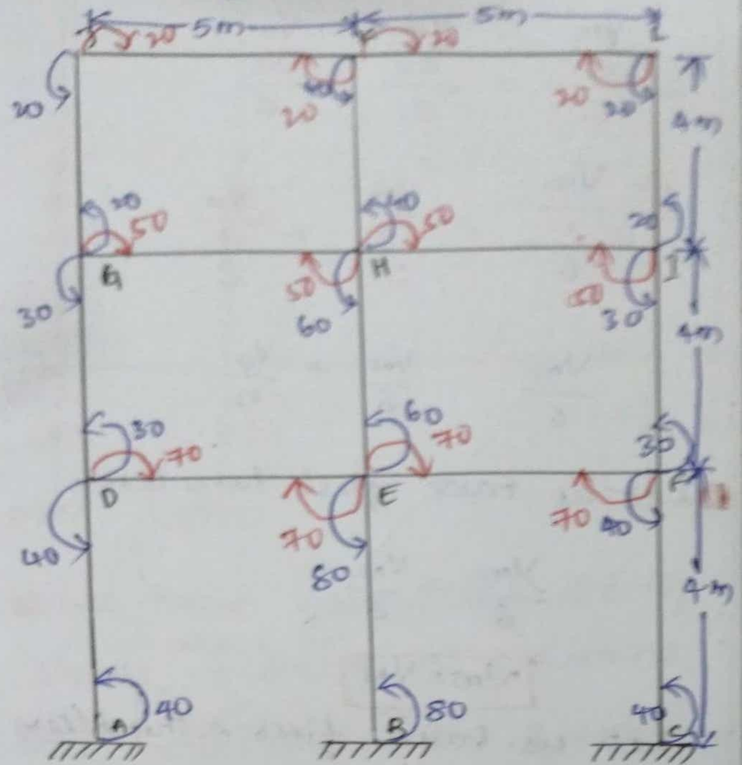
Now let us find the vertical reactions @ fixed ends. As the force is upward the reaction should be ↓ds @ B, $V_B = 0$ @ C the force is downward so the reaction

Should be ↑d

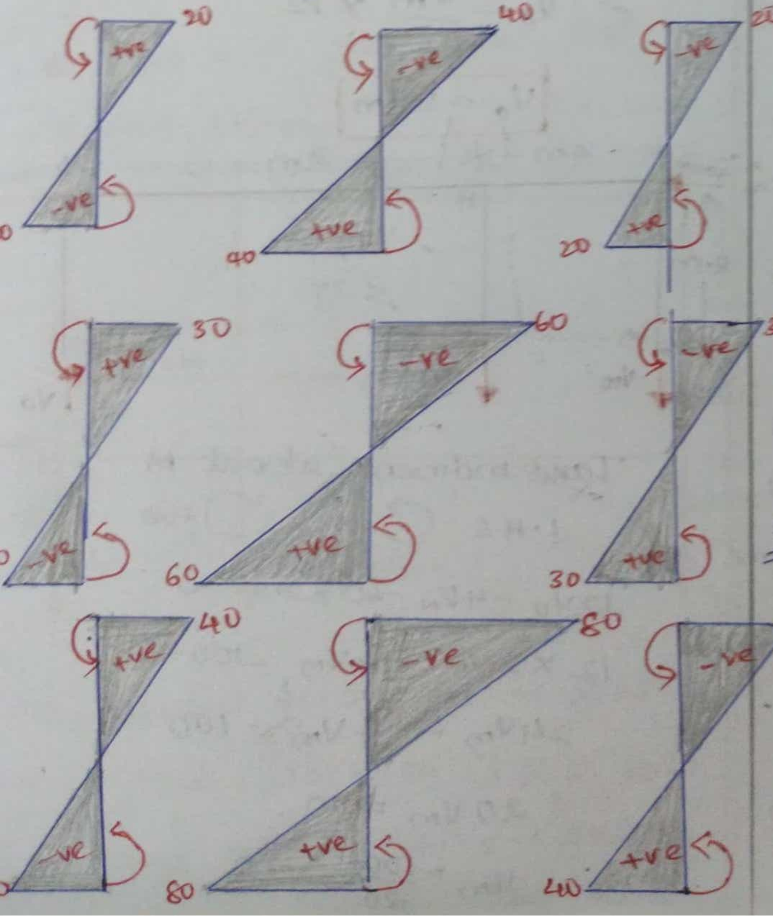
$V_A = 56KN$ (↓)

$V_B = 0$

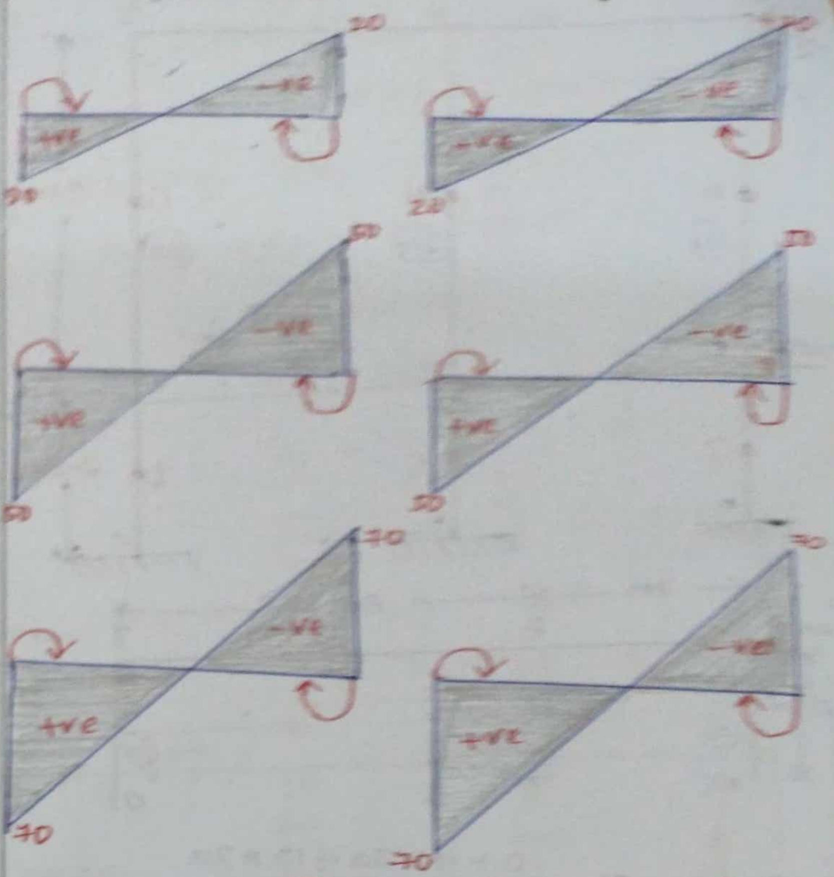
$V_C = 56KN$ (↑)



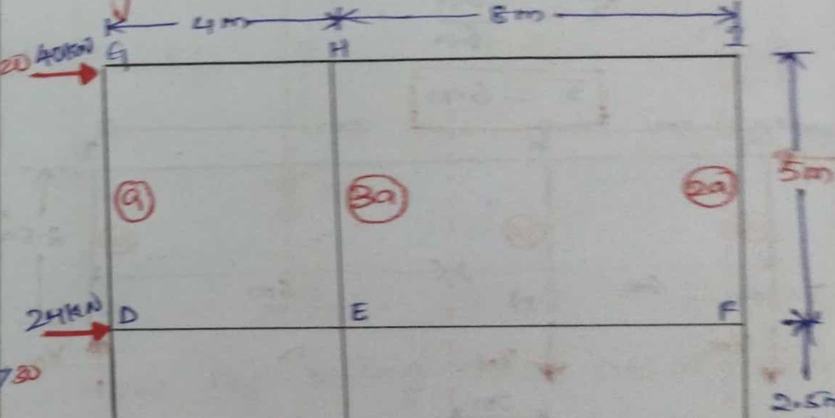
BMD for columns.



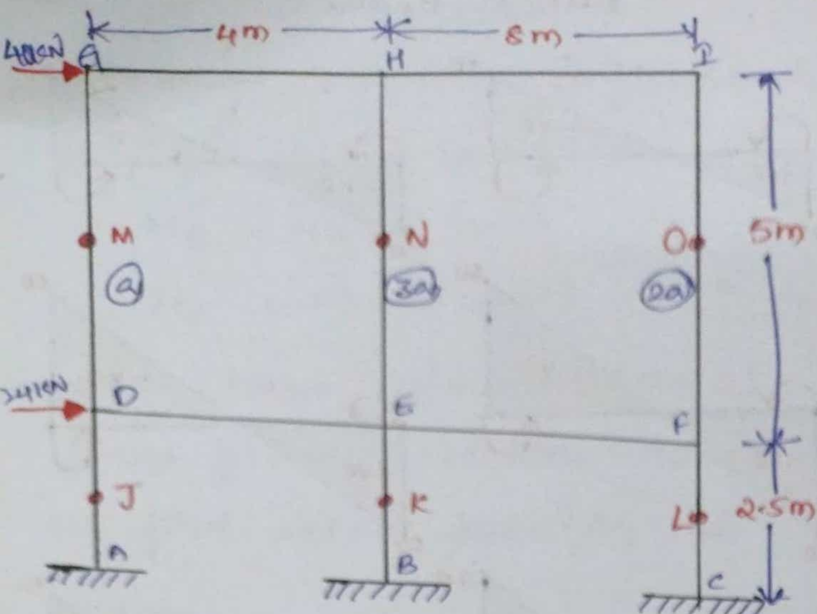
BMD for Beams

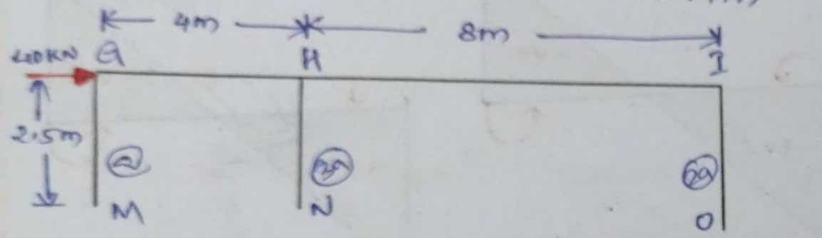


Problem no. 4 :- Analyse the frame using cantilever method.



Sol: Here in this frame the columns have different cross-sectional areas, in both the storeys. The first column has the area (a) and second column (3a) and third column (2a).

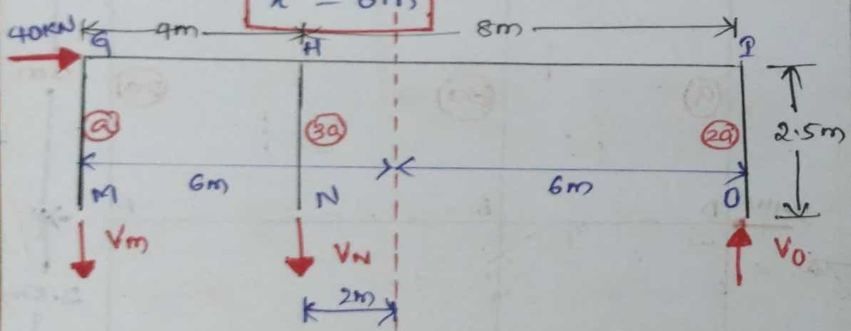




$$\bar{x} = \frac{0 + 4\times 3a + 12\times 2a}{a+3a+2a} = \frac{12a+24a}{6a}$$

$$\bar{x} = \frac{36a}{6a}$$

$$\boxed{\bar{x} = 6m}$$



The axial stresses in the columns are proportional to their distances from the centroidal axis of the frame

■ We know

$$\text{stress} = \frac{\text{Force}}{\text{Area}}$$

For the member MG, stress is $\frac{V_m}{a}$

& HN is $\frac{V_N}{3a}$ & OI is $\frac{V_O}{2a}$

■ lets us apply the centroidal distances for the members

$$\frac{\frac{V_m}{a}}{6} = \frac{\frac{V_N}{3a}}{2} = \frac{\frac{V_O}{2a}}{6}$$

$$\frac{\frac{V_m}{1}}{6} = \frac{\frac{V_N}{3}}{2} = \frac{\frac{V_O}{2}}{6}$$

$$\frac{V_m}{6} = \frac{V_N}{6} = \frac{V_O}{12}$$

■ Let us take first two terms

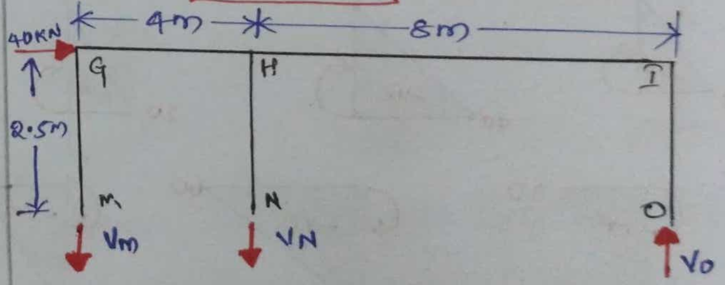
$$\frac{V_m}{\cancel{6}} = \frac{V_N}{\cancel{6}}$$

$$\boxed{V_m = V_N}$$

Let us take first & third term

$$V_O = \frac{V_m}{\cancel{6}} \times \cancel{12}^{\,2}$$

$$\boxed{V_O = 2V_m}$$



Take moment about M

L.H.S ↻ −ve ↺ +ve

$$12V_O - 4V_N - 40\times 2.5 = 0$$

$$12\times 2V_m - 4V_m - 100 = 0$$

$$24V_m - 4V_m = 100$$

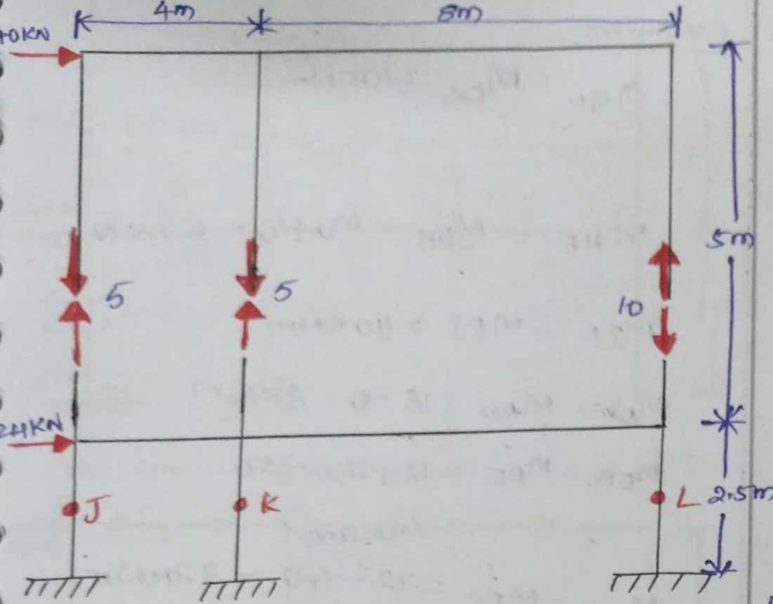
$$20V_m = 100$$

$$V_m = \frac{100}{20}$$

$\boxed{V_M = 5KN = V_N}$

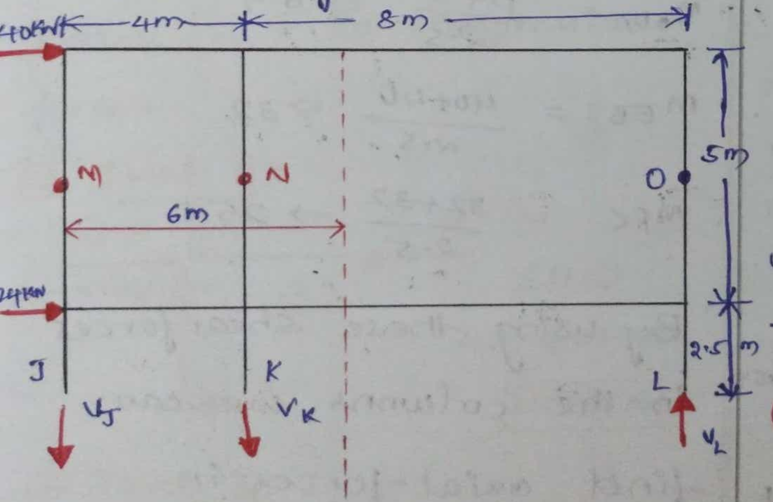
$V_0 = 2V_m$

$V_0 = 2\times 5 \Rightarrow \boxed{V_0 = 10KN}$



■ we have calculated the top storey above centre of the columns and below the centre of columns it will be opposite direction.

■ Now will calculate the second storey.



The axial forces in M & N are same likewise J & K will be same and also V_0 is $2V_m$ same $V_K = V_J$ $V_L = 2V_J$

Take moment about J

L.H.S (↻ -ve ↺) +ve

$12V_L - 4V_K - 40\times \overset{5+1.25}{6.25} - 24\times 1.25 = 0$

$12\times 2V_J - 4V_J - 250 - 30 = 0$

$24V_J - 4V_J - 250 - 30 = 0$

$20V_J - 280 = 0$

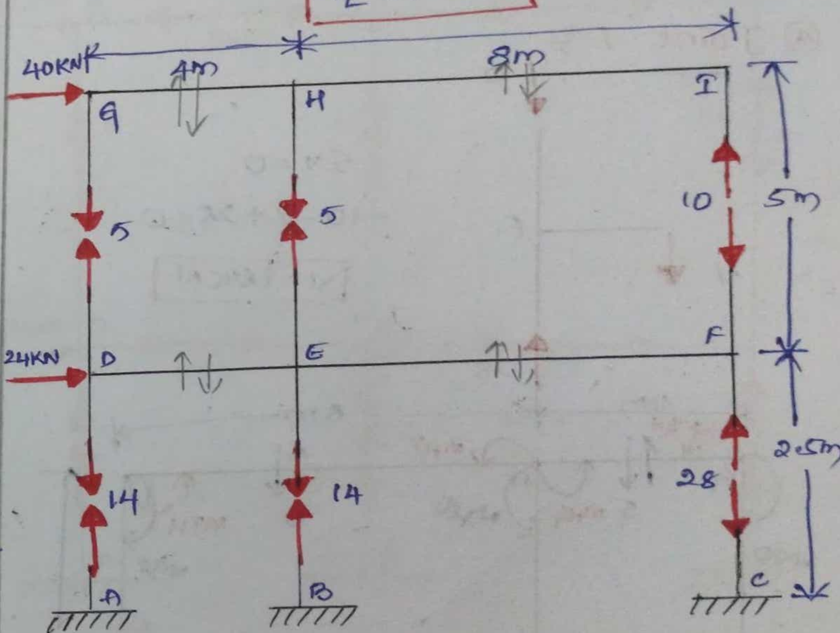
$20V_J = 280$

$V_J = \frac{280}{20}$

$\boxed{V_J = 14KN}$

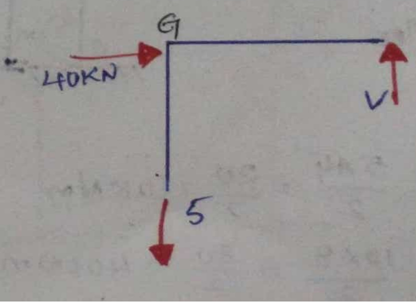
$V_L = 2V_J = 2\times 14$

$\boxed{V_L = 28KN}$



Now by using these axial forces we can find the shear forces in the beams.

@ Joint G :-

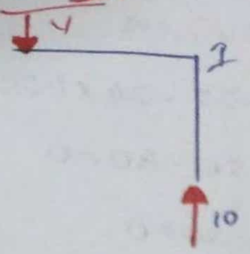


$\Sigma V = 0$

$-5 + V = 0$

$\boxed{V = 5KN}$

@ Joint I :-

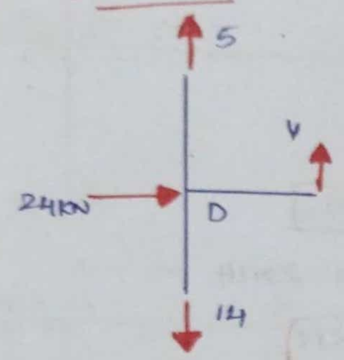


$\Sigma V = 0$

$10 - V = 0$

$\boxed{V = 10KN}$

@ Joint D :-

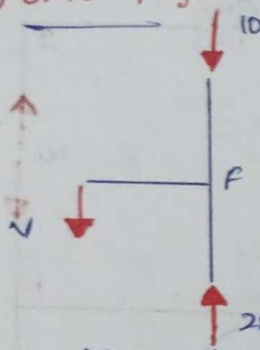


$\Sigma V = 0$

$-14 + 5 + V = 0$

$\boxed{V = 9KN}$

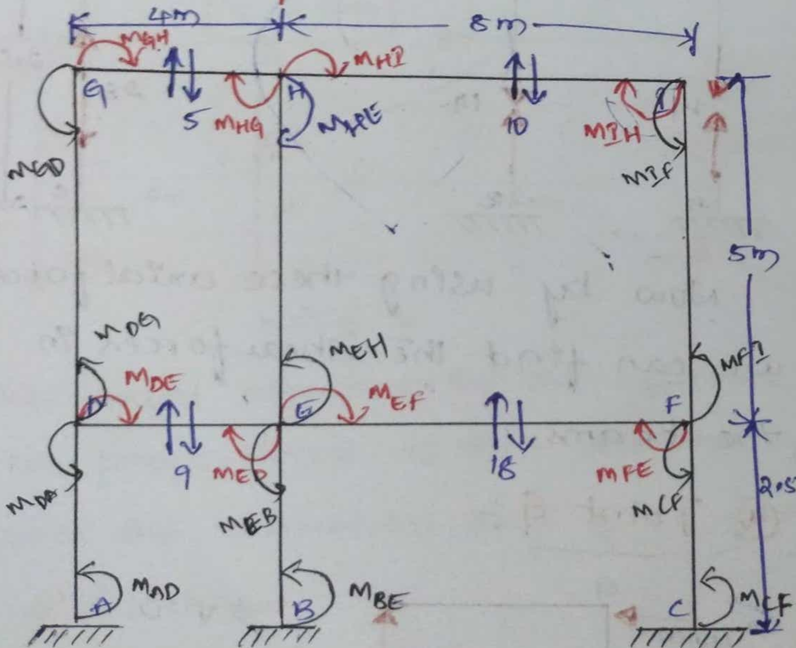
@ Joint F :-



$\Sigma V = 0$

$-10 - V + 28 = 0$

$\boxed{V = 18KN}$



Beams

$M_{GH} = M_{HG} = \frac{5 \times 4}{2} = \frac{20}{2} = 10KNm$

$M_{HI} = M_{IH} = \frac{10 \times 8}{2} = \frac{80}{2} = 40KNm$

$M_{DE} = M_{ED} = \frac{9 \times 4}{2} = 18KNm$

$M_{EF} = M_{FE} = \frac{18 \times 8}{2} = 72KNm$

columns

$M_{GD} = M_{DG} = 10KNm$

(if $M_{GH} = 10$ then M_{GD} & M_{DG} also to be same)

$M_{HE} = M_{EH} = 40 + 10 = 50KNm$

$M_{IF} = M_{FI} = 40KNm$

$M_{DA} = M_{AD} = 18 - 10 = 8KNm$

$M_{EB} = M_{BE} = 18 + 72 - 50$

$= 40KNm$

$M_{CF} = M_{FC} = 72 - 40 = 32KNm$

shearforce in column

$M_{GD} = \frac{10+10}{5} \Rightarrow \frac{20}{5} \Rightarrow 4$

$M_{HE} = \frac{50+50}{5} = \frac{100}{5} \Rightarrow 20$

$M_{IF} = \frac{40+40}{5} = \frac{80}{5} \Rightarrow 16$

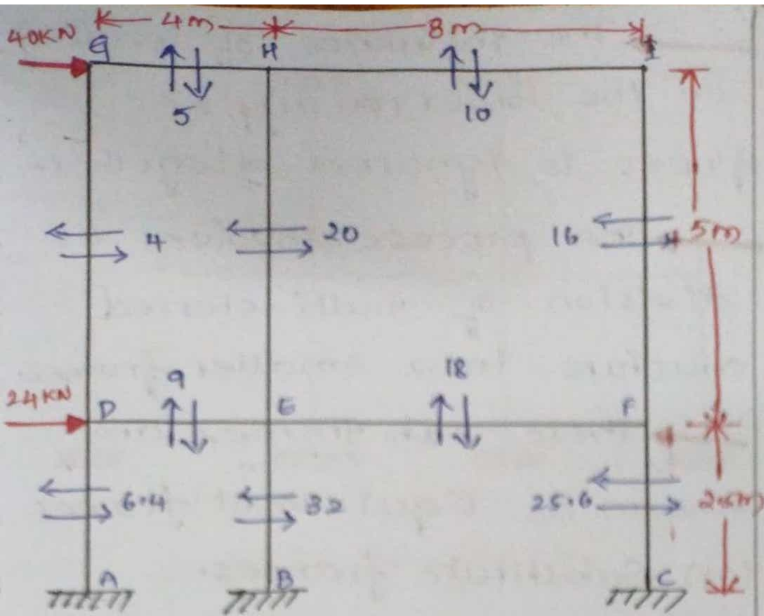
$M_{DA} = \frac{8+8}{2.5} \Rightarrow 6.4$

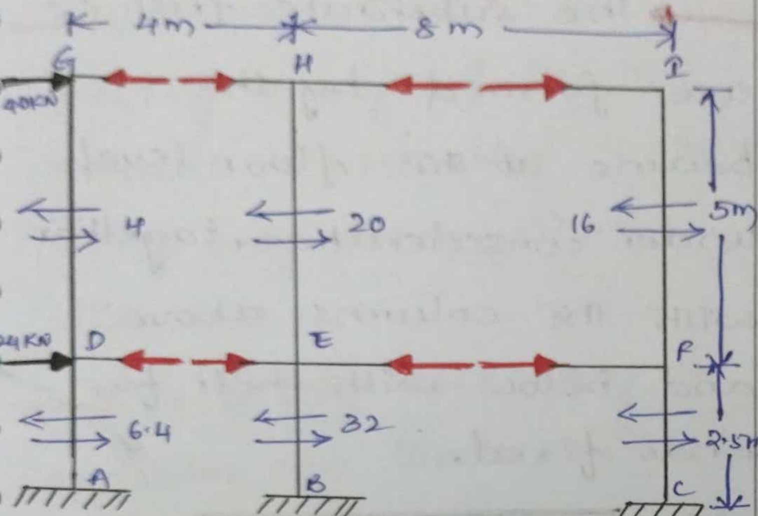
$M_{EB} = \frac{40+40}{2.5} \Rightarrow 32$

$M_{FC} = \frac{32+32}{2.5} \Rightarrow 25.6$

figure →

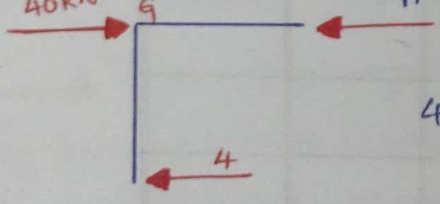
By using these shearforces in the columns we can find axial forces in the beams.





To find the axial force in beams w.r.to compressive forces.

@ Joint G:

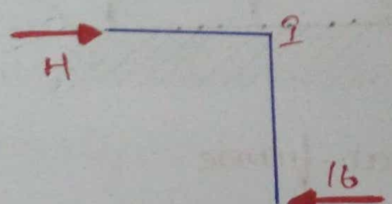


$\Sigma H = 0$

$40 - 4 - H = 0$

$\boxed{H = 36KN}$

@ Joint I:

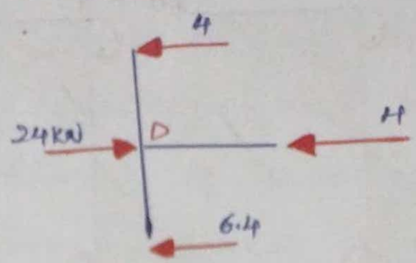


$\Sigma H = 0$

$H - 16 = 0$

$\boxed{H = 16KN}$

@ Joint D:

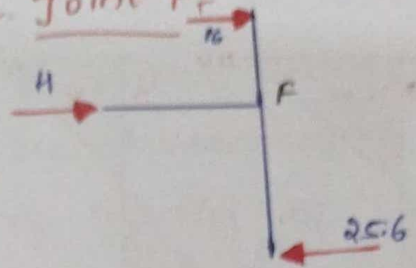


$\Sigma H = 0$

$4 + 24 - 6.4 - H = 0$

$\boxed{H = 21.6KN}$

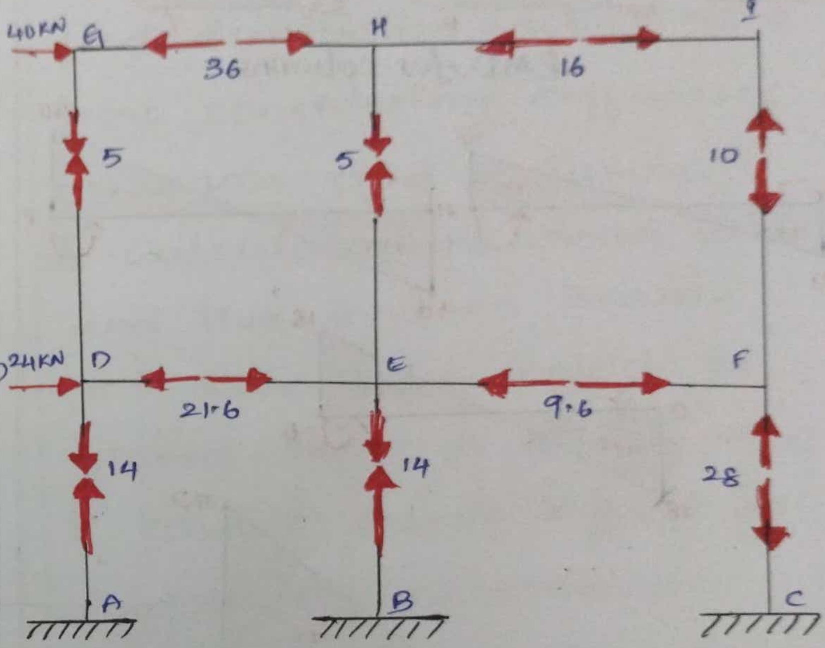
@ Joint F:

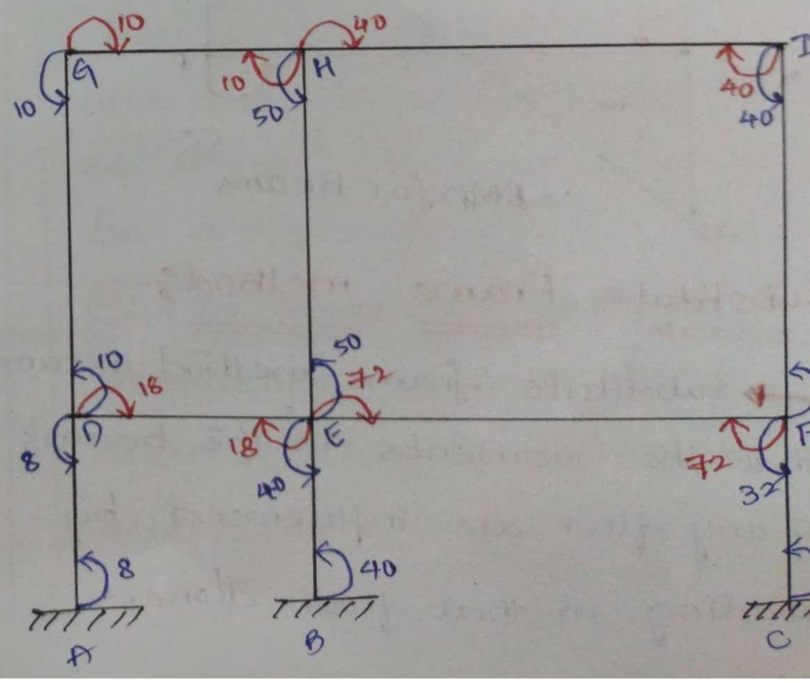


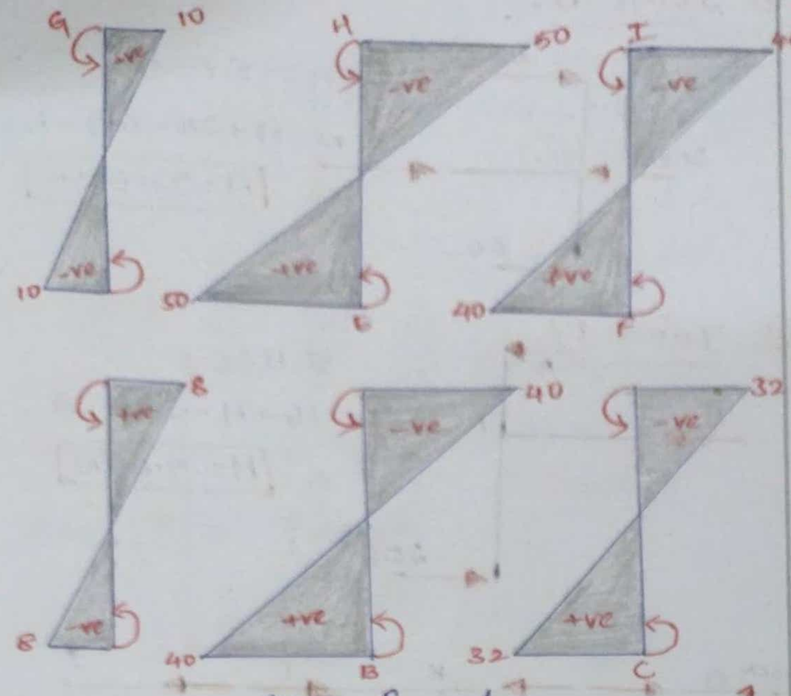
$\Sigma H = 0$

$16 + H - 25.6 = 0$

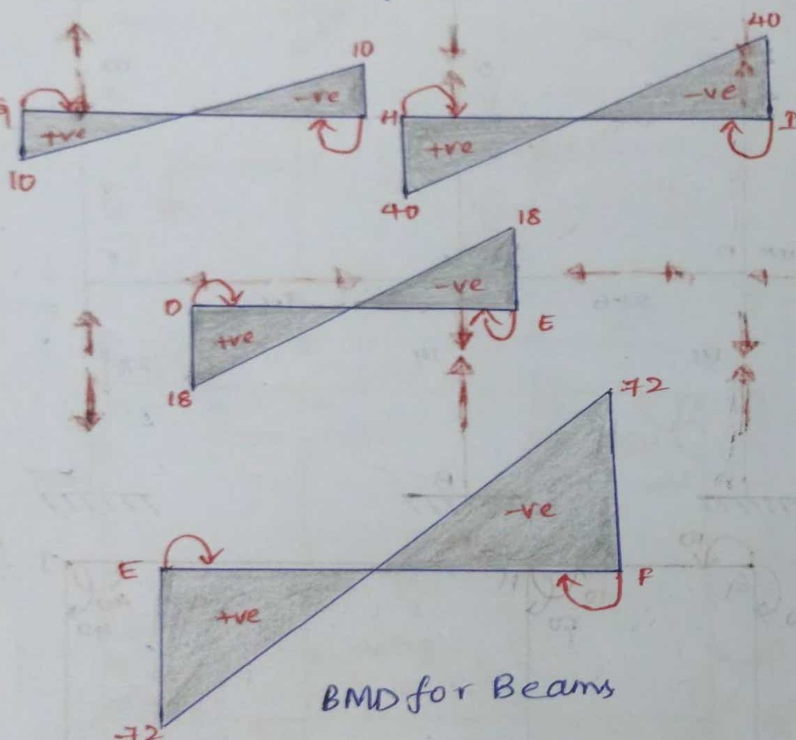
$\boxed{H = 9.6KN}$







BMD for columns.



BMD for Beams

Substitute Frame method:

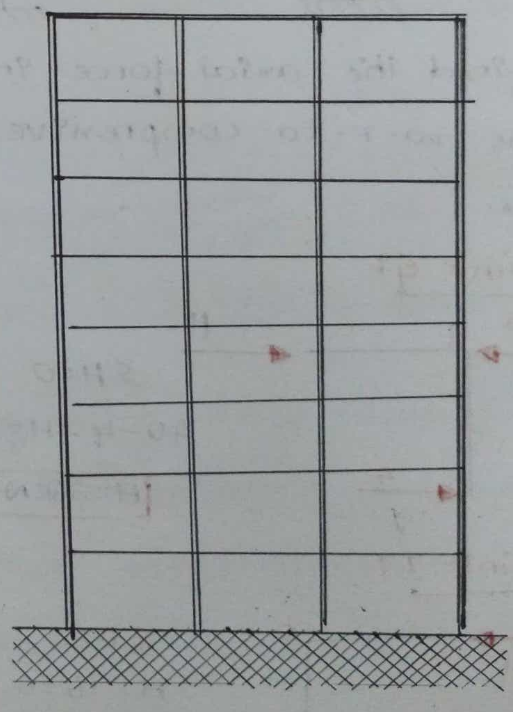
→ Substitute frame method assumes that the moments in the beams of any floor are influenced by loading on that floor alone.

→ The influence of loading on the lower (or) upper floors is ignored altogether.

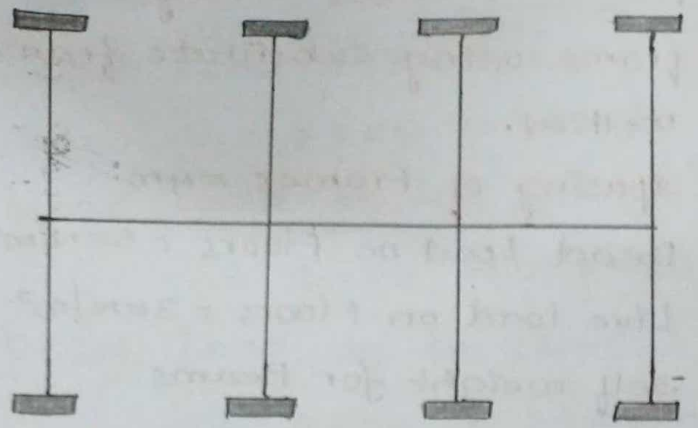
→ The process involves the division of multi-storied structure into smaller frames.

→ These sub frames are known as Equivalent frames (or) Substitute frames.

→ The substitute frames are formed by the beams at the floor level under consideration, together with the columns above and below with their far ends fixed.



Actual frame



Substitute frame.

→ The sub frames are usually analyzed by the moment distribution method, using only two cycle of distribution.

→ It is only necessary to consider the loads on the two nearest spans on each side of the point.

→ The distributed B.M are not carried over far ends of the columns in this process.

→ The moments in the columns are computed at each floor level independently are retained at that floor irrespective of further analysis.

Steps for the Analysis:-

1. Select a substitute frame, by taking floor beam with columns of lower and upper storeys fixed at far ends.

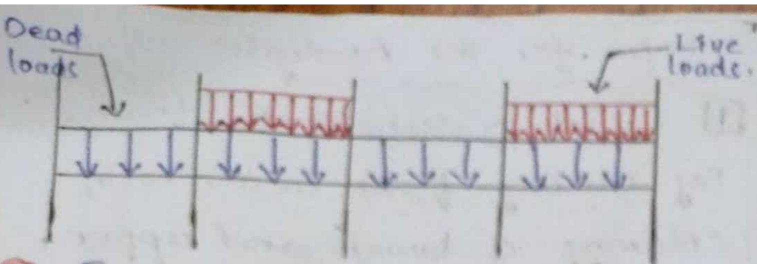
2. Cross sectional dimensions of beams and columns may be chosen such that moment of inertia of beam is 1.5 to 2 times that of a column & find distribution factors at a joint considering stiffnesses of beams and columns.

3. Calculate the dead load and live load on beam.

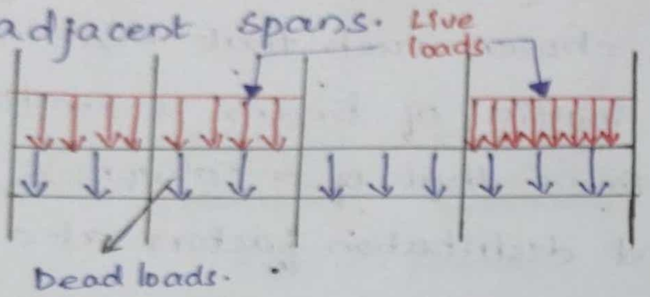
→ Live load should be placed in such a way that it causes worst effect at the section considered.

[Live loads are placed alternate & adjacent spans as shown below should be adopted]

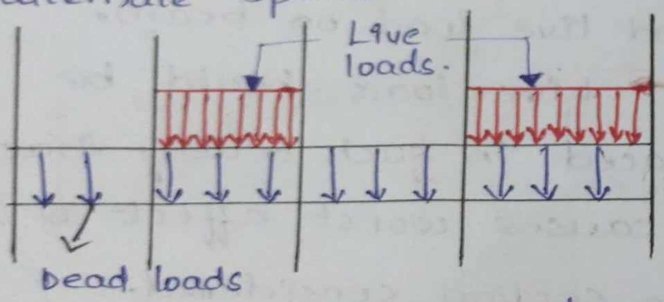
(a) For maximum positive moments at mid point of span: Live loads are on alternate spans.



(b) For maximum negative moments at the point of support: Live loads are on adjacent spans.



(c) For maximum moments in columns: Live loads are on alternate spans.



[4] Find the initial fixed end moments and analyse this frame by moment distribution method.

[5] Finally draw shear and moment diagram indicating values at critical section.

Problem no. (5): Analyse the frame using substitute frame method.

Spacing of frames = 4m

Dead Load on floors = 5KN/m²

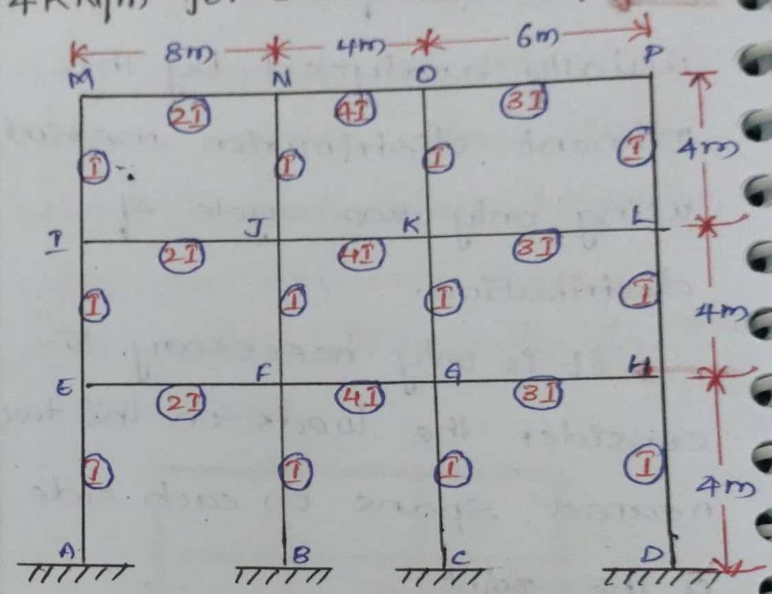
Live load on floors = 3KN/m²

Self weight for Beams

7KN/m for beams with span 8m

1KN/m for beams with span 4m

4KN/m for beams with span 6m.



Sol: Spacing of Frames = 4m

Dead Load on Floors = 5KN/m²

Live Load on Floors = 3KN/m²

Self weight for Beams

7KN/m for beams with span 8m

1KN/m for beams with span 4m

4KN/m for beams with span 6m

Live load on per meter run of girder = 3×4 = 12KN/m.

Dead load on per meter run of

girder = 5×4 = 20 KN/m

Dead Load on 8m span

$= 7 + 20 \Rightarrow 27\ KN/m$

Dead Load on 4m span.

$= 1 + 20 \Rightarrow 21\ KN/m$

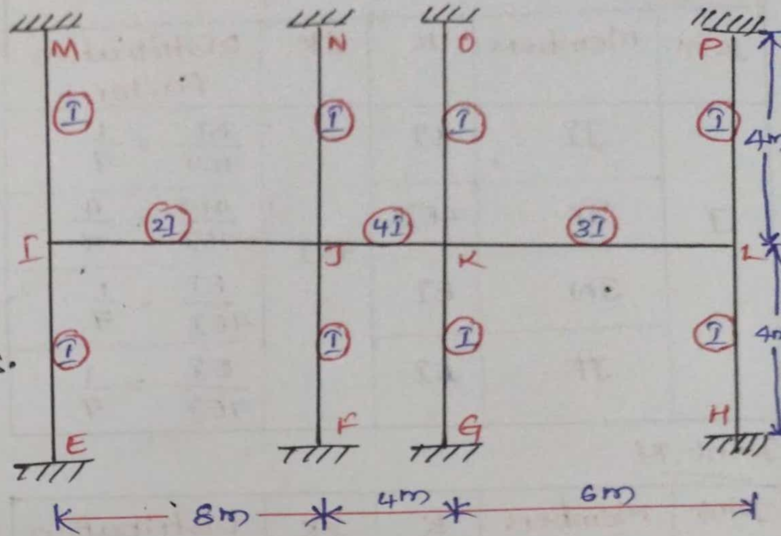
Dead Load on 6m span

$= 4 + 20 \Rightarrow 24\ KN/m$

Substitute frame method:-

- This method is only applicable for the multistorey frames subjected to only vertical loads.
- This method assumes that the moments in the beams of any floor are influenced by loading on that floor alone.
- The influence of loading on the lower on upper floors is ignored altogether.
- Columns are considered to be fixed at far ends.
- Analysis is done by the moment distribution method, using only two cycle of distribution. So this method is also called "Two cycle method."
- Here in this problem Analysis of second floor is only done. Similar to analysis can be done for all the floors to get complete solution.

When we take the second floor for the analysis we have to also the columns above and below.



We know that in substitute frame the far ends of the columns should be fixed.

Stiffness :- $K = \frac{4EI}{L}$ } to find

- In this frame all of the columns are having same moment of Inertia

K for all the columns $= \frac{4EI}{4} \Rightarrow EI$

K for beam IJ $= \frac{4E(2I)}{8} \Rightarrow EI$

K for beam JK $= \frac{4E(4I)}{4} \Rightarrow 4EI$

K for beam KL $= \frac{4E(3I)}{6} = 2EI$

Joint I:

Joint	Members	K	ΣK	Distribution Factor
I	IM	EI	3EI	$\frac{EI}{3EI} = \frac{1}{3}$
	IE	EI		$\frac{EI}{3EI} = \frac{1}{3}$
	IJ	EI		$\frac{EI}{3EI} = \frac{1}{3}$

Joint J:

Joint	Members	K	ΣK	Distribution Factor
J	JI	EI	7EI	$\frac{EI}{7EI} = \frac{1}{7}$
	JK	4EI		$\frac{4EI}{7EI} = \frac{4}{7}$
	JN	EI		$\frac{EI}{7EI} = \frac{1}{7}$
	JF	EI		$\frac{EI}{7EI} = \frac{1}{7}$

Joint K:

Joint	Members	K	ΣK	Distribution Factor
K	KJ	4EI	8EI	$\frac{4EI}{8EI} = 0.5$
	KL	2EI		$\frac{2EI}{8EI} = 0.25$
	KO	EI		$\frac{EI}{8EI} = 0.125$
	KG	EI		$\frac{EI}{8EI} = 0.125$

Joint L:

Joint	Members	K	ΣK	Distribution Factor
L	LK	2EI	4EI	$\frac{2EI}{4EI} = 0.5$
	LP	EI		$\frac{EI}{4EI} = 0.25$
	LH	EI		$\frac{EI}{4EI} = 0.25$

To find fixed end moments

Live load = 12 kN/m

Dead load on 8m span = 27 kN/m

Dead load on 4m span = 21 kN/m

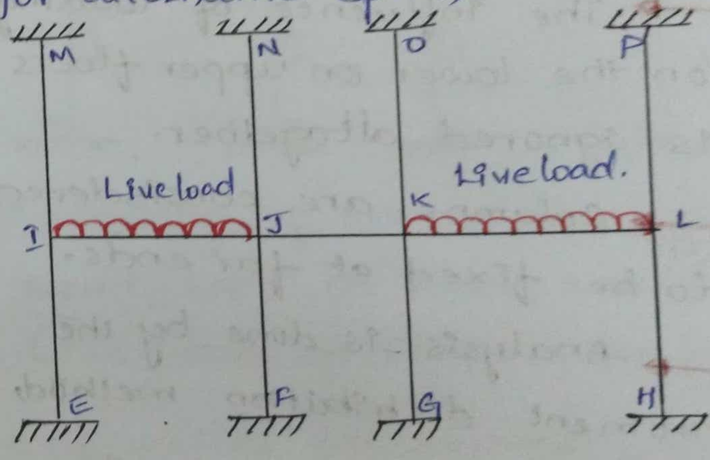
Dead load on 6m span = 24 kN/m

Span	Length	LL	DL	Total load	FEM due to DL FEM $= \frac{wl^2}{12}$	FEM due to Total load
IJ	8m	12	27	27+12 = 39	$\frac{27 \times 8^2}{12}$ = 144	$\frac{39 \times 8^2}{12}$ = 208
JK	4m	12	21	21+12 = 33	$\frac{21 \times 4^2}{12}$ = 28	$\frac{33 \times 4^2}{12}$ = 44
KL	6m	12	24	12+24 = 36	$\frac{24 \times 6^2}{12}$ = 72	$\frac{36 \times 6^2}{12}$ = 108

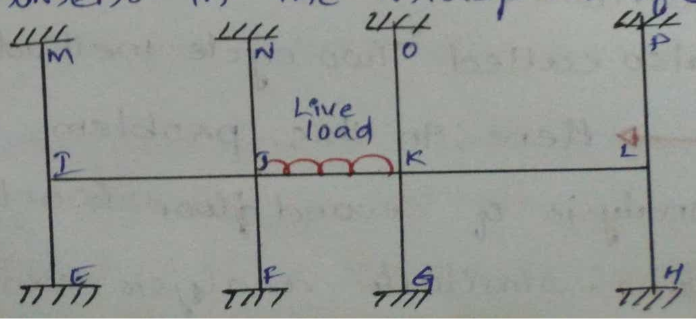
Design moments in Beams:

To find the maximum moment in the midspan of IJ

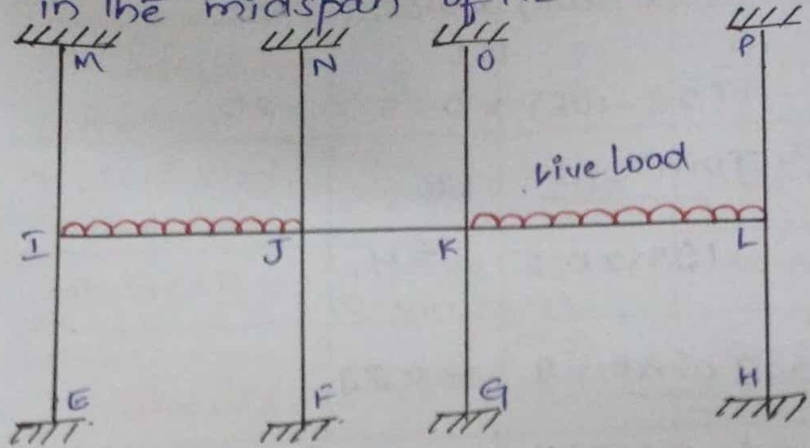
In all of the members there will be dead load in addition to that live load is added. (this will be applied for alternative span)



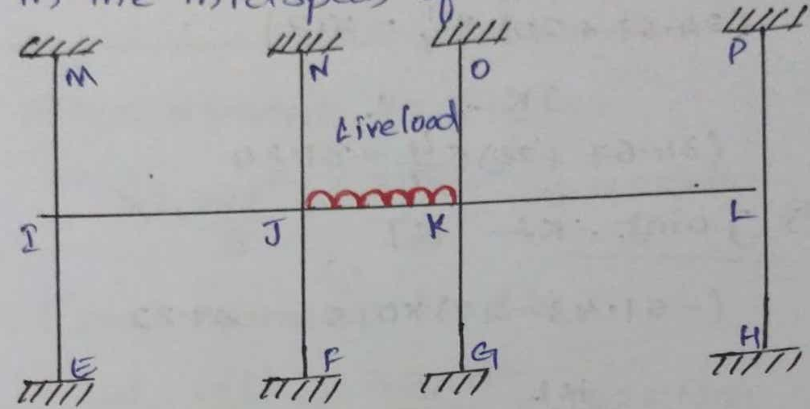
To find the maximum moment in the midspan of JK



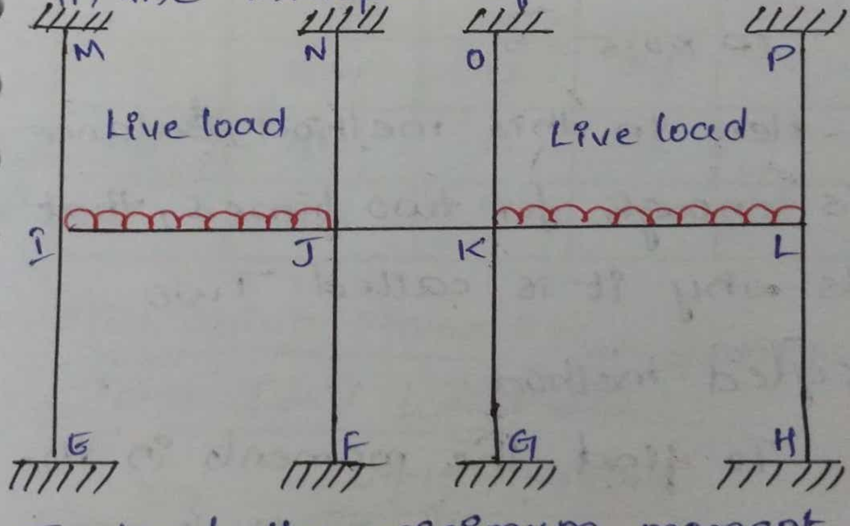
To find the maximum moment in the midspan of KL.



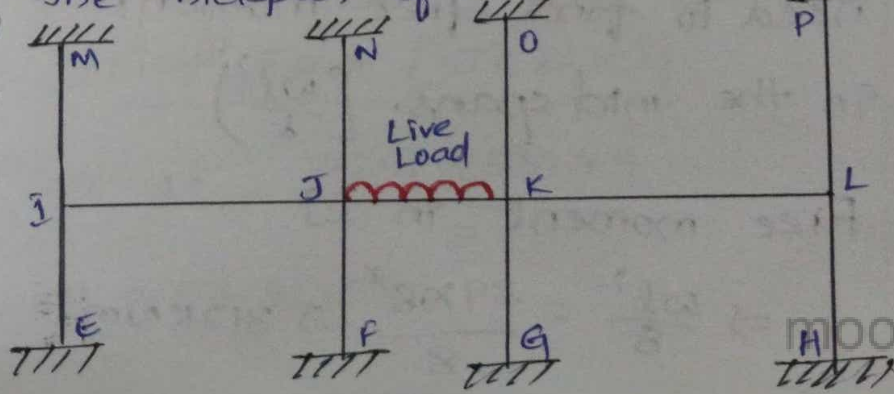
To find the minimum moment in the midspan of IJ



To find the minimum moment in the midspan of JK

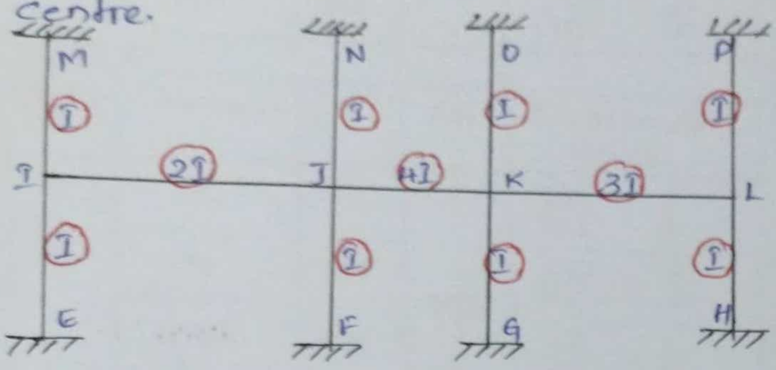


To find the minimum moment in the midspan of KL



→ For all the three max^m JK min^n IJ & KL have the same principle. (same centre)

→ Similarly max^m moment IJ, min^n JK & max^m KL also same centre.



Member	I	J		K		L
	IJ	JI	JK	KJ	KL	LK
Distribution Factor	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{1}{2}$	0.25	$\frac{1}{2}$
Fixed End moments	-208	208	-28	28	-108	108
	ACW	CW	ACW	CW	ACW	CW

Span	FEM due to DL	FEM due to Total load
IJ	144	208
JK	28	44
KL	72	108

only for IJ & KL we are applying the live load. JK – DL only.

(Balance – 1)

@ Joint I:- IJ

$-208 \times \frac{1}{3} = -69.33$

@ Joint J:- JI

$(208-28) \times \frac{1}{7} = 25.71$

@ Joint J:- JK

$(208-28) \times \frac{4}{7} = 102.86$

@ Joint K:- KJ

$(28-108) \times 0.5 = -40$

KL

$(28-108) \times 0.25 = -20$

@ Joint L:- LK

$108 \times 0.5 = 54$

(Balance – 2)

@ Joint I → IJ

$(-12.85) \times \frac{1}{3} = -4.28$

@ Joint J:- JI

$(34.67+20) \times \frac{1}{7} = 7.81$

JK

$(34.67+20) \times \frac{4}{7} = 31.24$

@ Joint K:- KJ

$(-51.43-27) \times 0.5 = -39.22$

KL

$(-51.43-27) \times 0.5 = -19.61$

@ Joint L:- LK

$10 \times 0.5 = 5$

Here in this method, Balance is enough for two times, that is why it is called Two cycled method.

To find the moments in the midspan, for that first we need to find free moments in the mid spans. $\left[\frac{wl^2}{8}\right)$

Free moment in IJ

$\Rightarrow \frac{wl^2}{8} = \frac{39 \times 8^2}{8} \Rightarrow 312\text{ kNm}$

Member	I	J		K		L
	IJ	JI	JK	KJ	KL	LK
Distribution Factor	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{1}{2}$	0.25	$\frac{1}{2}$
Fixed end Moments	-208	208	-28	28	-108	108
Balance (1)	+69.33	-25.71	-102.86	40	20	-54
carry over Moments	-12.85	34.67	20	-51.43	-27	10
Balance (2)	4.28	-7.81	-31.24	39.22	19.61	-5
Total	-147.24	209.15	-142.1	55.79	-95.39	59

Free moment in JK

$\Rightarrow \frac{wl^2}{8} = \frac{21 \times 4^2}{8} \Rightarrow 42 KNm$

Free moment in LK

$\Rightarrow \frac{wl^2}{8} = \frac{36 \times 6^2}{8} = 162 KNm$

Span	Length	LL	DL	Total load
IJ	8m	12	27	39
JK	4m	12	21	33
KL	6m	12	24	36

Mid point Moment:-

span. IJ

Joint J & I [IJ & JI]

$\Rightarrow 312 - \frac{209.15 + 147.24}{2}$

$\Rightarrow 133.81 KNm$

spanJK

Joint J & K [JK & KJ]

$\Rightarrow 42 - \frac{142.1 + 55.79}{2}$

$\Rightarrow -56.95 KNm$

span KL

Joint K & L [KL & LK]

$\Rightarrow 162 - \frac{95.39 + 59}{2}$

$\Rightarrow 84.81 KNm$

To find max^m moment in the midspan of JK & min^m moment in IJ & KL we need to use same structure and two tables.

(Balance -1) for next table.

@ Joint I:-

$IJ \Rightarrow -144 \times \frac{1}{3} = -48$

@ Joint J:-

$JI \Rightarrow (144 - 44) \times \frac{1}{7} = 14.29$

$JK \Rightarrow (144 - 44) \times \frac{4}{7} = 57.14$

@ Joint K:-

$KJ = (44 - 72) \times 0.5 = -14$

$KL = (44 - 72) \times 0.25 = -7$

@ Joint L:-

$LK = 72 \times 0.5 = 36.$

(Balance -2)

@ Joint -I:-

$IJ = (-7.15) \times \frac{1}{3} = -2.38$

Member	I	J		K		L
	IJ	JI	JK	KJ	KL	LK
Distribution Factor	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{1}{2}$	0.25	$\frac{1}{2}$
Fixed End Moments	−144	144	−44	44	−72	72
Balance	48	−14.29	−57.14	14	7	−36
Carry over moments	−7.15	24	7	−28.57	−18	3.5
Balance	2.38	−4.43	−17.71	23.29	11.64	−1.75
Total	−100.77	149.28	−111.85	52.72	−71.36	37.75
Mid-point Moment	$216 - \frac{100.77+149.28}{2}$ $= 90.98$ KNm		$66 - \frac{111.85+52.72}{2}$ $= -16.29$ KNm		$108 - \frac{71.36+37.75}{2}$ $= 53.45$ KNm	

@ Joint J:

$JI = (24+7) \times \frac{1}{7} = 4.43$

$JK = (24+7) \times \frac{4}{7} = 17.71$

@ Joint K:

$(-28.57 - 18) \times 0.5 = -23.29$

$KJ = (-28.57 - 18) \times 0.5 = -23.29$

$KL = (-28.57 - 18) \times 0.25 = -11.64$

@ Joint L:

$LK = (3.5 \times 0.5) = 1.75$

Free moment IJ:

$$\frac{wl^2}{8} = \frac{27 \times 8^2}{8} = 216 \text{ KNm}$$

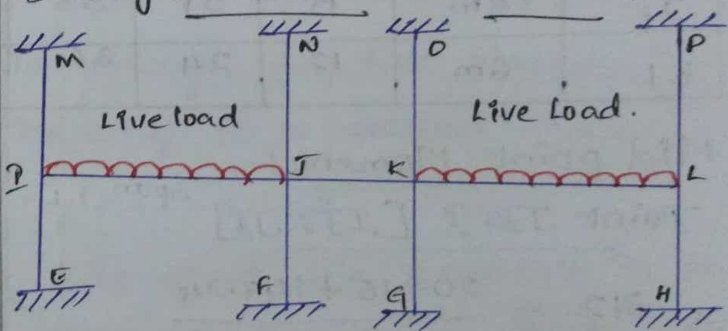
Free moment JK:

$$\frac{wl^2}{8} = \frac{33 \times 4^2}{8} = 66 \text{ KNm}$$

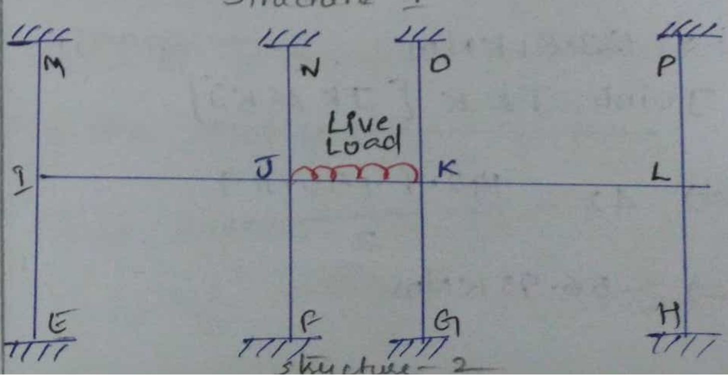
Free moment in LK:

$$\frac{wl^2}{8} = \frac{24 \times 6^2}{8} = 108 \text{ KNm}$$

Design moments in columns



Structure 1



Structure - 2

we have only these 2 possibilities for the structure one we have already made MDM method in which we have to select the FEM & carryover moments

Member	I	J		K		L
FEM	-208	208	-28	28	-108	108
carry over moment	-0.85	34.67	20	-51.43	-27	10
FEM + COM	adding -220.85	234.67		-158.43		118

column moments

= -(FEM+COM) × DF

column moment at top	73.62	-33.52	19.80	-29.5
column moments at bottom	73.62	-33.52	19.80	-29.5

Members	DF
IM & IE	1/3
JF & JN	1/7
KG & KO	0.125
LP & LH	0.25

Even for structure two we have MDM but we need to calculate FEM & COM.

Member	I	J		K		L
FEM	-144	144	-44	44	-72	72
carry over moment	-7.15	24	7	-28.57	-18	3.5
FEM + COM	-151.15	131		-74.57		75.5
column moment at top	50.38	-18.71		9.32		-18.88
column moments at bottom	50.38	-18.71		9.32		-18.88

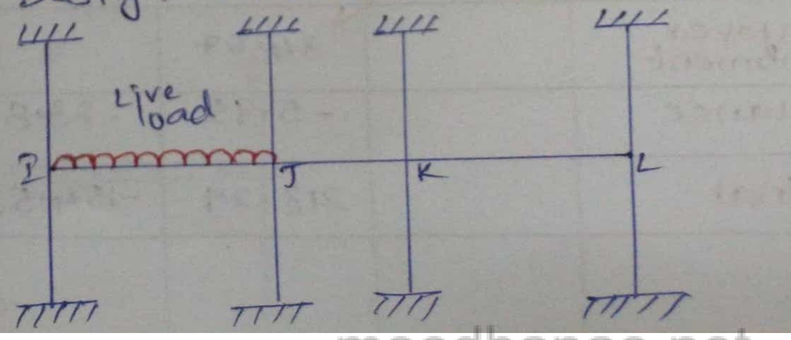
Members	DF
IM & IE	1/3
JF & JN	1/7
KG & KO	0.125
LP & LH	0.25

We have calculated two types of column moments to select the maximum values, now compare two tables and select the maximum values.

Design moments in columns a

Joint	I	J	K	L
At top	73.62	-33.52	19.80	-29.5
At Bottom	73.62	-33.52	19.80	-29.5

Design moments max^m at Joint



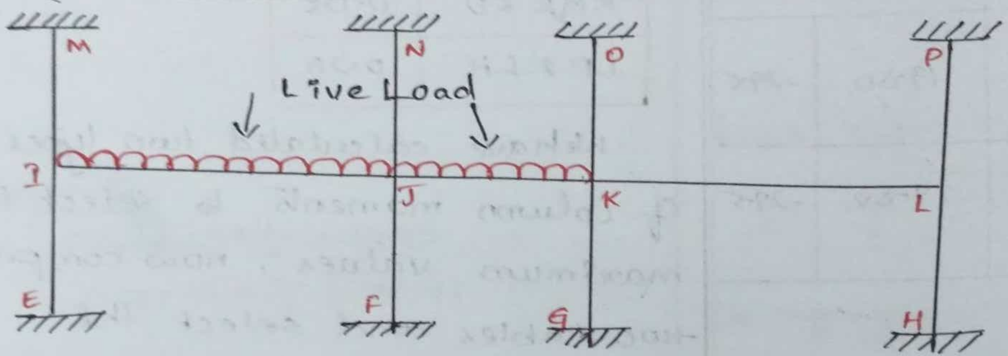
Member	I	J		K		L
	IJ	JI	JK	KJ	KL	LK
Distribution Factor	1/3	1/7	4/7	0.5	0.25	0.5
Fixed end Moments	−208	208	−28	28	72	72
Balance 1	69.83	−25.71	−102.86	No need because we are doing only in Joint I		
Carry over moment	−12.86					
Balance	4.29					
Final	−147.24					

Balance 1

$-208 \times \frac{1}{3} = -69.33$ — IJ $(208-28) \times \frac{4}{7} = 102.86 \to$ JK.

$(208-28) \times \frac{1}{7} = 25.71$ — JI

To find maximum Moment at Joint -J :-



As Joint 'J' connecting two spans IJ & JK so we have to apply liveload for both spans.

Member	I	J		K		L
	IJ	JI	JK	KJ	KL	LK
Distribution Factor	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{1}{2}$	0.25	$\frac{1}{2}$
Fixed End Moments	−208	208	−44	44	−72	72
Balance	69.83	−23.43	−93.71	14	7	
carryover moment		34.67	7			
Balance		−5.95	−23.81			
Final		213.29	−154.52			

$IJ \rightarrow (-208) \times \frac{1}{3} = -69.33$

$JI \rightarrow (208-44) \times \frac{1}{7} = 23.43$

$JK \rightarrow (208-44) \times \frac{4}{7} = 93.71$

$KJ \rightarrow (44-72) \times 0.5 = -14$

Balance 1

$KL \rightarrow (44-72) \times 0.25 = -7$

Balance 2

$JI \rightarrow (34.67+7) \times \frac{1}{7} = 5.95$

$JK \rightarrow (34.67+7) \times \frac{4}{7} = 23.81$

$JI \rightarrow (144-44) \times \frac{1}{7} = 14.29$

$JK \rightarrow (144-44) \times \frac{4}{7} = 57.14$

$KJ \rightarrow (44-108) \times 0.5 = -32$

$KL \rightarrow (44-108) \times 0.25 = -16$

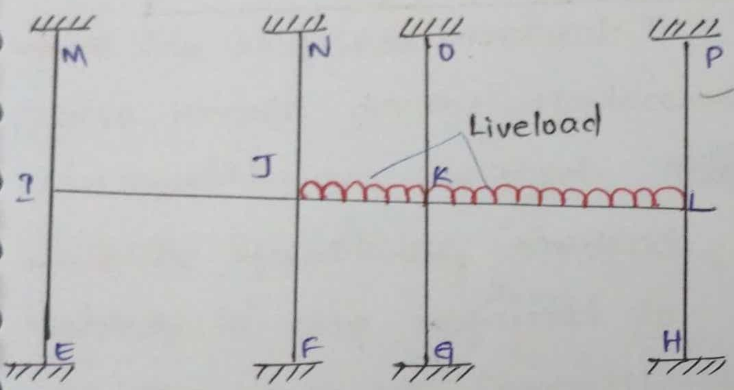
$LK \rightarrow (108 \times 0.5) = 54$

Balance -2

$KJ \rightarrow (-28.57-27) \times 0.5 = -27.79$

$KL \rightarrow (-28.57-27) \times 0.25 = -13.89$

To find Maximum moment at Joint K:



Balance -1 Joint 'L'

$KJ \rightarrow (28-108) \times 0.5 = -40$

$(28-108) \times 0.25 = -20 \rightarrow KL$

$LK \rightarrow (108 \times 0.5) = 54$

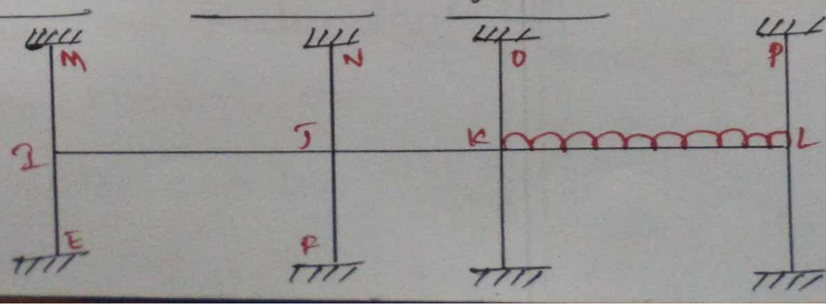
Balance -2

$LK \rightarrow 10 \times 0.5 = 5$

(P.T.O)

Member	I	J		K		L
	IJ	JI	JK	KJ	KL	LK
Distribution Factor	1/3	1/7	4/7	1/2	0.25	1/2
Fixed End Moments	-144	144	-44	44	-108	108
Balance		-14.29	-57.14	32	16	-54
Carry over moment				-28.57	-27	
Balance				27.79	13.89	
Final				75.22	-105.11	

To find maximum moment at Joint L:



Member	I	J		K		L
	IJ	JI	JK	KJ	KL	LK
Distribution Factor	1/3	1/7	4/7	0.5	0.25	0.5
Fixed End moments	-144	144	-28	28	-108	108
Balance				40	20	-54
Carry over moment						10
Balance						-5
Final						59