UNIT- IV

DISPLACEMENT METHOD OF ANALYSIS: SLOPE DEFLECTION ANDMOMENT DISTRIBUTION

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure. In the analysis of indeterminate structure it is necessary to satisfy theequilibrium equations (implying that the structure is in equilibrium) compatibility equations(requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis

2. Displacement method of analysis

In the force method of analysis, primary unknown are forces. In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium. In the displacement method of analysis, the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations.

The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3. Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2. Moment distribution method 3. Kani's method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy- kinematic indeterminacy
Governing equations-compatibility equations	Governing equations-equilibrium equations
Force displacement relations- flexibility matrix	Force displacement relations- stiffness matrix

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements. The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames.

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In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known.

The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

FUNDAMENTAL SLOPE-DEFLECTION EQUATIONS:

The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments in terms of its three degrees of freedom, namely its angular displacements and linear displacement which could be caused by relative settlements between the supports. Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement will be considered positive since this displacement causes the chord of the span and the span's chord angle to rotate clockwise. The slope deflection equations can be obtained by using principle of superposition by considering separately the moments developed at each supports due to each of the displacements & then the loads.

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beamsubjected to given load & end moments so we can work out the reactions & drawthe bending moment & shear force diagram.

Loads and displacements are vector quantities and hence a proper coordinate system is required to specify their correct sense of direction. Consider a planar truss, In this truss each node is identified by a number and each member is identified by a number enclosed in a circle. The displacements and loads acting on the truss are defined with respect to global co-ordinate system *xyz.*

The same co ordinate system is used to define each of the loads and displacements of all loads. In a global co-ordinate system, each node of a planer truss can have only two displacements: one along *x* -axis and another along *y*-axis. The truss shown in figure has eight displacements. Each displacement (degree of freedom) in a truss is shown by a number in the figure at the joint.

The direction of the displacements is shown by an arrow at the node. However out of eight displacements, five are unknown. The displacements indicated by numbers 6, 7 and 8 are zero due to support conditions.

The displacements denoted by numbers 1-5 are known as unconstrained degrees of freedom of the truss and displacements denoted by 6-8 represent constrained degrees of freedom. In this course, unknown displacements are denoted by lower numbers and the known displacements are denoted by higher code numbers.

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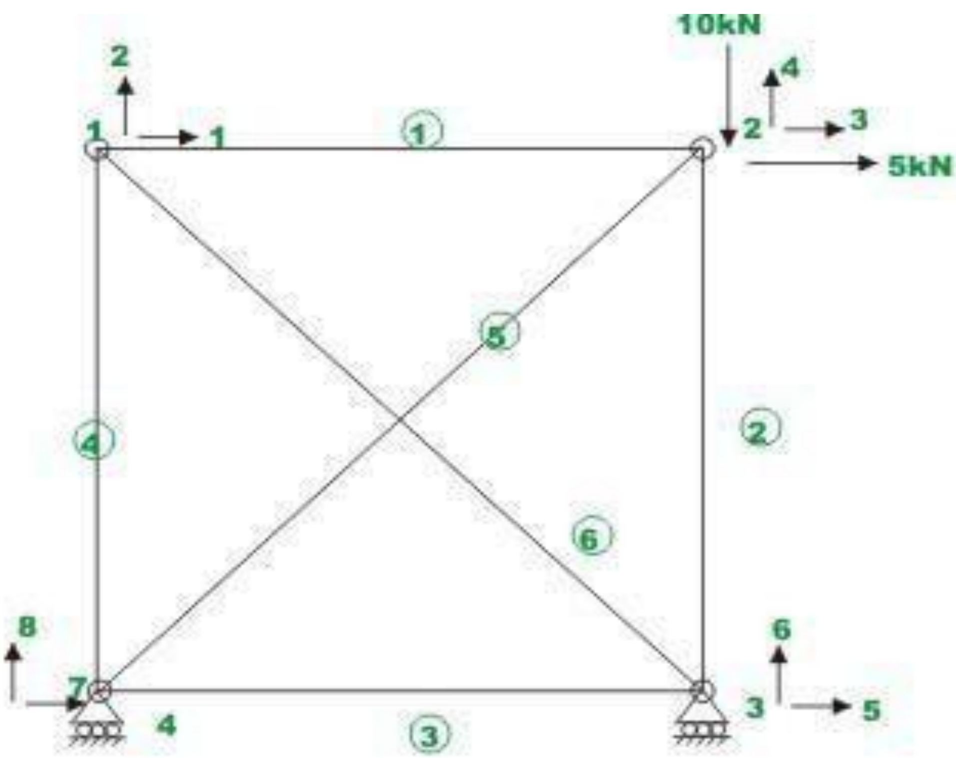

MEMBER STIFFNESS MATRIX INTRODUCTION

To analyse the truss shown in, the structural stiffness matrix *K* need to be evaluated for the given truss. This may be achieved by suitably adding all the member stiffness matrices *k*', which is used to express the force-displacement relation of the member in local co-ordinate system. Since all members are oriented at different directions, it is required to transform member displacements and forces from the local co-ordinate system to global co-ordinate system so that a global load-displacement relation may be written for the complete truss.

MEMBER STIFFNESS MATRIX ANALYSIS

Consider a member of the truss in local co-ordinate system *x*'*y*'. As the loads are applied along the centroidal axis, only possible

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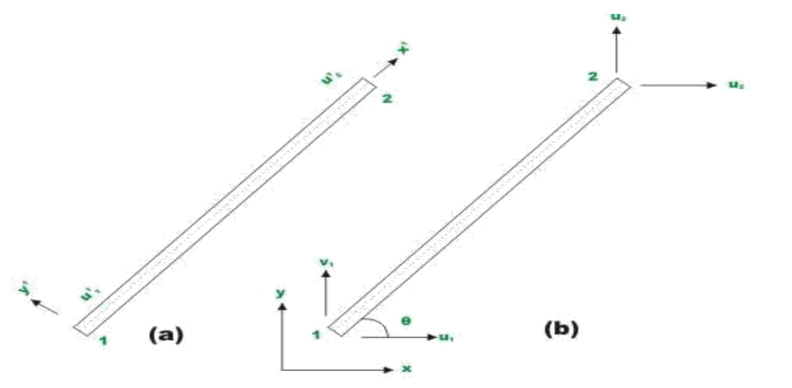
refers to node 1 of the truss member and subscript 2 refers to node 2 of the truss member. Give displacement u'_1 at node 1 of the member in the positive *x*' direction, keeping all other displacements to zero. This displacement in turn **TRANSFORMATION FROM LOCAL TO GLOBAL CO-ORDINATE SYSTEM.**

Displacement Transformation Matrix

A truss member is shown in local and global co ordinate system in Figure. Let *x*' *y*' *z*'be in local co ordinate system and*xyz* be the global co ordinate system.



The nodes of the truss member be identified by 1 and 2. Let u'_1 and u'_2 be the displacement of nodes 1 and 2 in local co ordinate system. In global co ordinate system, each node has two degrees of freedom. Thus, u_1,v_1 and u_2,v_2 are the nodal displacements at nodes 1 and 2 respectively along x - and y - directions.

Let the truss member be inclined to x axis by θ as shown in figure. It is observed from the figure that u'_1 is equal to the projection of u_1 on x' axis plus projection of $v_1$on x'-axis. Thus, (vide Fig. 24.7)

$u'_1 \square\ u_1\cos\theta \square\ v_1\sin\theta$

$u'_2 \square\ u_2\cos\theta \square\ v_2\sin\theta$This may be written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

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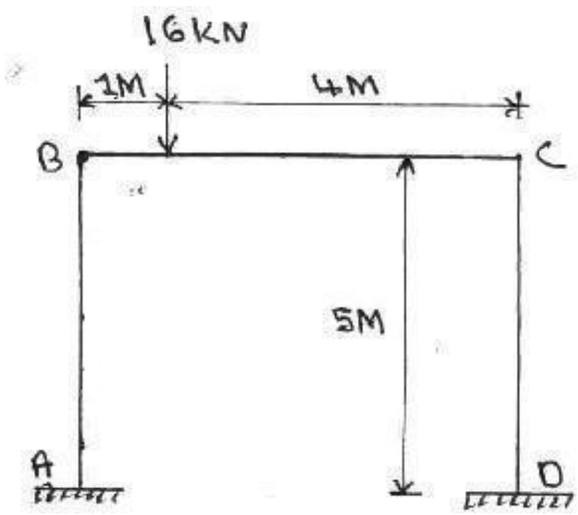
MOMENT DISTRIBUTION METHOD FOR FRAMES WITH SIDE SWAY

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.

Non SwayAnalysis:



First consider the frame without side sway

$$M_{FAB=}\ M_{FBA=}\ M_{FCD=}\ 0$$

$$M_{FBC=} -\frac{16\times1\times4^2}{5^2}=-10.24KNm$$

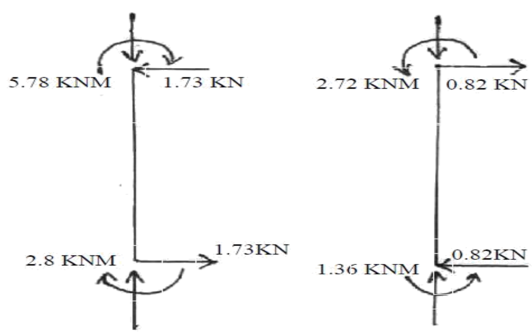
$$M_{FCB=} \frac{16\times4\times1^2}{5^2}=2.56KNm$$

DISTRIBUTION FACTOR

Jt.	Member	Relative stiffness K	ΣK	DF = $\frac{K}{\sum K}$
B	BA BC	I/5 = 0.2 I I/5 = 0.2 I	0.4 I	0.5 0.5
C	CB CD	I/5 = 0.2I I/5 = 0.2I	0.4I	0.5 0.5

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

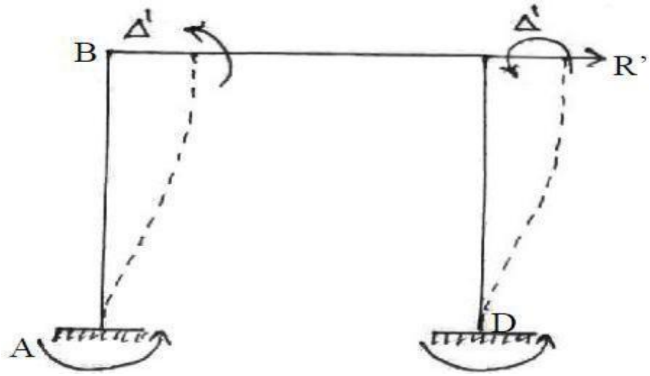
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	0	0.5	0.5	0.5	0.5	0
FEM	0		-10.24	2.56	0	0
Balance		5.12	5.12	-1.28	-1.28	
CO	2.56		-0.64	2.56		-0.64
Balance		0.32	0.32	-0.08	-0.08	
CO	0.16		-0.64	0.16		-0.64
Balance		0.32	0.32	-0.08	-0.08	
C.O	0.16		-0.04	0.16		-0.04
Balance		0.02	0.02	-0.08	-0.08	
C.O	0.01					-0.04
Final moments	**2.89**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.36**



By seeing of the FBD of columns R = 1.73 – 0.82

(Using F_x =0 for entire frame) = 0.91 KN ⟵

Now apply R = 0.91 KN acting opposite as shown in the above figure for the sway analysis. Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect as shown in the following figure.



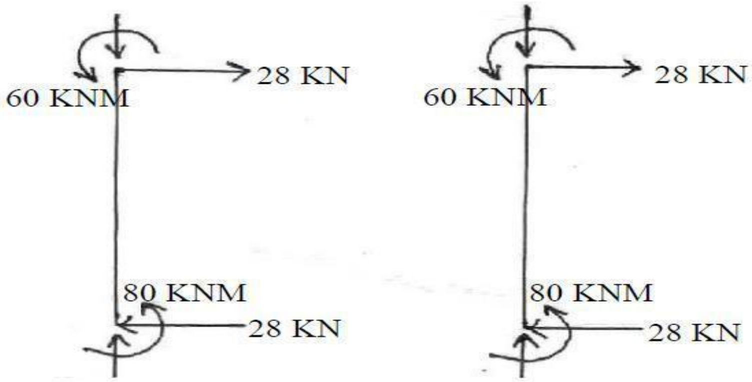
Since both ends are fixed, columns are of same length & I and assuming joints B & C are temporarily restrained from rotating and resulting fixed end moment are assumed.

$$M'_{AB} = M'_{BA} = M'_{CD} = M'_{DC} = \frac{6EI}{L^2}\Delta$$
$$M'_{BA} = -100KNm$$
$$M'_{AB} = M'_{CD} = M'_{DC} = -100KNm$$

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	01	0.5	0.5	0.5	0.5	0
FEM	–100	–100	0	0	–100	–100
Balance		50	50	50	50	
CO	25		25	25		25
Balance		–12.5	–12.5	–12.5	–12.5	
CO	-6.25		–6.25	-6.25		–6.25
Balance		3.125	3.125	3.125	3.125	
C.O	1.56		1.56	1.56		1.56
Balance		–0.78	–0.78	–0.78	–0.78	
C.O	-0.39		–0.39	–0.39		0.39
Balance		0.195	0.195	0.195	0.195	
C.O	0.1					0.1
Final moments	**- 80**	**- 60**	**60**	**60**	**– 60**	**– 80**

Free body diagram of columns



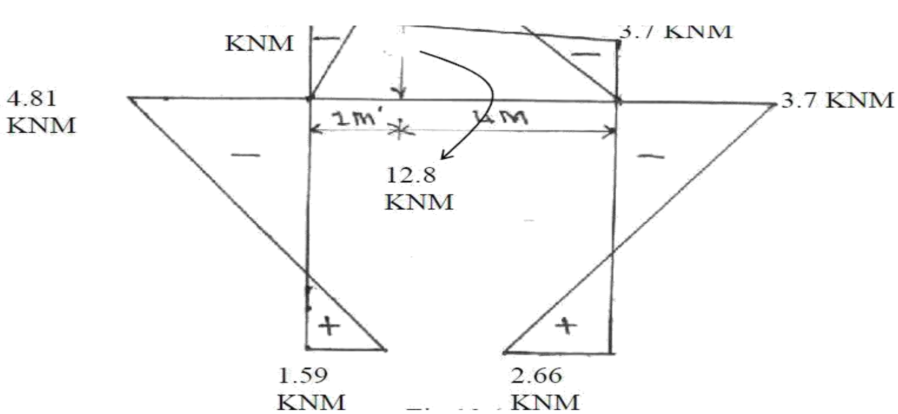
Using F x = 0 for the entire frame R = 28 + 28 = 56 KN

Hence R' = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Moments calculated for R = 0.91KN, as shown below.

$$M_{AB} = 2.89 + \frac{0.91}{56}(-80) = 1.59KNm$$
$$M_{BA} = 5.78 + \frac{0.91}{56}(-60) = 4.81KNm$$

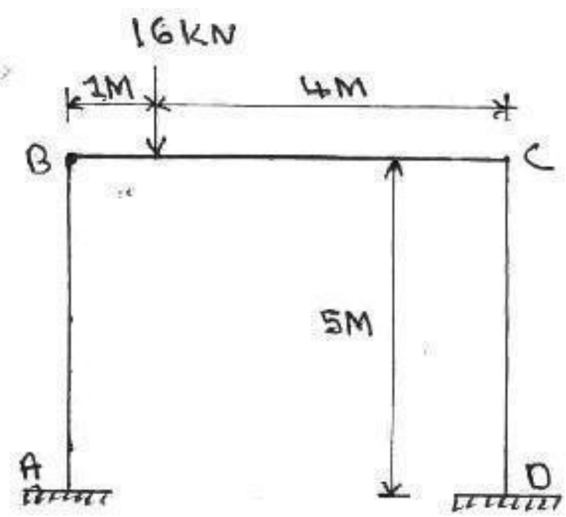
$$M_{BC} = -5.78 + \frac{0.91}{56}(60) = -4.81KNm$$
$$M_{CB} = 2.72 + \frac{0.91}{56}(60) = 3.7KNm$$
$$M_{CD} = -2.72 + \frac{0.91}{56}(-60) = -3.7KNm$$
$$M_{DC} = -1.36 + \frac{0.91}{56}(-80) = -2.66KNm$$



BMD

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.



Non SwayAnalysis:

First consider the frame without side sway

$$M_{FAB} = M_{FBA} = M_{FCD} = 0$$

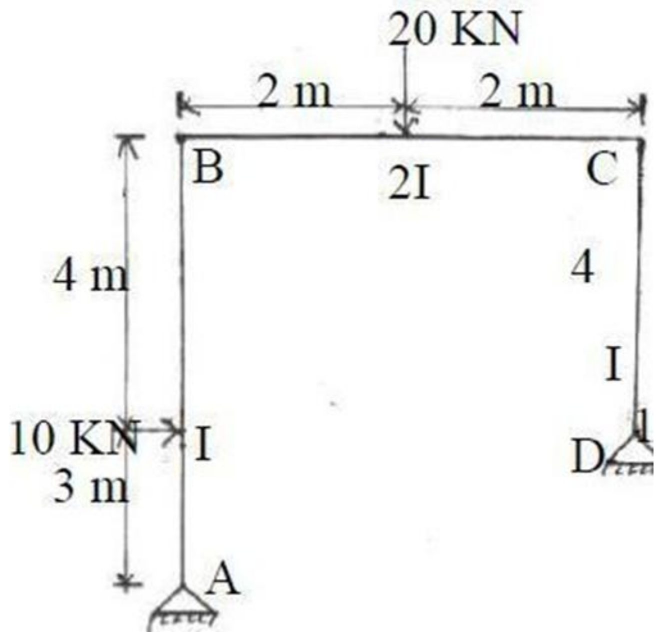
$$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24KNm$$

$$M_{FCB} = \frac{16\times4\times1^2}{5^2} = 2.56KNm$$

Hence R' = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Q) Analysis the rigid frame shown in figure by moment distribution method and draw BMD



A. Non Sway Analysis:

First consider the frame held from side sway

FEMS

$$M_{FAB=} -\frac{10\times3\times4^2}{7^2} = -9.8KNm$$

$$M_{FBA} = \frac{10\times4\times3^2}{7^2} = 7.3KNm$$

$$M_{FBC=} -\frac{20\times4}{8} = -10KNm$$

$$M_{FCB=} \frac{20\times4}{8} = 10KNm$$

$$M_{FCD=}\ M_{FDC=}\ 0$$

Calculation of distribution factors:

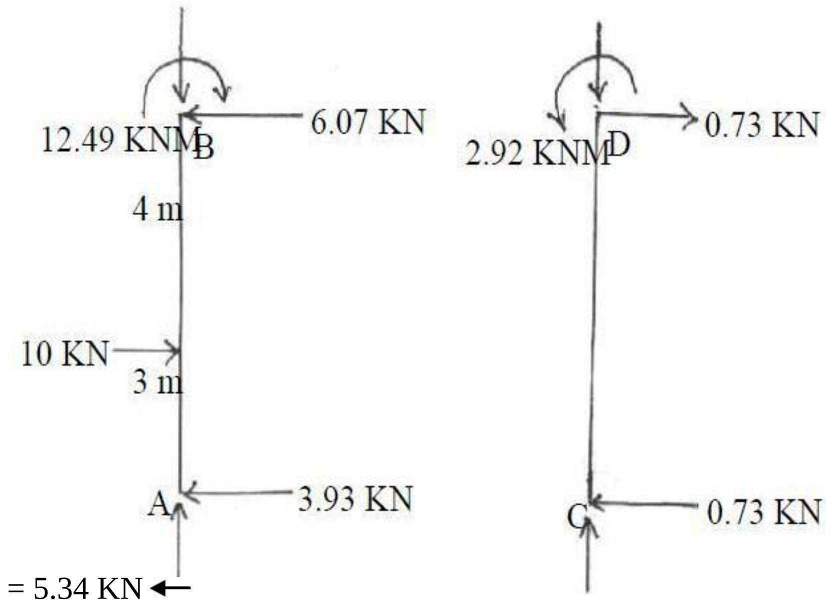
Joint	Member	Relative stiffness k	Σk	$DF = \frac{K}{\sum K}$
B	BA	$\frac{3}{4} \times \frac{I}{7} = 0.11I$	0.61 I	0.18
	BC	2I/4 = 0.5I		0.82
C	CB	2I/4 = 0.5I		0.72
	CD	$\frac{3}{4} \times \frac{I}{4} = 0.19\ I$	0.69 I	0.28

DISTRIBUTION FACTOR

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	-9.8	7.3	-10	10	0	0
Release jt. 'D'	+9.8 →					
CO		4.9				
Initial moments	0	12.2	-10	10	0	0
Balance		-0.4	-1.8	-7.2	-2.8	
CO			-3.6	-0.9		
Balance		0.65	2.95	0.65	0.25	
C.O			0.33	1.48		
Balance		-0.06	-0.27	-1.07	-0.41	
C.O			-0.54	-0.14		
Balance		0.1	0.44	0.1	0.04	
Final moments	**0**	**12.49**	**-12.49**	**2.92**	**-2.92**	**0**

FREE BODY DIAGRAM OF COLUMNS



Applying Fx = 0 for frame as a Whole, R = 10 – 3.93 – 0.73

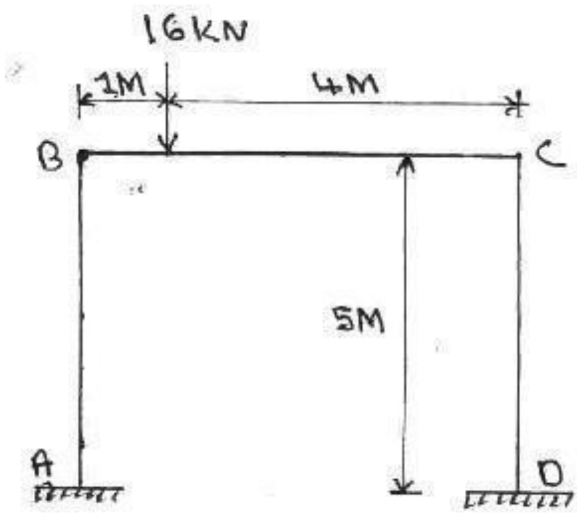
Now apply R = 5.34KN acting opposite

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.



Non SwayAnalysis:

First consider the frame without side sway

$$M_{FAB=}\ M_{FBA=}\ M_{FCD=}\ 0$$

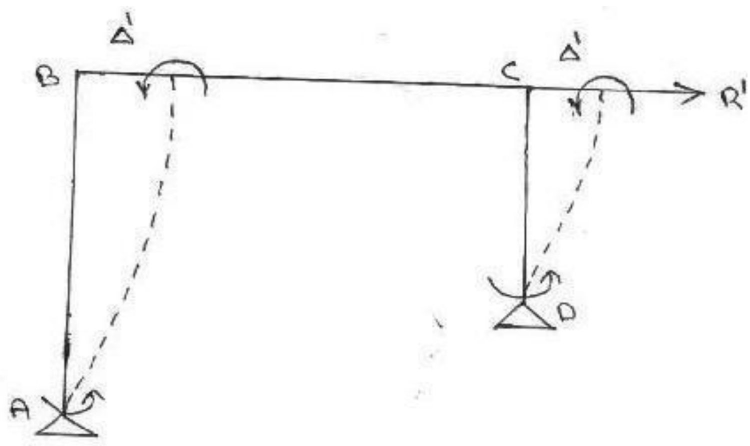
$$M_{FBC=} -\frac{16\times1\times4^2}{5^2} = -10.24KNm$$

$$M_{FCB=} \frac{16\times4\times1^2}{5^2} = 2.56KNm$$

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

Sway analysis: For this we will assume a force R' is applied at C causing the frame todeflect as shown in figure



Since ends A & D are hinged and columns AB & CD are of different lengths

$M'_{BA} = -\frac{3EI}{L_1^2}\Delta'$, $M'_{CD} = -\frac{3EI}{L_2^2}\Delta'$,

$$\frac{M'_{BA}}{M'_{CD}} = \frac{\frac{3EI}{L_1^2}\Delta'}{\frac{3EI}{L_2^2}\Delta'} = \frac{L_2^2}{L_1^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

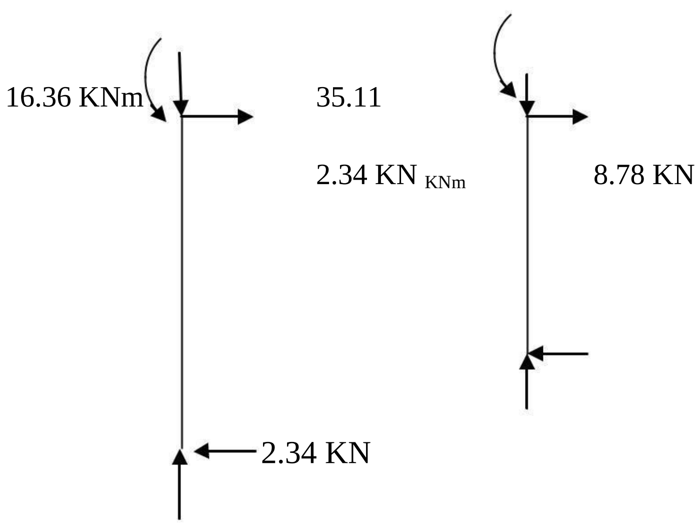
Assume

$$M'_{BA} = -16KNm, M'_{AB} = 0$$
$$M'_{CD} = -49KNm, \; M'_{DC} = 0$$

MOMENT DISTRIBUTION FOR SWAY ANALYSIS

Joint	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	
D.F	1	0.18	0.82	0.72	0.28	1	
FEM	0	-16	0	0	-49	0	
Balance		2.88	13.12	35.28	13.72		
CO			17.64	6.56			
Balance		-3.18	-14.46	-4.72	-1.84		
CO			-2.36	-7.23			
Balance		0.42	1.94	5.21	2.02		
C.O			2.61	0.97			
Balance		-0.47	-2.14	-0.7	-0.27		
C.O			0.35	-1.07			
Balance		0.06	0.29	0.77	0.3		
C.O			0.39	0.15			
Balance		-0.07	-0.32	-0.11	-0.04		
Final moments	**0**	**-16.36**	**16.36**	**35.11**	**-35.11**	**0**	

FREE BODY DIAGRAMS OF COLUMNS AB &CD



Using Fx = 0 for the entire frame

$R' = 11.12$ k →

Hence $R' = 11.12$ KN creates the sway moments shown in the above moment distribution table. Corresponding moments caused by R = 5.34 kN can be determined by proportion. Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.

$$M_{AB} = 0$$

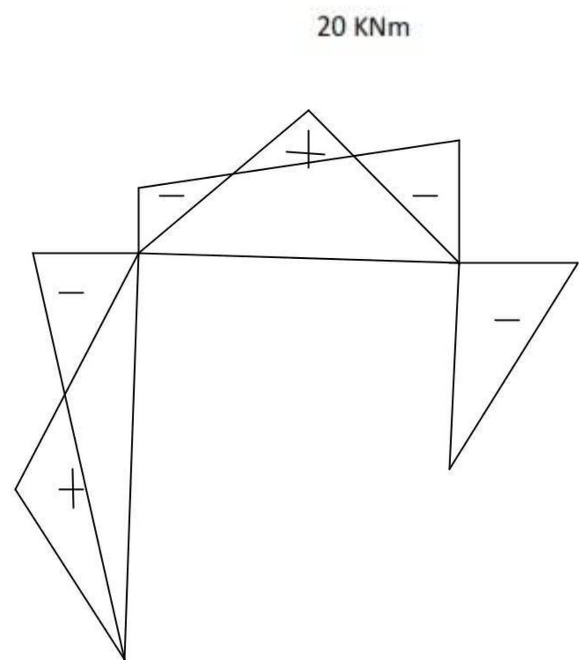
$$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63\text{KNm}$$

$$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63\text{KNm}$$

$$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78\text{KNm}$$

$$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78\text{KNm}$$

$$M_{DC} = 0$$



B.M.D

APPROXIMATE LATERAL LOAD ANALYSIS BY PORTAL METHOD

Portal Frame

Portal frames, used in several Civil Engineering structures like buildings, factories, bridges have the primary purpose of transferring horizontal loads applied at their tops to their foundations. Structural requirements usually necessitate the use of statically indeterminate layout for portal frames, and approximate solutions are often used in their analyses.

Assumptions for the Approximate Solution

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general be consistent.

Thus, the first step in the approximate analysis of structures is to find its degree of statically indeterminacy (dosi) and then to make appropriate number of assumptions.

For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a vertical axis of symmetry that are loaded horizontally at thetop

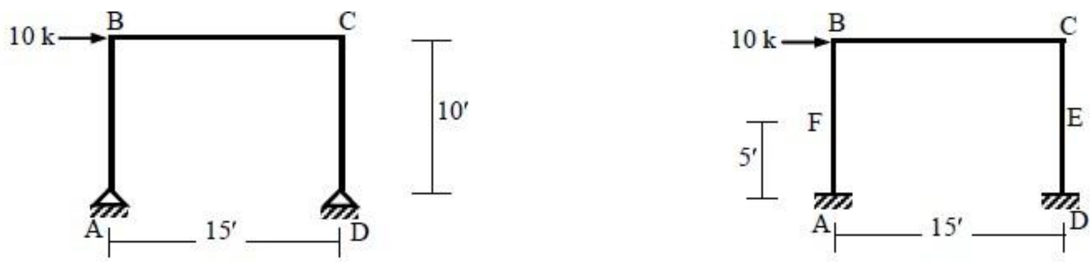
1. The horizontal support reactions areequal
2. There is a point of inflection at the center of the unsupported height of each fixed basedcolumn

Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2 is used if dosi 1.

Some additional assumptions can be made in order to solve the structure approximately for different loading and support conditions.

3. Horizontal body forces not applied at the top of a column can be divided into two forces (i.e., applied at the top and bottom of the column) based on simple supports
4. For hinged and fixed supports, the horizontal reactions for fixed supports can be assumed to be four times the horizontal reactions for hinged supports Example

Draw the axial force, shear force and bending moment diagrams of the frames loaded as shown below.



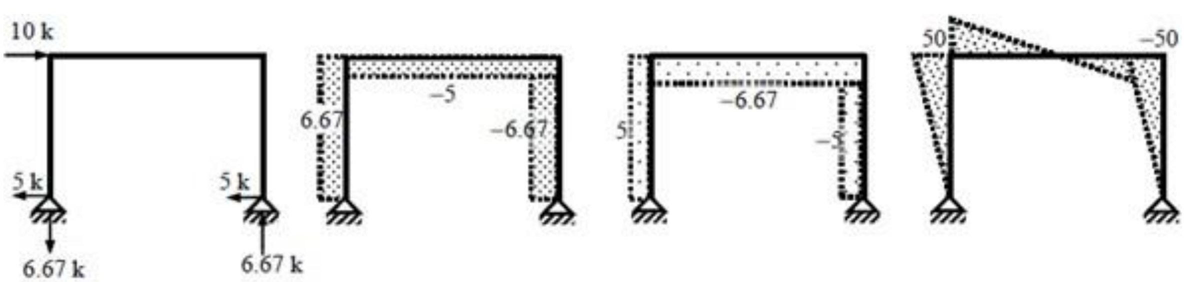
(i) (ii)

Solution

(i) For this frame, dosi = 3 × 3 + 4 – 3 × 4 = 1; i.e., Assumption 1 ⇒ $H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

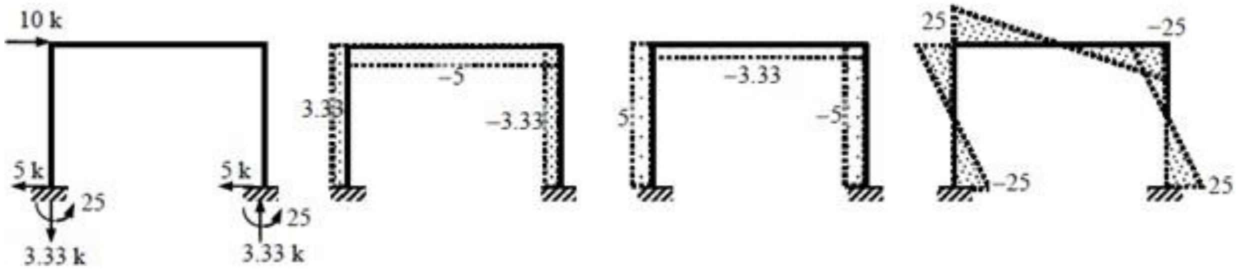
(ii) dosi = 3 × 3 + 6 – 3 × 4 = 3

Assumption 1 ⇒ $H_A = H_D = 10/2 = 5$ k, Assumption 2 ⇒ $BM_E = BM_F = 0$

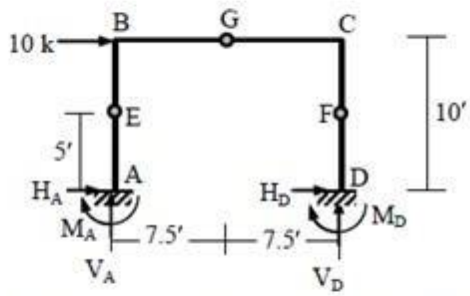
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

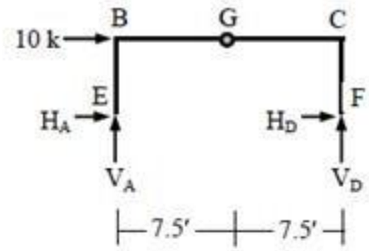
$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)





dosi = 3 × 4 + 6 – 3 × 5 – 1 = 2; ∴ Assumption 1 and 2 ⇒ $BM_E = BM_F = 0$

∴ $BM_E = 0$ (bottom) ⇒ $-H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$

Also $BM_E = 0$ (free body of EBCF) ⇒ $10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

∴ $\Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

$BM_G = 0$ (between E and G) ⇒ $V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k ⇒ $M_A = 5H_A = -25$

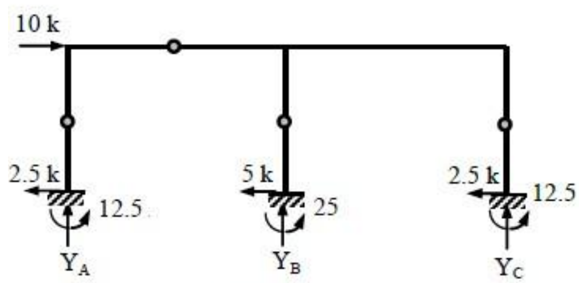
$\Sigma F_x = 0$ (entire structure) ⇒ $H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k ⇒ $M_D = 5H_D = -25$

(iv) dosi = 3 × 5 + 9 – 3 × 6 = 6 ⇒ 6 Assumptions needed to solve the structure

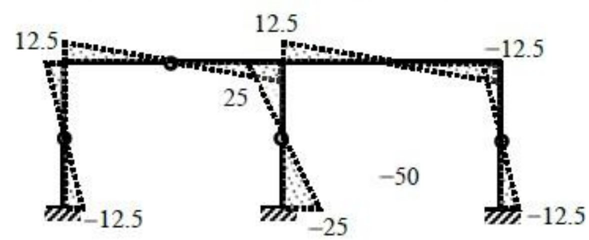
Assumption 1 and 2 ⇒ $H_A: H_B: H_C = 1: 2: 1 \Rightarrow H_A = 10/4 = 2.5$ k, $H_B = 5$ k, $H_C = 2.5$ k

∴ $M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

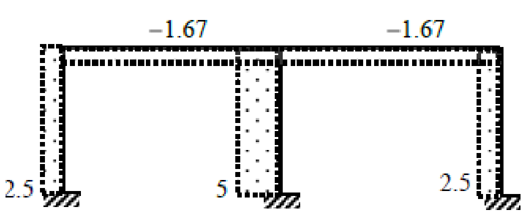
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



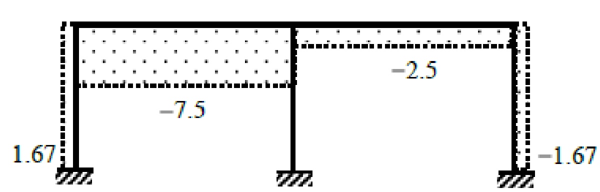
Reactions



BMD



SFD (k)



AFD (k)

Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statically indeterminacy of the structure.

Assumptions

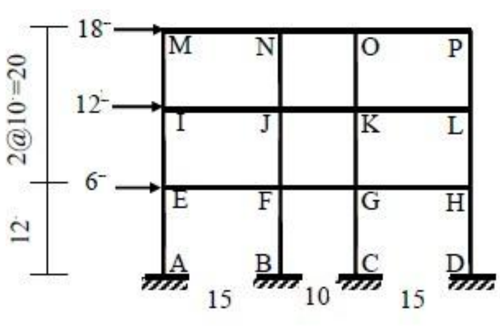
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

1. The shear force in an interior column is twice the shear force in an exterior column.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

$\therefore$ Shear force in (V) columns IM, JN, KO, LP are

[18 × 1/(1 + 2 + 2 + 1) =] 3, [18 × 2/(1 + 2 + 2 + 1) =] 6, 6, 3 respectively. Similarly,

$V_{EI} = 30\times1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36\times1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

Bending moments are

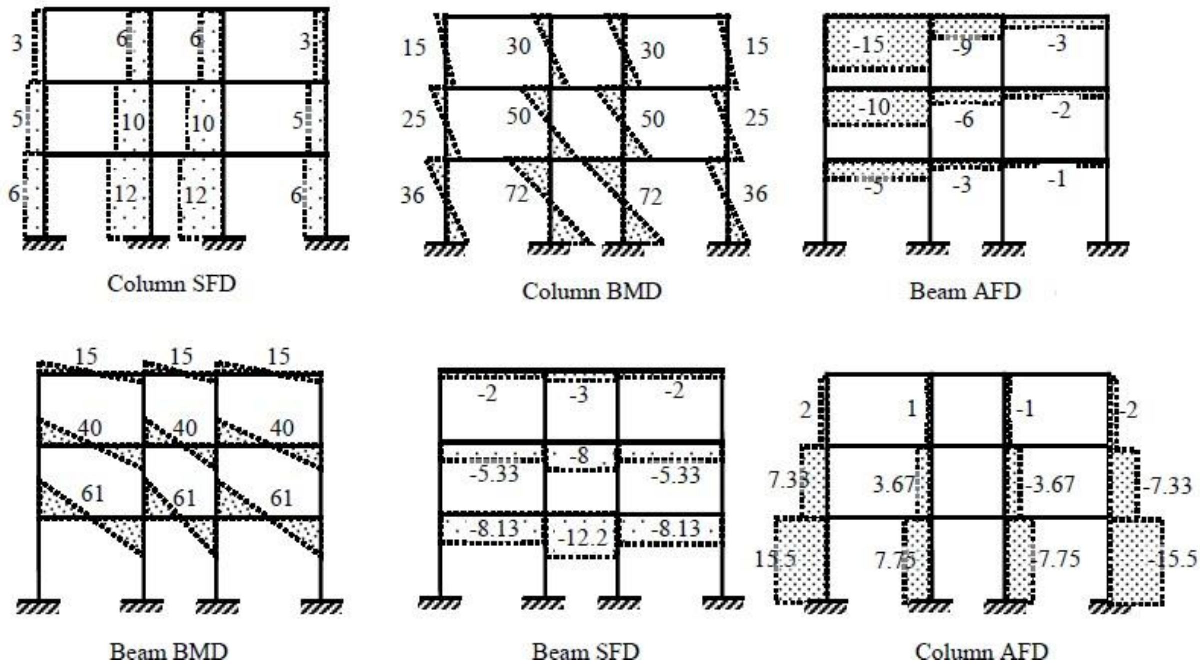
$M_{IM} = 3\times10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

$M_{EI} = 5\times10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6\times10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures.

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members.

The *Cantilever Method* attempts to rectify this limitation by considering the cross-sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

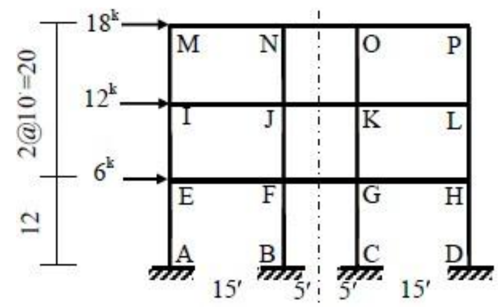
The Cantilever Method is based on three assumptions

1. The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of thestorey.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming that the axial stresses can be obtained by a method analogous to that used for determining the distribution of normal stresses on a transverse section of a cantilever beam. Assumption 2 and 3 are based on observing the deflected shape of the structure.

Example

Use the Cantilever Method to draw the axial force, shear force and bending moment diagrams of the three -storied frame structure loaded as shown below.



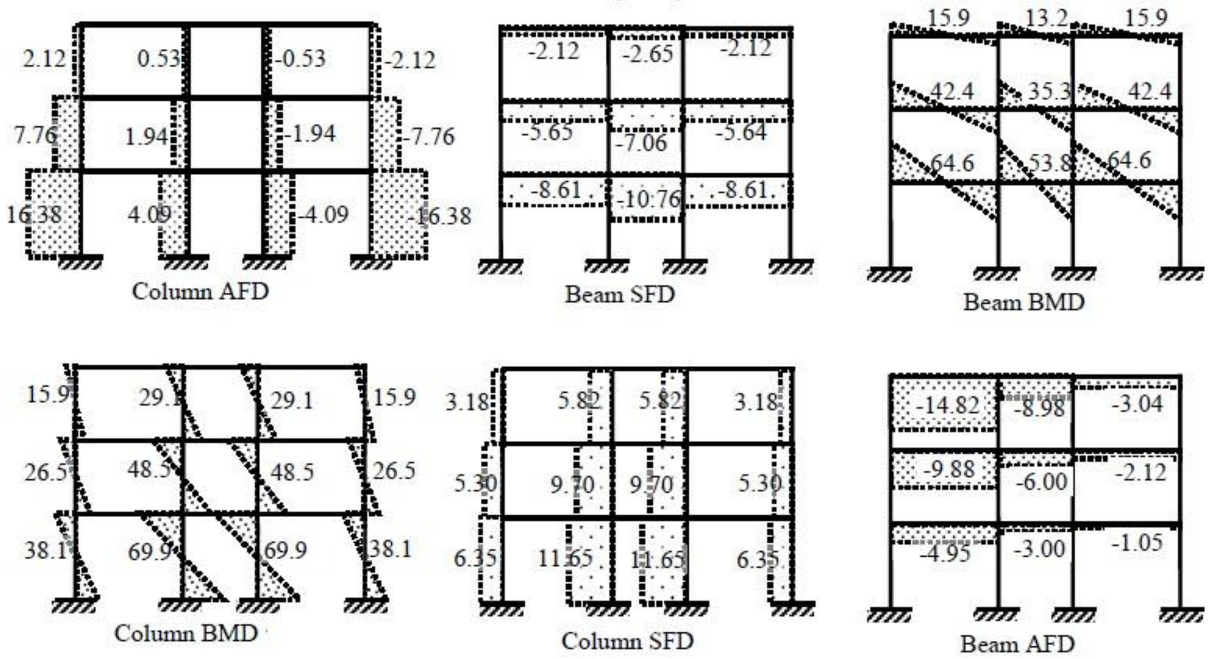
The dotted line is the column centerline (at all floors)

∴ Column axial forces are at the ratio of 20: 5: −5: −20.

∴ Axial force in (P) columns IM, JN, KO, LP are $[18 \times 5 \times 20/\{20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 2.12, $[18 \times 5 \times 5/(20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 0.53, −0.53, −2.12 respectively.

Similarly, $P_{EI} = 330\times20/(850) = 7.76$, $P_{FJ} = 1.94$, $P_{GK} = -1.94$, $P_{HL} = -7.76$; and $P_{AE} = 696\times20/(850) = 16.38$, $P_{BF} = 4.09$, $P_{CG} = -4.09$, $P_{DH} = 16.38$

The rest of the calculations follow from the free-body diagrams



Introducing direction cosines $l \cos\theta; m \sin\theta$; the above equation is written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \qquad (24.10a)$$

Or,

$$\{u'\} = [T]\{u\} \qquad (24.10b)$$

In the above equation T is the displacement transformation matrix whichtransforms the four global displacement components to two displacement component in local coordinate system.