

UNIT -V

INFLUENCE LINES AND MOVING LOADS

Influence Lines

An influence line represents the variation of either the reaction, shear, moment or deflection at a specific point in a member as a concentrated force moves over the member. Example bridges, industrial crane rails, conveyors, etc

Influence lines are important in the design of structures that resist large live loads. → In our work up to this point, we have discussed analysis techniques for structures subjected to dead or fixed loads.

We learned that shear and moment diagrams are important in determining the maximum internal force in a structure. → If a structure is subjected to a live or moving load, the variation in shear and moment is best described using influence lines.

Since beams or girders are usually major load-carrying members in large structures, it is important to draw influence lines for reaction, shear, and moment at specified points. → Once an influence line has been drawn, it is possible to locate the live loads on the beam so that the maximum value of the reaction, shear, or moment is produced. → This is very important in the design procedure.

Concentrated Force - Since we use a unit force (a dimensionless load), the value of the function (reaction, shear, or moment) can be found by multiplying the ordinate of the influence line at the position x by the magnitude of the actual force P .

One can tell at a glance where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.

Influence lines for statically determinate structures are piecewise linear.

statically indeterminate example

shear & moment diagrams:

effect of fixed loads at all points along the axis of the member, influence lines:

effect of a moving load only at a specified point on the member

Rounded Aggregate

The rounded aggregates are completely shaped by attrition and available in the form of seashore gravel. Rounded aggregates result the minimum percentage of voids (32 – 33%) hence gives more workability. They require lesser amount of water-cement ratio. They are not considered for high strength concrete because of poor interlocking behavior and weak bond strength.

Irregular Aggregates

The irregular or partly rounded aggregates are partly shaped by attrition and these are available in the form of pit sands and gravel. Irregular aggregates may result 35- 37% of voids. These will give lesser workability when compared to rounded aggregates. The bond strength is slightly higher than rounded aggregates but not as required for high strength concrete.

Angular Aggregates

The angular aggregates consist well defined edges formed at the intersection of roughly planar surfaces and these are obtained by crushing the rocks. Angular aggregates result maximum percentage of voids (38-45%) hence gives less workability. They give 10-20% more compressive strength due to development of stronger aggregate-mortar bond. So, these are useful in high strength concrete manufacturing.

Flaky Aggregates

When the aggregate thickness is small when compared with width and length of that aggregate it is said to be flaky aggregate. Or in the other, when the least dimension of aggregate is less than the 60% of its mean dimension then it is said to be flaky aggregate.

Elongated Aggregates

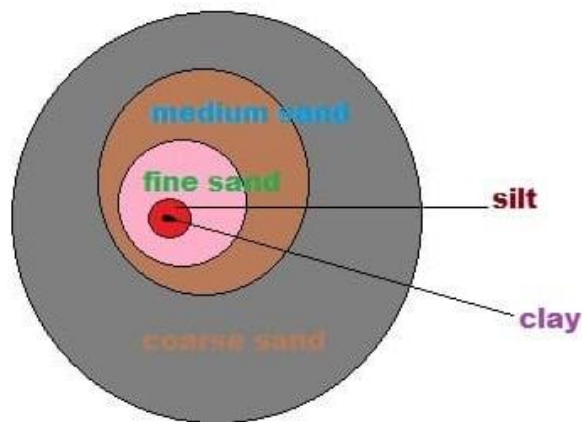
When the length of aggregate is larger than the other two dimensions then it is called elongated aggregate or the length of aggregate is greater than 180% of its mean dimension.

Flaky and Elongated Aggregates

When the aggregate length is larger than its width and width is larger than its thickness then it is said to be flaky and elongated aggregates. The above 3 types of aggregates are not suitable for concrete mixing. These are generally obtained from the poorly crushed rocks.

Classification of Aggregates Based on Size

Aggregates are available in nature in different sizes. The size of aggregate used may be related to the mix proportions, type of work etc. the size distribution of aggregates is called grading of aggregates.



Following are the classification of aggregates based on size:

Aggregates are classified into 2 types according to size

1. Fine aggregate
2. Coarse aggregate

Fine Aggregate

When the aggregate is sieved through 4.75mm sieve, the aggregate passed through it called as fine aggregate. Natural sand is generally used as fine aggregate, silt and clay are also come under this category. The soft deposit consisting of sand, silt and clay is termed as loam. The purpose of the fine aggregate is to fill the voids in the coarse aggregate and to act as a workability agent.

Coarse Aggregate

When the aggregate is sieved through 4.75mm sieve, the aggregate retained is called coarse aggregate. Gravel, cobble and boulders come under this category. The maximum size aggregate used may be dependent upon some conditions. In general, 40mm size aggregate used for normal strengths and 20mm size is used for high strength concrete. the size range of various coarse aggregates given below.

Grading of Aggregates

Grading is the particle-size distribution of an aggregate as determined by a sieve analysis using wire mesh sieves with square openings. As per IS:2386(Part-1)

Fine aggregate—6 standard sieves with openings from 150 μm to 4.75 mm.

Coarse aggregate—5 sieves with openings from 4.75mm to 80 mm.

Gradation (grain size analysis)

Grain size distribution for concrete mixes that will provide a dense strong mixture. Ensure that the voids between the larger particles are filled with medium particles. The remaining voids are filled with still smaller particles until the smallest voids are filled with a small amount of fines. Ensure maximum density and strength using a maximum density curve

Good Gradation:

Concrete with good gradation will have fewer voids to be filled with cement paste (economical mix) Concrete with good gradation will have fewer voids for water to permeate (durability)

Particle size distribution affects:

1. Workability
2. Mixproportioning

Fine Aggregate effect on concrete:

2. Over sanded (More than required sand)
 - Over cohesive mix.
 - Water reducers may be less effective.
 - Air entrainment may be more effective.

3. Under sanded (deficit of sand)

- Prone to bleed and segregation.
- May get high levels of water reduction.
- Air entrainers may be less effective.

Shape and surface texture of aggregates:

The shape of aggregate is an important characteristic since it affects the workability of concrete.

It is difficult to measure the shape of irregular shaped aggregates. Not only the type of parent rock but also the type of crusher used also affects the shape of the aggregate produced.

Good Granite rocks found near Bangalore will yield cuboidal aggregates. Many rocks contain planes of jointing which is characteristic of its formation and hence tend to yield more flaky aggregates.

The shape of the aggregates produced is also dependent on type of crusher and the reduction ratio of the crusher.

Quartzite which does not possess cleavage planes tend to produce cubical shape aggregates.

From the standpoint of economy in cement requirement for a given water cement ratio rounded aggregates are preferable to angular aggregates.

On the other hand, the additional cement required for angular aggregates is offset to some extent by the higher strengths and some times greater durability as a result of greater Interlocking texture of the hardened concrete.

Flat particles in concrete will have objectionable influence on the workability of concrete, cement requirement, strength and durability.

In general excessively flaky aggregates make poor concrete.

While discussing the shape of the aggregates, the texture of the aggregate also enters the discussion because of its close association with the shape.

Generally round aggregates are smooth textured and angular aggregates are rough textured. Therefore some engineers argue against round aggregates from the point of bond strength between aggregates and cement.

But the angular aggregates are superior to rounded aggregates from the following two points:

Angular aggregates exhibit a better interlocking effect in concrete, which property makes it superior in concrete used for road and pavements.

The total surface area of rough textured angular aggregate is more than smooth rounded aggregates for the given volume.

By having greater surface area, the angular aggregates may show higher bond strength than rounded aggregates.

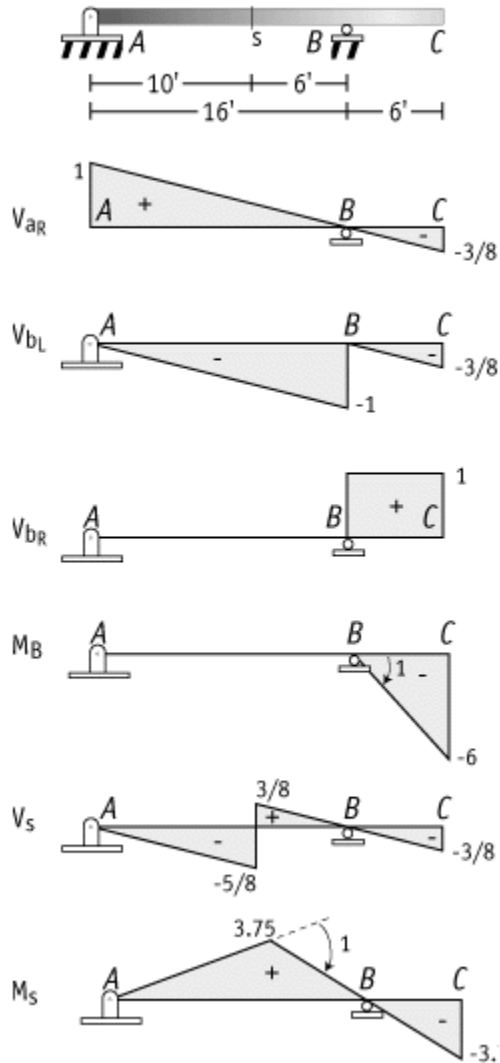
The shape of the aggregates becomes all the more important in case of high strength and high performance concrete where very low water/cement ratio is required to be used . In such cases cubical aggregates are required for better workability.

Surface texture is the property, the measure of which depends upon the relative degree to which particle surface are polished or dull, smooth or rough.

Surface texture depends upon hardness, grain size, pore structure, structure of the rock and the degree to which the forces acting on it have smoothed the surface or roughened. Experience and laboratory experiments have shown that the adhesion between cement paste and the aggregate is influenced by several complex factors in

Procedure of Analysis

1. tabulate values
2. influence-line equations



Assumptions for the Approximate Solution

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are n more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the n th degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make n additional independent assumptions.

A solution based on statics will not be possible by making fewer than n

assumptions, while more than n assumptions will not in general be consistent.

Thus, the first step in the approximate analysis of structures is to find its degree of statical indeterminacy (dosi) and then to make appropriate number of assumptions.

For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a vertical axis of symmetry that are loaded horizontally at the top

The horizontal support reactions are equal

There is a point of inflection at the center of the unsupported height of each fixed based column

Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2 is used if dosi is even.

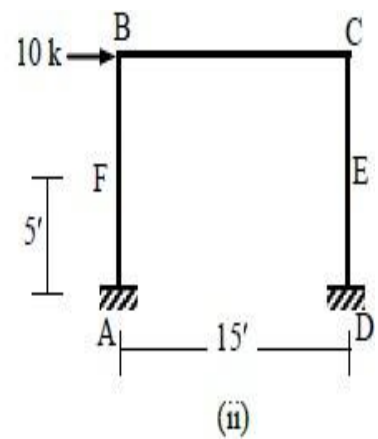
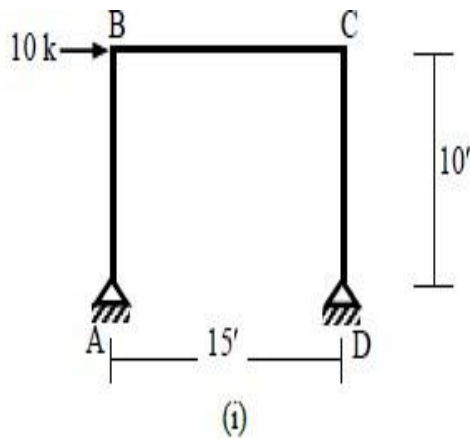
Some additional assumptions can be made in order to solve the structure approximately for different loading and support conditions.

Horizontal body forces not applied at the top of a column can be divided into two forces (i.e., applied at the top and bottom of the column) based on simple supports

For hinged and fixed supports, the horizontal reactions for fixed supports can be assumed to be four times the horizontal reactions for hinged supports Example

Draw the axial force, shear force and bending moment diagrams of the frames loaded as shown below.

Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make n additional independent assumptions. A solution based on statics will not be possible by making fewer than n assumptions, while more than n assumptions will not in general be consistent.

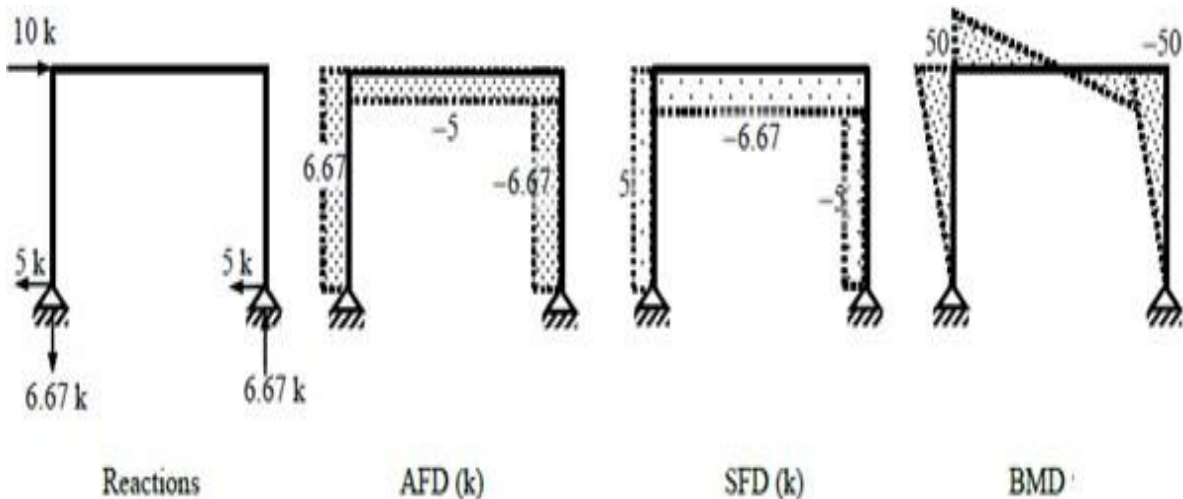


Solution

(i) For this frame, $dosi = 3 \times 3 + 4 - 3 \times 4 = 1$; i.e., Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5 \text{ k}$

$$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67 \text{ k}$$

$$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67 \text{ k}$$



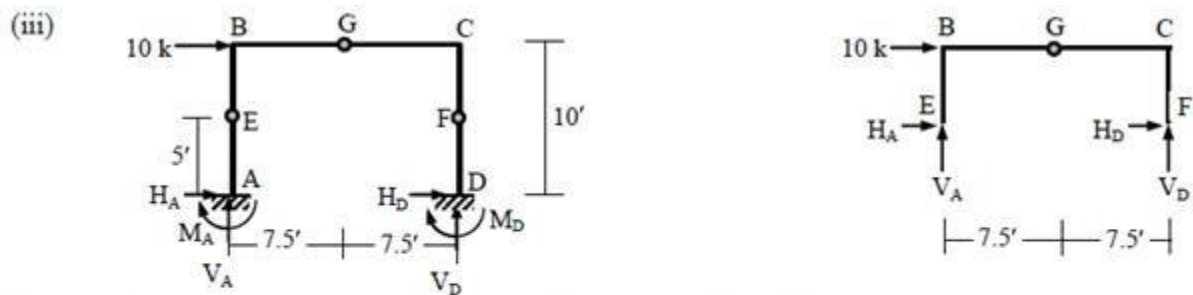
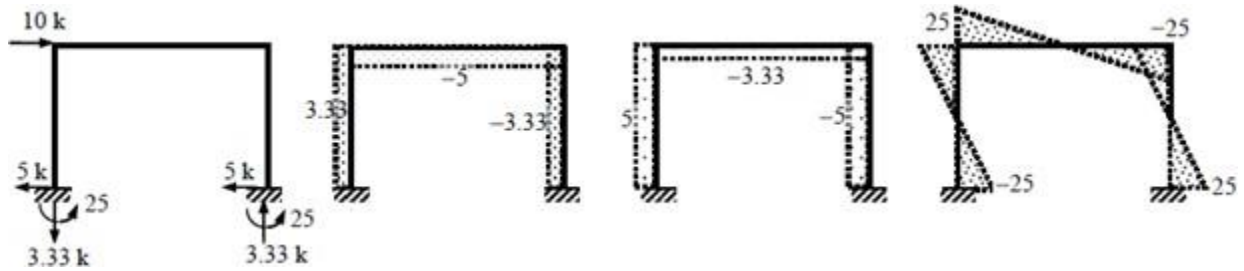
(ii) $dosi = 3 \times 3 + 6 - 3 \times 4 = 3$

Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5 \text{ k}$, Assumption 2 $\Rightarrow BM_E = BM_F = 0$

$$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25 \text{ k-ft; Similarly } BM_E = 0 \Rightarrow M_D = -25$$

$$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33 \text{ k}$$

$$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33 \text{ k}$$

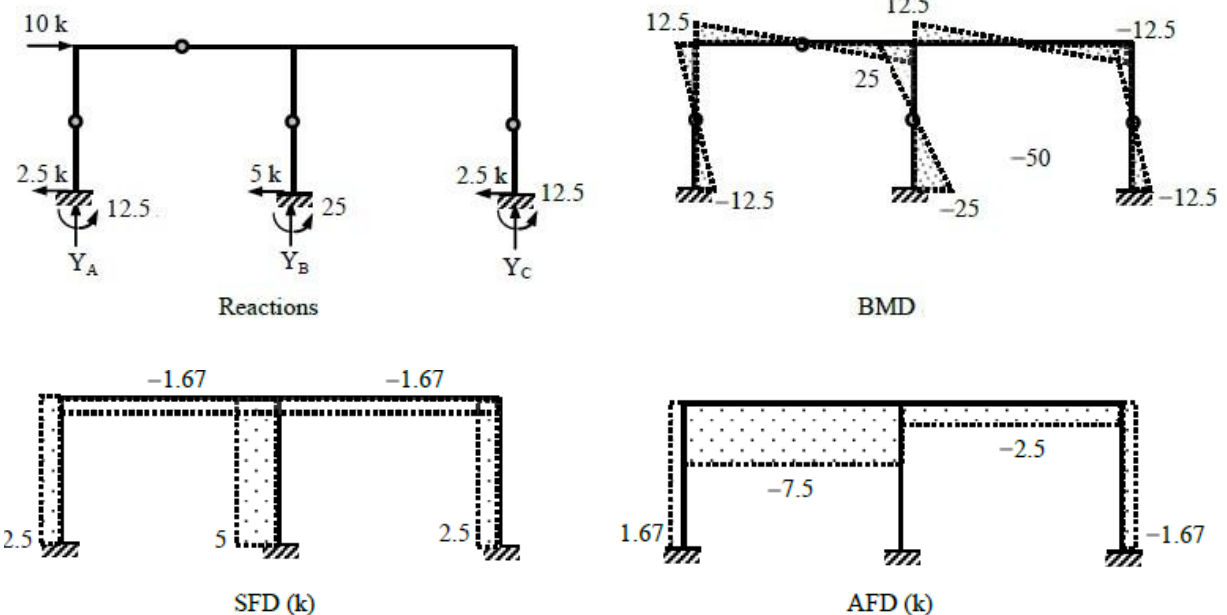


$dosi = 3 \times 4 + 6 - 3 \times 5 - 1 = 2$; \therefore Assumption 1 and 2 $\Rightarrow BM_E = BM_F = 0$
 $\therefore BM_E = 0$ (bottom) $\Rightarrow -H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$
 Also $BM_E = 0$ (free body of EBCF) $\Rightarrow 10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33 \text{ k}$
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$BM_G = 0$ (between E and G) $\Rightarrow V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5 \text{ k} \Rightarrow M_A = 5H_A = -25$
 $\sum F_x = 0$ (entire structure) $\Rightarrow H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5 \text{ k} \Rightarrow M_D = 5H_D = -25$

(iv) $dosi = 3 \times 5 + 9 - 3 \times 6 = 6 \Rightarrow 6$ Assumptions needed to solve the structure
 Assumption 1 and 2 $\Rightarrow H_A : H_B : H_C = 1 : 2 : 1 \Rightarrow H_A = 10/4 = 2.5 \text{ k}, H_B = 5 \text{ k}, H_C = 2.5 \text{ k}$
 $\therefore M_A = M_C = 2.5 \times 5 = 12.5 \text{ k-ft}, M_B = 5 \times 5 = 25$

The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statical indeterminacy of the structure.

Assumptions

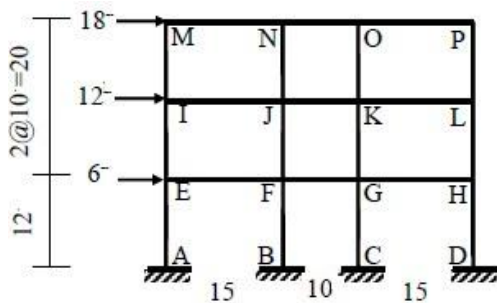
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

4. The shear force in an interior column is twice the shear force in an exterior column.
5. There is a point of inflection at the center of each column.
6. There is a point of inflection at the center

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of the structure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are $[18 \times 1/(1 + 2 + 2 + 1) =] 3^l$, $[18 \times 2/(1 + 2 + 2 + 1) =] 6^l$, 6^l , 3^l respectively. Similarly,

$V_{EI} = 30 \times 1/(6) = 5^l$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5^l$; and $V_{AE} = 36 \times 1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

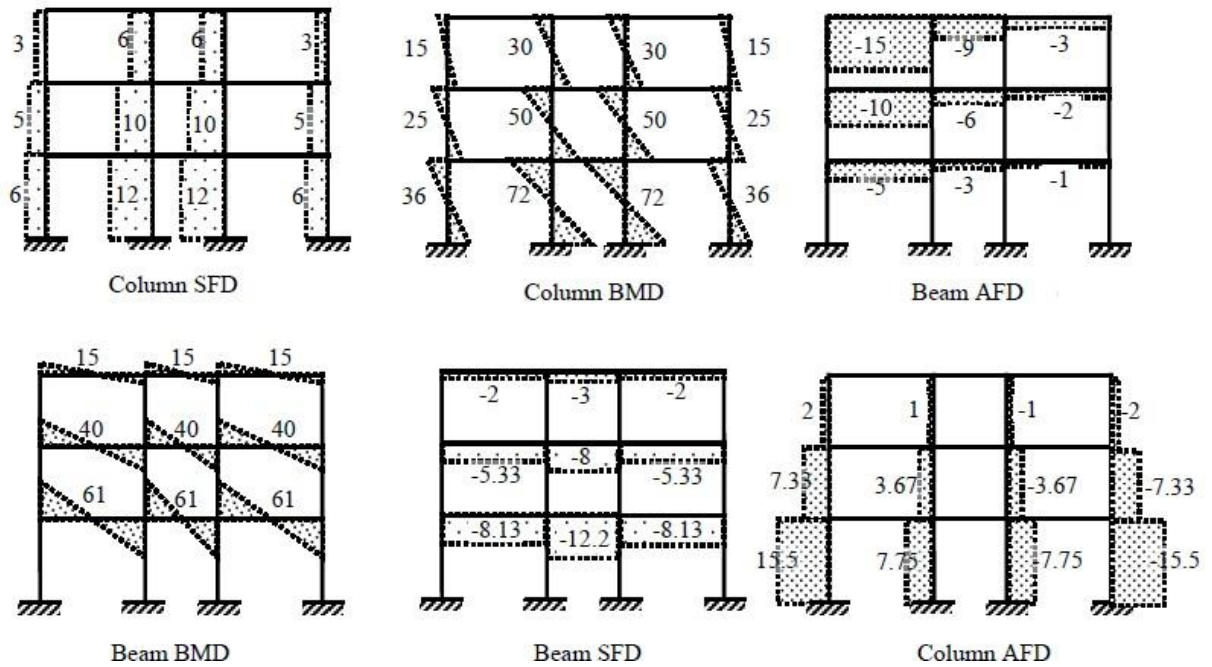
Bending moments are

$M_{IM} = 3 \times 10/2 = 15^l$, $M_{JN} = 30^l$, $M_{KO} = 30^l$, $M_{LP} = 15^l$

$M_{EI} = 5 \times 10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6 \times 10/2 = 30^l$, $M_{BF} = 60$, $M_{CG} = 60$, $M_{DH} = 30^l$

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross-sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

The Cantilever Method is based on three assumptions

The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of the storey.

There is a point of inflection at the center of each column.

There is a point of inflection at the center of each beam. Assumption 1 is based on assuming that the axial stresses can be obtained by a method analogous to that used for determining the distribution of normal stresses on a transverse section of a cantilever beam. Assumption

2 and 3 are based on observing the deflected shape of the structure.

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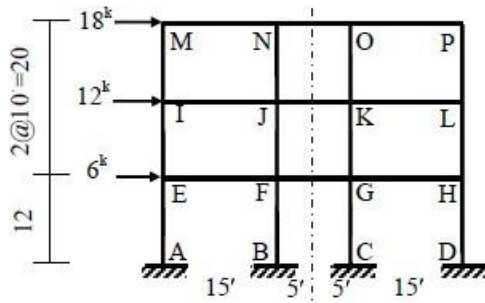
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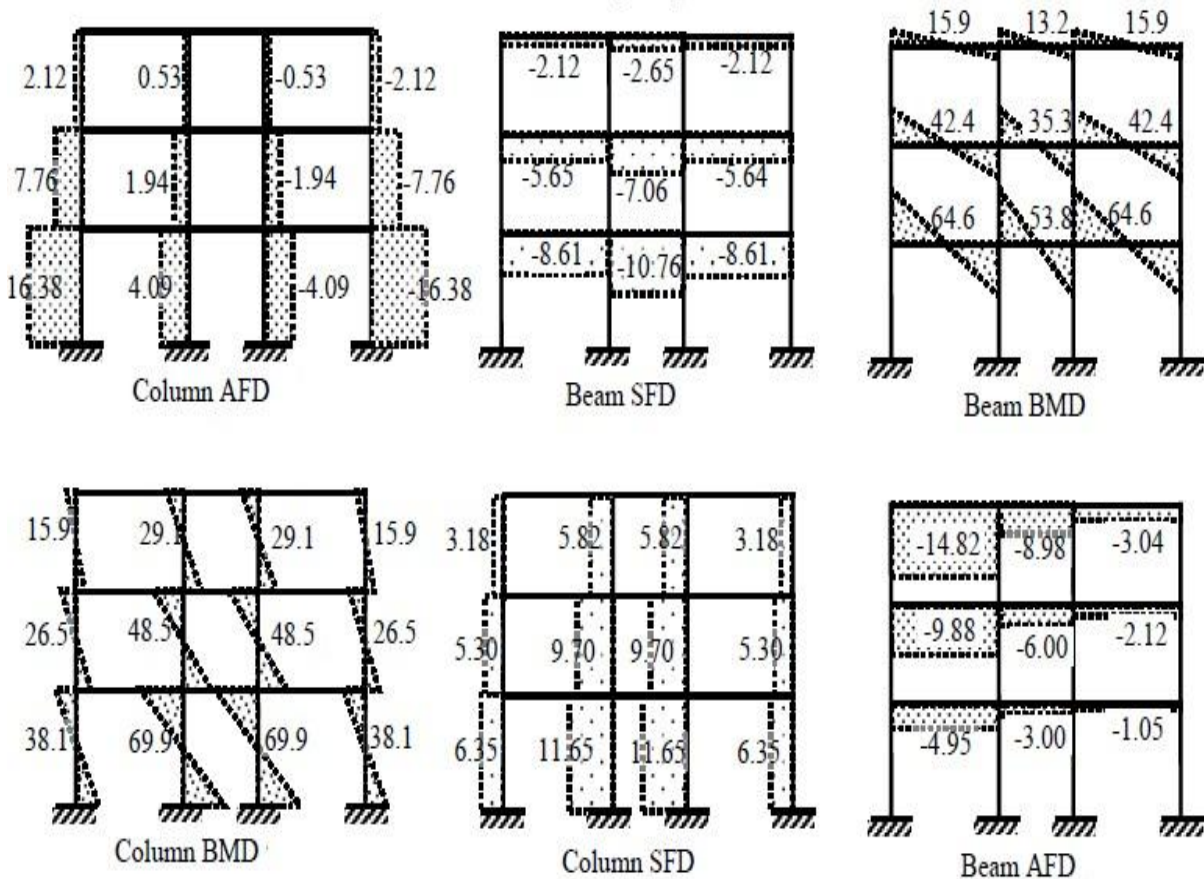
Example

Use the Cantilever Method to draw the axial force, shear force and bending moment diagrams of the three -storied frame structure loaded as shown below.



The dotted line is the column centerline (at all floors)
 \therefore Column axial forces are at the ratio of 20: 5: -5: -20.
 \therefore Axial force in (P) columns IM, JN, KO, LP are
 $[18 \times 5 \times 20 / \{20^2 + 5^2 + (-5)^2 + (-20)^2\}] = 2.12$, $[18 \times 5 \times 5 / \{20^2 + 5^2 + (-5)^2 + (-20)^2\}] = 0.53$, -0.53 , -2.12 respectively.
 Similarly, $P_{EI} = 330 \times 20 / (850) = 7.76$, $P_{FJ} = 1.94$, $P_{GK} = -1.94$, $P_{HL} = -7.76$; and
 $P_{AE} = 696 \times 20 / (850) = 16.38$, $P_{BF} = 4.09$, $P_{CG} = -4.09$, $P_{DH} = 16.38$

The rest of the calculations follow from the free-body diagrams



Introducing direction cosines $l = \cos\theta; m = \sin\theta$; the above equation is written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

Or, $\{u'\} = [T] \{u\}$

In the above equation T is the displacement transformation matrix which transforms the four global displacement components to two displacement component in local coordinate system

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beams subjected to given load & end moments so we can work out the reactions & draw the bending moment & shear force diagram.

Loads and displacements are vector quantities and hence a proper coordinate system is required to specify their correct sense of direction. Consider a planar truss, In this truss each node is identified by a number and each member is identified by a number enclosed in a circle. The displacements and loads acting on the truss are defined with respect to global coordinate system xyz .

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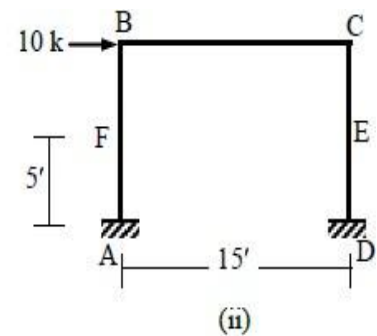
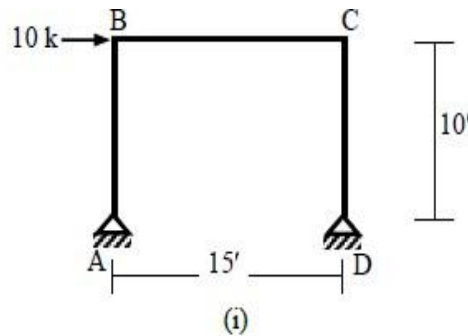
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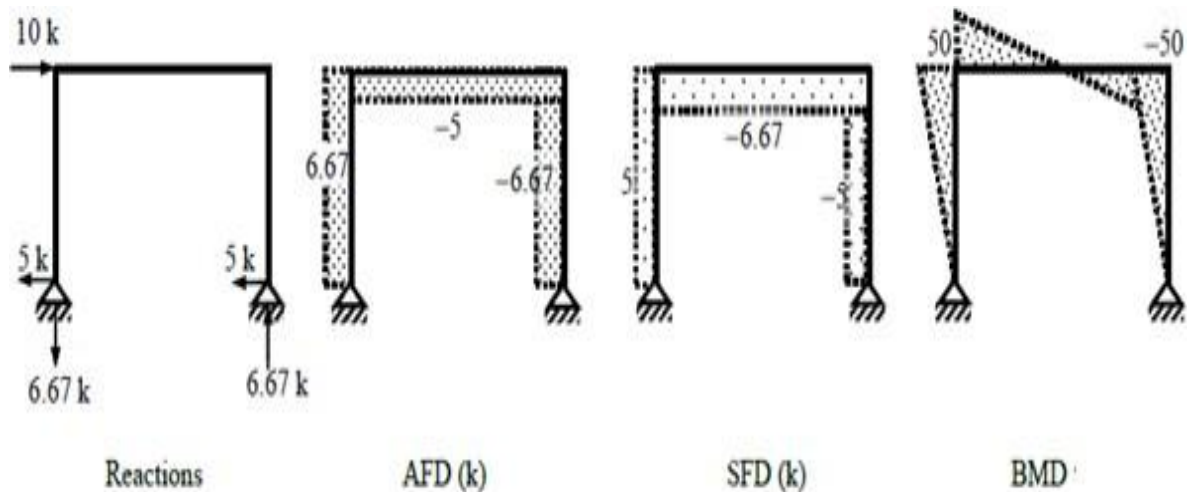


Solution

(i) For this frame, $dosi = 3 \times 3 + 4 - 3 \times 4 = 1$; i.e., Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5 \text{ k}$

$$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67 \text{ k}$$

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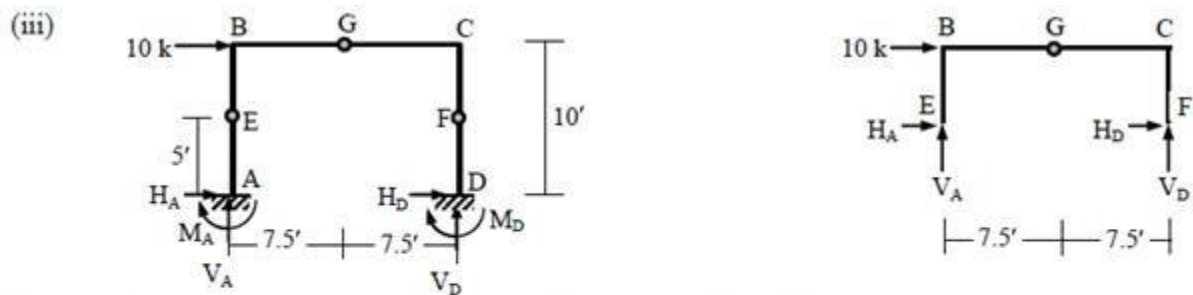
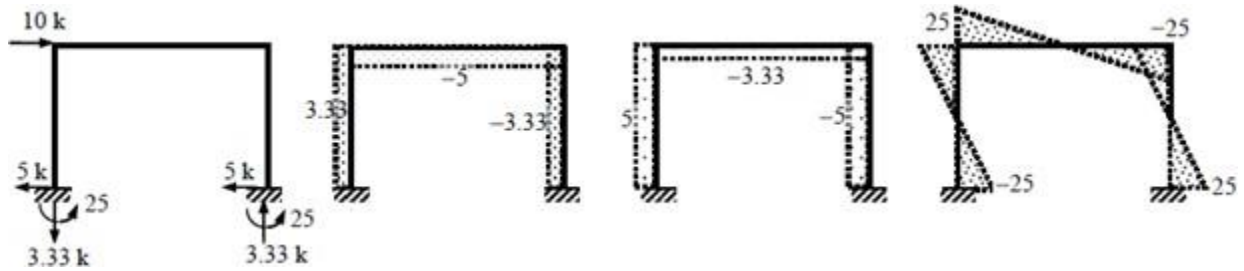
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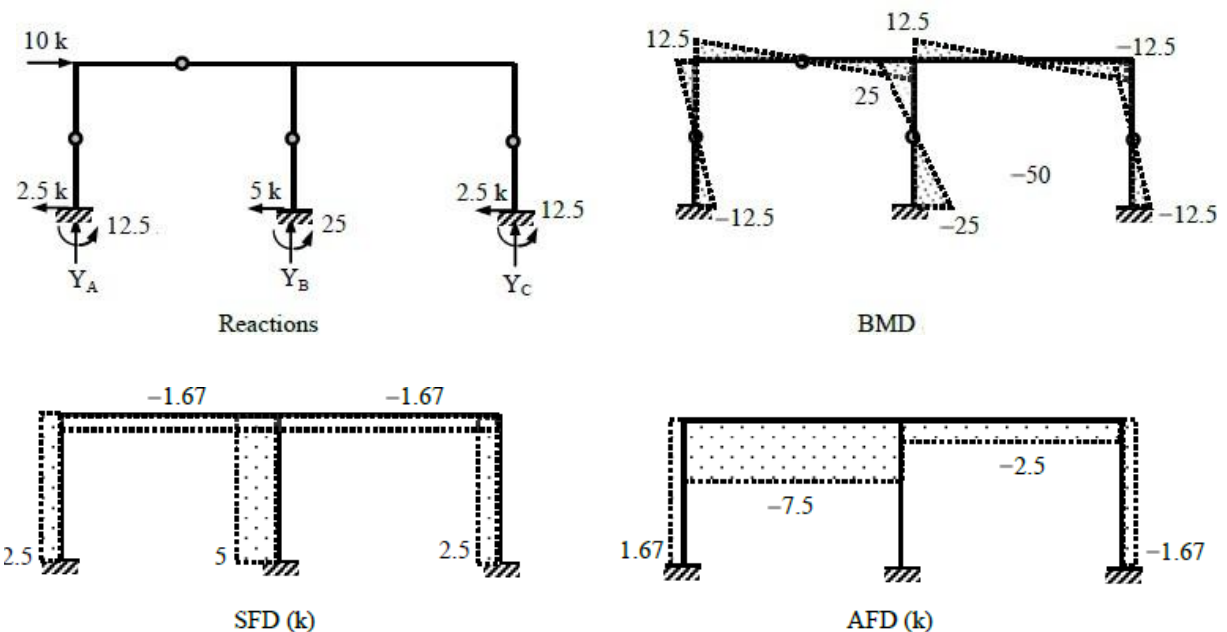


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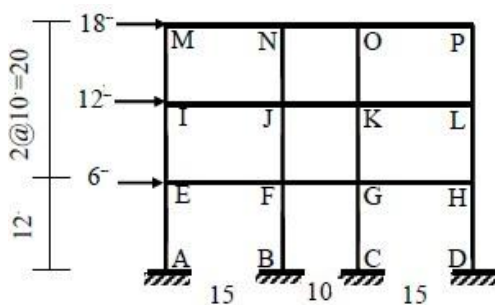
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Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are $[18 \times 1/(1 + 2 + 2 + 1) =] 3^l$, $[18 \times 2/(1 + 2 + 2 + 1) =] 6^l$, 6^l , 3^l respectively. Similarly,

$V_{EI} = 30 \times 1/(6) = 5^-$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5^-$; and $V_{AE} = 36 \times 1/(6) = 6^-$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6^-$

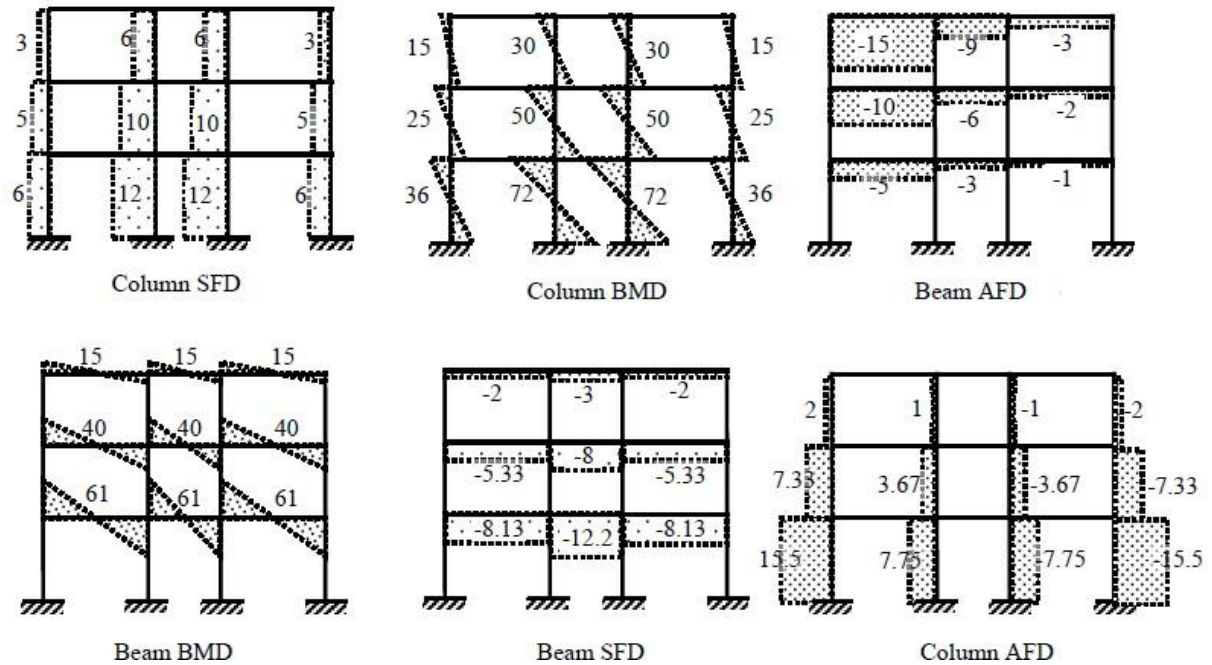
Bending moments are

$M_{IM} = 3 \times 10/2 = 15^l$, $M_{JN} = 30^-$, $M_{KO} = 30^l$, $M_{LP} = 15^-$

$M_{EI} = 5 \times 10/2 = 25^-$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25^-$

$M_{AE} = 6 \times 10/2 = 30^-$, $M_{BF} = 60$, $M_{CG} = 60$, $M_{DH} = 30^-$

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross-sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

The Cantilever Method is based on three assumptions

The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of the storey.

There is a point of inflection at the center of each column.

There is a point of inflection at the center of each beam.