mood-book

Boolean Algebra:

→ It deals with logical expressions which ~~the~~ involves 0's and 1's and follows some set of rules, laws (or) theorems.

Properties of boolean Algebra:

(i) Commutative property: $A+B = B+A$

$A \cdot B = B \cdot A$

(ii) Associative property: $A+(B+C) = (A+B)+C$

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$

(iii) Distributive property: $A+(BC) = (A+B)(A+C)$

$A \cdot (B+C) = (AB)+(AC)$

(iv) Absorption laws:

(a) $A+AB = A$

(b) $A \cdot (A+B) = A$

(c) $A+\overline{A}B = A+B$

(d) $A \cdot (\overline{A}+B) = AB$

(v) Consensus laws: $AB+\overline{A}C+BC = AB+\overline{A}C$

(vi)

OR rules	AND rules
$A+0 = A$	$A \cdot 0 = 0$
$A+1 = 1$	$A \cdot 1 = A$
$A+A = A$	$A \cdot A = A$
$A+\overline{A} = 1$	$A \cdot \overline{A} = 0$
	$\overline{\overline{A}} = A$

i) Duality Principle:

The dual of the boolean expression is obtained by interchanging multiplication and addition and also interchanging 0s and 1s.

$$F = A\bar{B} + \bar{A}B$$

$$F^d = (A+\bar{B}) \cdot (\bar{A}+B)$$

Minimise the boolean expressions:

i) $AB + \bar{A}B + B = Y$

$= B(A + \bar{A} + 1)$

$= B(1+1)$ (∵ from properties)

$= B$

ii) $AB + A\bar{B} = Y$

$= A(B+\bar{B})$

$= A(1)$

$= A$

(iii) $ABC + \bar{A}B + AB\bar{C}$

$= AB(C+\bar{C}) + \bar{A}B$

$= AB + \bar{A}B$

$= B(A+\bar{A}) = B$

(iv) $\bar{A}BC + AC$

$= C(\bar{A}B + A)$

$= C(A+B)$ (∵ A·L)

$= AC + BC$

(v) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

$= \bar{A}\bar{B}(\bar{C}+C) + \bar{A}B\bar{C} + A\bar{B}(\bar{C}+C) + AB(\bar{C}+C)$

$= \bar{A}\bar{B} + \bar{A}B\bar{C} + A\bar{B} + AB$

$= \bar{A}(\bar{B}+B\bar{C}) + A(\bar{B}+B)$

$= \bar{A}(\bar{B}+\bar{C}) + A$ $[∵ (A+\bar{A}B) = A+B]$

$Y = \bar{A}\bar{B} + \bar{A}\bar{C} + A$

$= \bar{A}\bar{B} + (A+\bar{C})$ (∵ from A·L) $[\overline{A+\bar{A}B} = \bar{A}+\bar{B}]$

$Y = A + \bar{B} + \bar{C}$

$Y = A + \overline{BC}$

Any logical fn can be expressed in two forms

(i) Sum of products (SOP)

(ii) Product of sum (POS)

Product term:

→ AND fn is referred to as a fn. The variables in a product term can be either in true form (or) compliment form.

Ex: $AB\bar{C}$

Sum term:

→ OR fn is referred to a sum. The variables in a sumterm can be either in true form (or) complement form.

Ex: $A+\bar{B}+C$

SOP:

→ The logical sum of two or more logical products.

$$Y = \bar{A}\bar{B} + A\bar{B} + \bar{A}B$$

POS:

→ The logical product of two or more logical sums.

$$Y = (A+B+C)(\bar{A}+B+C)$$

Minterm:

A productterm containing all the n variables either in true or compliment term is called a minterm.

A two variable fns has 4 minterms ($2^2 = 2^n$).

	A	B	minterm	
0-	0	0	m_0	$\bar{A}\bar{B}$
1-	0	1	m_1	$\bar{A}B$
2-	1	0	m_2	$A\bar{B}$
3-	1	1	m_3	AB

true form - 1
complement - 0 (SOP)

A Three variable fns has 8 minterms ($2^3 = 8$)

	A	B	C	mi	
0-	0	0	0	m_0	$\bar{A}\bar{B}\bar{C}$
1-	0	0	1	m_1	$\bar{A}\bar{B}C$
2-	0	1	0	m_2	$\bar{A}B\bar{C}$
3-	0	1	1	m_3	$\bar{A}BC$
4-	1	0	0	m_4	$A\bar{B}\bar{C}$
5-	1	0	1	m_5	$A\bar{B}C$
6-	1	1	0	m_6	$AB\bar{C}$
7-	1	1	1	m_7	ABC

Maxterm:

A Sumterm containing all the 'n' variables either in true (or) complement term is called a maxterm.

A	B	maxterm	
0	0	M_0	$A+B$
0	1	M_1	$A+\bar{B}$
1	0	M_2	$\bar{A}+B$
1	1	M_3	$\bar{A}+B$

True form - 0
Compliment - 1 (POS)

A	B	C	Mi	
0	0	0	M_0	$A+B+C$
0	0	1	M_1	$A+B+\bar{C}$
0	1	0	M_2	$A+\bar{B}+C$
0	1	1	M_3	$A+\bar{B}+\bar{C}$
1	0	0	M_4	$\bar{A}+B+C$
1	0	1	M_5	$\bar{A}+B+\bar{C}$
1	1	0	M_6	$\bar{A}+\bar{B}+C$
1	1	1	M_7	$\bar{A}+\bar{B}+\bar{C}$

Canonical SOP form:

1. $Y(A,B) = A+B$

$= A(B+\bar{B}) + B(A+\bar{A})$

$= AB + A\bar{B} + AB + \bar{A}B$

$= AB + A\bar{B} + \bar{A}B \Rightarrow m_1 + m_2 + m_3 = \sum_m (m_1, m_2, m_3)$

2. $Y(A,B,C) = \bar{A}B + AB\bar{C} + \bar{A}BC$

$= \bar{A}B(C+\bar{C}) + AB\bar{C} + \bar{A}BC$

$= \bar{A}BC + \bar{A}B\bar{C} + AB\bar{C} + \bar{A}BC$

$= m_3 + m_2 + m_6 + m_3 \Rightarrow \sum m(m_2, m_3, m_6)$

3. $Y(A,B,C) = A+BC$

$= A(B+\bar{B})\cdot(C+\bar{C}) + (A+\bar{A})BC$

$= ABC + A\bar{B}\bar{C} + ABC + \bar{A}BC + AB\bar{C} + A\bar{B}C$

$= ABC + A\bar{B}\bar{C} + \bar{A}BC + AB\bar{C} + A\bar{B}C$

$= m_7 + m_4 + m_3 + m_6 + m_5$

$= \sum m(m_3, m_4, m_5, m_6, m_7)$

4. $Y(A,B,C,D) = AB+CD$

$= AB(C+\bar{C})(D+\bar{D}) + CD(A+\bar{A})(B+\bar{B})$

$= (ABC + AB\bar{C})(D+\bar{D}) + ADC + \bar{A}CD(B+\bar{B})$

$= ABCD + AB\bar{C}\bar{D} + ADCB + \bar{A}\bar{B}CD + ABC\bar{D} + A\bar{B}CD$
$+ \bar{A}BCD + AB\bar{C}D$

$= ABCD + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + ABC\bar{D} + A\bar{B}CD + \bar{A}BCD + AB\bar{C}D$

$\Sigma = m_{15} + m_{12} + m_3 + m_{14} + m_{11} + m_7 + m_{13}$

$\Sigma m = (3, 7, 11, 12, 13, 14, 15)$

anonical POS form:

$Y(A,B) = A\cdot(\overline{A}+\overline{B})\cdot\overline{B}$

$= (A+0)\cdot(\overline{A}+\overline{B})\cdot(\overline{B}+0)$

$= (A+B\overline{B})\cdot(\overline{A}+\overline{B})\cdot(\overline{B}+A\overline{A})$

$= (A+B)\cdot(\overline{A}+\overline{B})\cdot(\overline{B}+A)$ $\quad (A+B)(\overline{B}+A)$

$= (A+\overline{B})\cdot(\overline{A}+B)\cdot(\overline{A}+\overline{B})$ $\quad (A+\overline{B})(\overline{A}+\overline{B})(\overline{B}+\overline{A})$,

$Y = (A+\overline{B})\cdot(\overline{A}+\overline{B})$

$Y = M_1\cdot M_3$

$Y = \Pi_M(1,3)$

eriving SOP and POS expression from truth table:

ax	min	A	B	C	Y
M_0	m_0	0	0	0	0
M_1	m_1	0	0	1	0
M_2	m_2	0	1	0	1
M_3	m_3	0	1	1	1
M_4	m_4	1	0	0	0
M_5	m_5	1	0	1	1
M_6	m_6	1	1	0	0
M_4	m_7	1	1	1	1

SOP : $Y = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$

$Y = \Sigma m(2,3,5,7)$

POS : $Y = (A+B+C)\cdot(A+B+\overline{C})\cdot(\overline{A}+B+C)\cdot(\overline{A}+\overline{B}+C)$

$Y = \Pi_M(0,1,4,6)$

Write the truthtable for SOP and POS.

1. $F(A,B,C) = \Sigma m(1,5,6,7)$

2. $F(A,B,C,D) = \Pi_M(0,1,5,7,8,9,11,13,15)$

	A	B	C	Y
m_0	0	0	0	0
m_1	0	0	1	1
m_2	0	1	0	0
m_3	0	1	1	0
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	1
m_7	1	1	1	1

SOP

1. $Y = (\bar{A}\bar{B}C) + (A\bar{B}C) + (AB\bar{C}) + (ABC)$

POS:

$Y = (A+B+C)\cdot(A+\bar{B}+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+C)$

	A	B	C	D	Y
M_0	0	0	0	0	0
M_1	0	0	0	1	0
M_2	0	0	1	0	1
M_3	0	0	1	1	1
M_4	0	1	0	0	1
M_5	0	1	0	1	0
M_6	0	1	1	0	1
M_7	0	1	1	1	0
M_8	1	0	0	0	0
M_9	1	0	0	1	0
M_{10}	1	0	1	0	1
M_{11}	1	0	1	1	0
M_{12}	1	1	0	0	1
M_{13}	1	1	0	1	0
M_{14}	1	1	1	0	1
M_{15}	1	1	1	1	0

POS:

$$Y = (A+B+C+D)\cdot(A+B+C+\bar{D})\cdot(A+\bar{B}+C+\bar{D})\cdot(A+\bar{B}+\bar{C}+\bar{D})\cdot(\bar{A}+B+C+D)\cdot(\bar{A}+B+C+\bar{D})\cdot(\bar{A}+B+\bar{C}+D)\cdot(\bar{A}+\bar{B}+C+\bar{D})\cdot(\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

SOP:

$$Y = (\bar{A}\bar{B}C\bar{D}) + (\bar{A}\bar{B}CD) + (\bar{A}B\bar{C}\bar{D}) + (\bar{A}BC\bar{D}) + (A\bar{B}C\bar{D}) + (AB\bar{C}\bar{D}) + (ABC\bar{D})$$

Convert SOP to POS:

1. $Y = (A + \bar{B}C)$

$= (A+\bar{B})(A+C)$

$= (A+\overline{B}+0)\cdot(A+C+0)$

$= (A+\overline{B}+C\overline{C})\cdot(A+C+B\overline{B})$

$= (A+\overline{B}+\overline{C})\cdot(A+C+\overline{B}) \Rightarrow \underset{0\ \ 1\ \ 1}{(A+\overline{B}+\overline{C})}\cdot\underset{0\ \ 1\ \ 0}{(A+\overline{B}+C)}$

$Y = M_3 \cdot M_2$

$Y = \pi_M(3,2)$

Simplify:

1. $Y = \overline{A}B + A\overline{B} + AB$

2. $Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + A\overline{B}C$

3. $Y = \overline{C}\overline{D} + \overline{A}\overline{B}\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D} + C\overline{D}$

4. $Y = AB + (A+B)(\overline{A}+B)$

5. $Y = \overline{(AB+\overline{C})\cdot(\overline{A}+\overline{B}+C)}$

1Sol:

$Y = \overline{A}B + A\overline{B} + AB$

$= \overline{A}B + A(\overline{B}+B)$

$= \overline{A}B + A$

$Y = A+B$

2Sol:

$Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + A\overline{B}C$

$= \overline{B}C(\overline{A}+A) + \overline{A}B\overline{C} + A\overline{B}\overline{C}$

$= \overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C}$

$= \overline{B}(C+A\overline{C}) + \overline{A}B\overline{C}$

$= \overline{B}(A+C) + \overline{A}B\overline{C}$

$Y = \overline{B}A + \overline{B}C + \overline{A}B\overline{C}$

Karnaugh maps (K-maps):

→ Kmap is a graphical representation that provides a systematic method for simplifying boolean expression.

→ It follows gray code (unit distance code).

2-Variable K-map:

→ For n-variable 2^n cells are required.

→ For 2-variable 2^2 cells = 4 cells.

A \ B		0 $\bar{B}$	1 B
0	$\bar{A}$	00 ₀	01 ₁
1	A	10 ₂	11 ₃

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}$	$\bar{A}B$
A	$A\bar{B}$	AB

3-Variable K-map:

→ $2^3 = 8$ Cells

A \ BC		00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
0	$\bar{A}$	000 $_0$	001 $_1$	011 $_3$	010 $_2$
1	A	100 $_4$	101 $_5$	111 $_7$	110 $_6$

AB \ C	$\bar{C}$	C
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}_0$	$\bar{A}\bar{B}C_1$
$\bar{A}B$	$\bar{A}B\bar{C}_2$	$\bar{A}BC_9$
AB	$AB\bar{C}_6$	ABC_7
$A\bar{B}$	$A\bar{B}\bar{C}_4$	$A\bar{B}C_5$

4-Variable K-maps:

→ $2^4 = 16$ cells

AB \ CD		00 $\bar{C}\bar{D}$	01 $\bar{C}D$	11 CD	10 $C\bar{D}$
00	$\bar{A}\bar{B}$	0000 $_0$	0001 $_1$	0011 $_3$	0010 $_2$
01	$\bar{A}B$	0100 $_4$	0101 $_5$	0111 $_7$	0110 $_6$
11	AB	1100 $_{12}$	1101 $_{13}$	1111 $_{15}$	1110 $_{14}$
10	$A\bar{B}$	1000 $_8$	1001 $_9$	1011 $_{11}$	1010 $_{10}$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
AB	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

Solving boolean expressions using K-maps:

1. $Y = \bar{A}B + A\bar{B} + AB$

$A \backslash B$	$\bar{B}$ (0)	B (1)
0 $\bar{A}$		1
1 A	1	1

1st pair $\begin{cases} A\bar{B} \\ AB \end{cases}$ $\quad$ $\left.\begin{matrix} \bar{A}B \\ AB \end{matrix}\right\}$ 2nd pair

Take common term

1st pair $= A$

2nd pair $= B$ $\Rightarrow A+B$

$Y = A+B$

- Grouping 1's should be done in the form of 2 (pair), 4 (quad), 8 (oct), 16
- Adjacent cells which have 1's can be grouped together in 2's power.

2. $Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$

$A \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$		1		1
A	1	1		

$Y = A\bar{B} + \bar{B}C + \bar{A}B\bar{C}$

$AB \backslash C$	$\bar{C}$	C
$\bar{A}\bar{B}$		1
$\bar{A}B$	1	
AB		
$A\bar{B}$	1	1

$\Rightarrow$ Extreme corners can be grouped

$Y = A\bar{B} + \bar{B}\bar{C} + \bar{A}B\bar{C}$

3. $Y = \bar{C}\bar{D} + \bar{A}B\bar{D} + \bar{A}BC\bar{D} + C\bar{D}$

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			1
$\bar{A}B$	1			1
AB	1			1
$A\bar{B}$	1			1

$Y = \bar{D}$

4. $Y = \Sigma m(0, 3, 4, 5, 7)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1 (0)	(1)	1 (3)	(2)
A	1 (4)	1 (5)	1 (7)	(6)

$Y = \bar{B}\bar{C} + \bar{A}\bar{B} + AC + BC +$

5. $Y = \Sigma m(0, 2, 4, 6, 8, 11, 13, 15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	(1)	(3)	1 (2)
$\bar{A}B$	1 (4)	(5)	(7)	1 (6)
AB	(12)	1 (13)	1 (15)	(14)
$A\bar{B}$	1 (8)	(9)	1 (11)	(10)

$Y = \bar{A}\bar{D} + ABD + ACD + \bar{B}\bar{C}\bar{D}$

Prime implicant:

Prime implicant is an implicant which cannot be minimised further.

Essential prime implicant:

Essential prime implicant is an implicant which is having atleast one minterm which is not covered by other P.I.

6. $Y = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$ and obtain PI & EPI

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	(0)	(1)	1 (3)	(2)
$\bar{A}B$	1 (4)	1 (5)	1 (7)	(6)
AB	(12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$	(8)	1 (9)	(10)	(11)

$Y = BD + \bar{A}B\bar{C} + \bar{A}CD + ABC + A\bar{C}D$ (PI = 5)

EPI = (3, 7)

(4, 5)

(15, 14)

(13, 9)

$Y = \Sigma m(0, 4, 5, 10, 11, 13, 15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1_{0}	$_{1}$	$_{3}$	$_{2}$
$\bar{A}B$	1_{4}	1_{5}	$_{7}$	$_{6}$
AB	$_{12}$	1_{13}	1_{15}	$_{14}$
$A\bar{B}$	$_{8}$	$_{9}$	1_{11}	1_{10}

$$Y = \bar{A}\bar{C}\bar{D} + \bar{A}B\bar{C} + B\bar{C}D + ABD + ACD + A\bar{B}C = P.I = 6$$

$$\left.\begin{aligned} EPI &= (11, 10) \\ &= (0, 4) \end{aligned}\right\} = 2$$

Rules followed for K-maps simplification:

- → Groups do not include any cell containing a 0.
- → Groups can be horizontal or vertical but not diagonal.
- → Groups must contain 1, 2, 4, 8 or 16 cells [2^n, n = no. of var]
- → Each group should be as large as possible.

Simplify using K-map in SOP and POS

$$Y(A, B, C) = \bar{A}BC + B\bar{C} + AB\bar{C} + A\bar{B}C$$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	$_{0}$	$_{1}$	1_{3}	1_{2}
A	$_{4}$	1_{5}	$_{7}$	1_{6}

SOP: $Y = \bar{A}B + B\bar{C} + A\bar{B}C$ P.I = 3

$$\left.\begin{aligned} EPI = &\ 3, 2 \\ &\ 2, 6 \end{aligned}\right\} = 2$$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0_{0}	0_{1}	$_{3}$	$_{2}$
A	0_{4}	$_{5}$	0_{7}	$_{6}$

POS: $Y = \overline{\bar{A}\bar{B} + \bar{B}\bar{C} + ABC}$

$= \overline{\bar{A}\bar{B}} + \overline{\bar{B}\bar{C}} + \overline{ABC} = (A+B) \cdot (B+C) \cdot (\bar{A}+\bar{B}+\bar{C})$

POS K-map

A \ BC	B+C 00	B+$\bar{C}$ 01	$\bar{B}$+$\bar{C}$ 11	$\bar{B}$+C 10
A 0	0	0		
$\bar{A}$ 1	0		0	

$$Y = (A+B) \cdot (B+C) \cdot (\bar{A}+\bar{B}+\bar{C})$$

$A \rightarrow 0$

$\bar{A} \rightarrow 1$

2. $Y(A,B,C,D) = \sum m(4,6,5,7,8,9,10,11,12,13,14,15)$

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	(0)	(1)	(3)	(2)
$\bar{A}B$	1 (4)	1 (5)	1 (7)	1 (6)
AB	1 (12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$	1 (8)	1 (9)	1 (11)	1 (10)

$Y = B + A$

3. $Y = \sum m(3,4,5,7,9,13,14,15)$

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	(0)	(1)	1 (3)	(2)
$\bar{A}B$	1 (4)	1 (5)	1 (7)	(6)
AB	(12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$	(8)	1 (9)	(11)	(10)

$Y = BD + \bar{A}B\bar{C} + \bar{A}CD + ABC + A\bar{C}D \Rightarrow SOP$

$AB \backslash CD$	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	0	0		0
$A+\bar{B}$				0
$\bar{A}+\bar{B}$	0			
$\bar{A}+B$	0		0	0

$Y = (A+B+C)\cdot(\bar{C}+D+A)\cdot(\bar{A}+C+D)\cdot(\bar{A}+B+\bar{C})\cdot(B+D)$
$\cancel{(B+C+D)}\cdot\cancel{(B+\bar{C}+D)}$

4. $Y = \Pi M(0,1,4,5,6,8,9,12,13,14)$

$AB \backslash CD$	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	0 (0)	0 (1)	(3)	(2)
$A+\bar{B}$	0 (4)	0 (5)	(7)	0 (6)
$\bar{A}+\bar{B}$	0 (12)	0 (13)	(15)	0 (14)
$\bar{A}+B$	0 (8)	0 (9)	(11)	(10)

$Y = C\cdot(\bar{B}+\cancel{C}+D)\cdot(\Rightarrow C\cdot(\bar{B}+D) \Rightarrow POS$

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$			1	1
$\bar{A}B$			1	
AB			1	
$A\bar{B}$			1	1

$Y = \bar{B}CD + \bar{A}\bar{B}C + A\bar{B}C + \bar{B}C\bar{D}$

K-map simplification using don't care combination:

In certain digital systems some input combinations never occur during the process of a usual operation bcoz these input conditions are guranteed never to occur, these are called as don't care combinations.

The function which assumes 1 (or) 0 is called don't care condition.

It is represented by d (or) x.

To simplify the expression don't care combinations can be used.

This is done to increase the no. of 1's in the selected group.

d (or) x need not be used in grouping if it doesnot cover a large no. of (ones) 1's.

1. $Y = \Sigma m(1,5,6,12,13,14) + d(4)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1 (1)	3	2
$\bar{A}B$	d (4)	1 (5)	7	1 (6)
AB	1 (12)	1 (13)	15	1 (14)
$A\bar{B}$	8	9	11	10

$Y = \cancel{BE}\; B\bar{C} + B\bar{D} + \bar{A}\bar{C}D$

2. $Y = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,5)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	d (0)	1 (1)	1 (3)	d (2)
$\bar{A}B$	4	d (5)	1 (7)	6
AB	12	13	1 (15)	14
$A\bar{B}$	8	9	1 (11)	10

$Y = \bar{A}\bar{B} + CD$

3. $Y = \Sigma m\,(0, 2, 3, 6, 7) + \Sigma d\,(8, 10, 11, 15)$ SOP & POS

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1_{0}	$_{1}$	1_{3}	1_{2}
$\bar{A}B$	$_{4}$	$_{5}$	1_{7}	1_{6}
AB	$_{12}$	$_{13}$	d_{15}	$_{14}$
$A\bar{B}$	d_{8}	$_{9}$	d_{11}	d_{10}

$Y = \bar{A}C + \bar{B}\bar{D}$ (SOP)

AB \ CD	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	$_{0}$	0	$_{3}$	$_{2}$
$A+\bar{B}$	0_{4}	0_{5}	$_{7}$	$_{6}$
$\bar{A}+\bar{B}$	0_{12}	0_{13}	d_{15}	0_{14}
$\bar{A}+B$	d_{8}	0_{9}	d_{11}	d_{10}

$Y = \bar{A}\cdot(C+\bar{D})\cdot(\bar{B}+C+\cancel{B})$ (POS)

4. $Y = \Sigma m(0, 1, 2, 3, 4, 5) + \Sigma d(10, 11, 12, 13, 14, 15)$ SOP & POS

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1_{0}	1	1_{3}	1_{2}
$\bar{A}B$	1_{4}	1_{5}	$_{7}$	$_{6}$
AB	d_{12}	d_{13}	d_{15}	d_{14}
$A\bar{B}$	$_{8}$	$_{9}$	d_{11}	d_{10}

$Y = \bar{A}\bar{B} + \bar{A}\bar{C}$ (SOP)

AB \ CD	$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	$_{0}$	$_{1}$	$_{3}$	$_{2}$
$A+\bar{B}$	$_{4}$	$_{5}$	0_{7}	0_{6}
$\bar{A}+\bar{B}$	d_{12}	d_{13}	d_{15}	d_{14}
$\bar{A}+B$	0_{8}	0_{9}	d_{11}	d_{10}

$Y = (\bar{B}+\bar{C})\cdot(\bar{A}+\cancel{\bar{C}})$ (POS)

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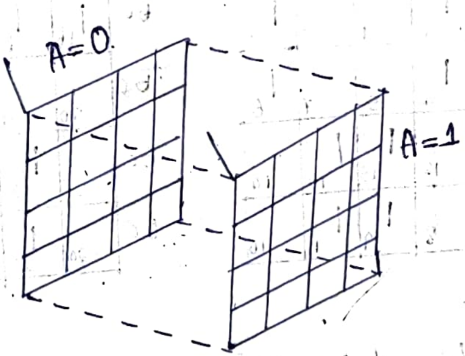
5-Variable K-map:

	A	B	C	D	E
0-	0	0	0	0	0
1-	0	0	0	0	1
2-	0	0	0	1	0
3-	0	0	0	1	1
4-	0	0	1	0	0
5-	0	0	1	0	1
6-	0	0	1	1	0
7-	0	0	1	1	1
8-	0	1	0	0	0
9-	0	1	0	0	1
10-	0	1	0	1	0
11-	0	1	0	1	1
12-	0	1	1	0	0
13-	0	1	1	0	1
14-	0	1	1	1	0
15-	0	1	1	1	1
16-	1	0	0	0	0
17-	1	0	0	0	1
⋮					
31-	1	1	1	1	1

(0-15) A=0

(16-31) A=1



A=0

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	0	1	3	2
$\bar{B}C$	4	5	7	6
BC	12	13	15	14
$B\bar{C}$	8	9	11	10

A=1

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	16	17	19	18
$\bar{B}C$	20	21	23	22
BC	28	29	31	30
$B\bar{C}$	24	25	27	26

1. $Y = \Sigma m(0,2,3,5,7,8,10,11,14,15,16,18,24,26,27,30,31)$

A=0

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	1 $_{0}$	$_{1}$	1 $_{3}$	1 $_{2}$
$\bar{B}C$	$_{4}$	1 $_{5}$	1 $_{7}$	$_{6}$
BC	$_{12}$	$_{13}$	1 $_{15}$	1 $_{14}$
$B\bar{C}$	1 $_{8}$	$_{9}$	1 $_{11}$	1 $_{10}$

A=1

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	1 $_{16}$	$_{17}$	$_{19}$	1 $_{18}$
$\bar{B}C$	$_{20}$	$_{21}$	$_{23}$	$_{22}$
BC	$_{28}$	$_{29}$	1 $_{31}$	1 $_{30}$
$B\bar{C}$	1 $_{24}$	$_{25}$	1 $_{27}$	1 $_{26}$

$$Y = \bar{C}\bar{E} + BD + \bar{A}DE + \bar{A}\bar{B}CE$$

2. $Y = \Sigma m(0,2,4,6,9,11,13,15,16,17,18,19,25,27,29,31)$

A=0

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	1 $_{0}$	$_{1}$	$_{3}$	1 $_{2}$
$\bar{B}C$	1 $_{4}$	$_{5}$	$_{7}$	1 $_{6}$
BC	$_{12}$	1 $_{13}$	1 $_{15}$	$_{14}$
$B\bar{C}$	$_{8}$	1 $_{9}$	1 $_{11}$	$_{10}$

A=1

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	1 $_{16}$	1 $_{17}$	1 $_{19}$	1 $_{18}$
$\bar{B}C$	$_{20}$	$_{21}$	$_{23}$	$_{22}$
BC	$_{28}$	1 $_{29}$	1 $_{31}$	$_{30}$
$B\bar{C}$	$_{24}$	1 $_{25}$	1 $_{27}$	$_{26}$

$$Y = BE + \bar{A}\bar{B}\bar{E} + A\bar{B}\bar{C}$$

3. $Y = \Sigma m(0,2,5,7,10,16,18,23,24,26,29) + \Sigma d(8,21)$

A=0

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	1 $_{0}$	$_{1}$	$_{3}$	1 $_{2}$
$\bar{B}C$	$_{4}$	1 $_{5}$	1 $_{7}$	$_{6}$
BC	$_{12}$	$_{13}$	$_{15}$	$_{14}$
$B\bar{C}$	d $_{8}$	$_{9}$	$_{11}$	1 $_{10}$

A=1

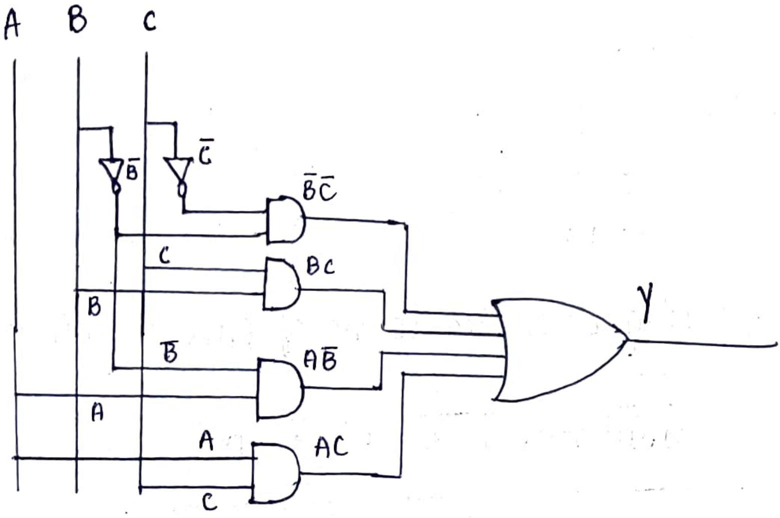
BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	1 $_{16}$	$_{17}$	$_{19}$	1 $_{18}$
$\bar{B}C$	$_{20}$	d $_{21}$	1 $_{23}$	$_{22}$
BC	$_{28}$	1 $_{29}$	$_{31}$	$_{30}$
$B\bar{C}$	1 $_{24}$	$_{25}$	$_{27}$	1 $_{26}$

$$Y = \bar{C}\bar{E} + \cancel{\bar{A}E} + \cancel{AEA}\ \cancel{A}\ \cancel{\bar{A}E\bar{B}C} + A\bar{B}CE + AC\bar{D}E$$

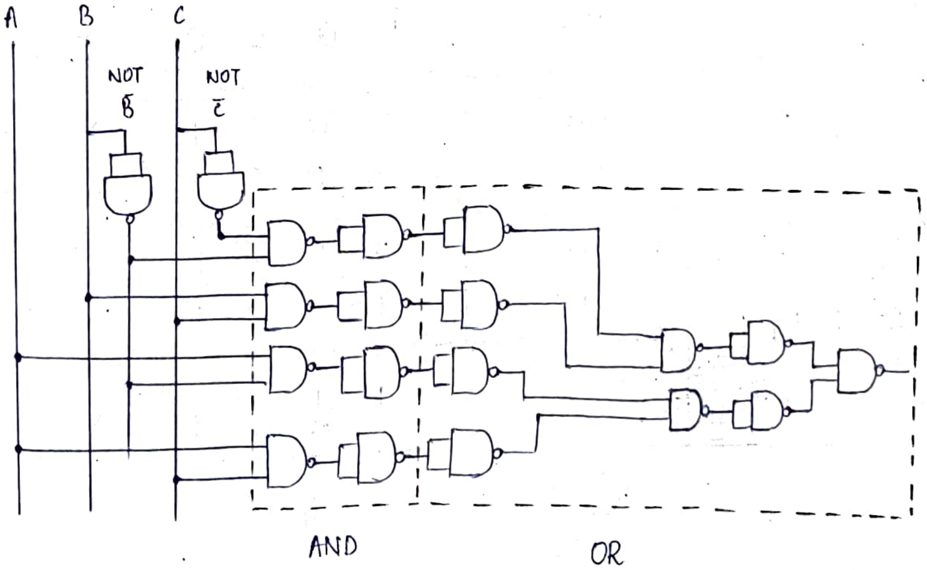
Simplify $Y = \Sigma_m (0,3,4,5,7)$ using K-map and draw the logic circuit.

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1 (0)	(1)	1 (3)	(2)
A	1 (4)	1 (5)	1 (7)	(6)

$Y = \bar{B}\bar{C} + BC + A\bar{B} + AC$



using NAND gates



* Tabular Method or Quine Mccluskey Method:

1. $Y = \sum_m (0,1,3,7,8,9,11,15)$

Step 1: Write binary numbers for given numbers

0	-	0000
1	-	0001
3	-	0011
7	-	0111
8	-	1000
9	-	1001
11	-	1011
15	-	1111

Step 2: Do grouping according to no. of ones in minterms

Group	minterms	A B C D	
0	m_0	0 0 0 0	✓
1	m_1	0 0 0 1	✓
	m_8	1 0 0 0	✓
2	m_3	0 0 1 1	✓
	m_9	1 0 0 1	✓
3	m_7	0 1 1 1	✓
	m_{11}	1 0 1 1	✓
4	m_{15}	1 1 1 1	✓

Step 3: Matched pairs should be grouped means the single bit change minterms should be considered when adjacent groups are compared.

Group	minterms of matched pairs	A B C D	
0	$m_0 - m_1$	0 0 0 -	✓
	$m_0 - m_8$	- 0 0 0	✓
1	$m_1 - m_3$	0 0 - 1	✓
	$m_1 - m_9$	- 0 0 1	✓
	$m_8 - m_9$	1 0 0 -	✓
2	$m_3 - m_7$	0 - 1 1	✓
	$m_3 - m_{11}$	- 0 1 1	✓
	$m_9 - m_{11}$	1 0 - 1	✓
3	$m_7 - m_{15}$	- 1 1 1	✓
	$m_{11} - m_{15}$	1 - 1 1	✓

Step 4: Further matched pairs should be grouped.

Group	Matched pairs	A B C D	
0	$m_0 - m_1 - m_8 - m_9$	- 0 0 -	$\bar{B}\bar{C}$
	$m_0 - m_8 - m_1 - m_9$	- 0 0 -	
1	$m_1 - m_3 - m_9 - m_{11}$	- 0 - 1	$\bar{B}D$
	$m_1 - m_9 - m_3 - m_{11}$	- 0 - 1	
2	$m_3 - m_7 - m_{11} - m_{15}$	- - 1 1	CD
	$m_3 - m_{11} - m_7 - m_{15}$	- - 1 1	

P.I table:

P.I	minterms involved	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	m_0 m_1 m_8 m_9	X	X			X	X		
$\bar{B}D$	m_1 m_3 m_9 m_{11}		X	X			X	X	
CD	m_3 m_7 m_{11} m_{15}			X	X			X	X

$$Y = \bar{B}\bar{C} + CD$$

using k map:

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	1 (3)	(2)
$\bar{A}B$	(4)	(5)	1 (7)	(6)
AB	(12)	(13)	1 (15)	(14)
$A\bar{B}$	1 (8)	1 (9)	1 (11)	(10)

$$Y = \bar{B}\bar{C} + CD$$