

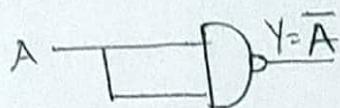
mood-book



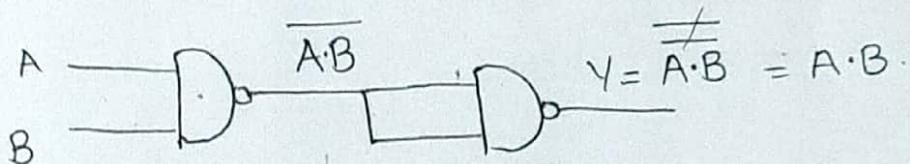
UNIT - IV

- ① @ Realization of all gates by NAND Gate
 ⑥ Realization of all gates by NOR Gate

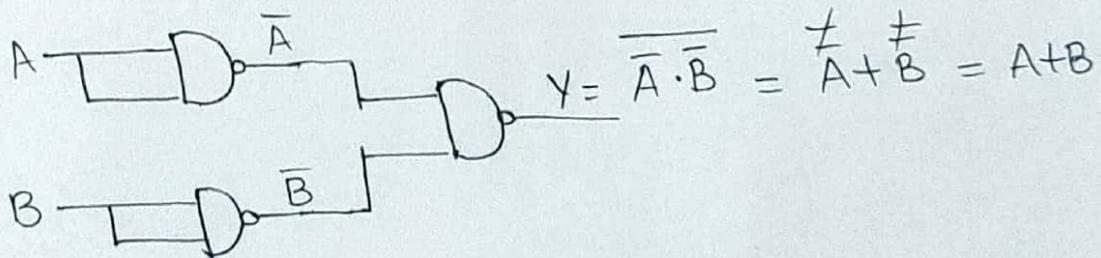
Amb: @ (i) NOT Gate



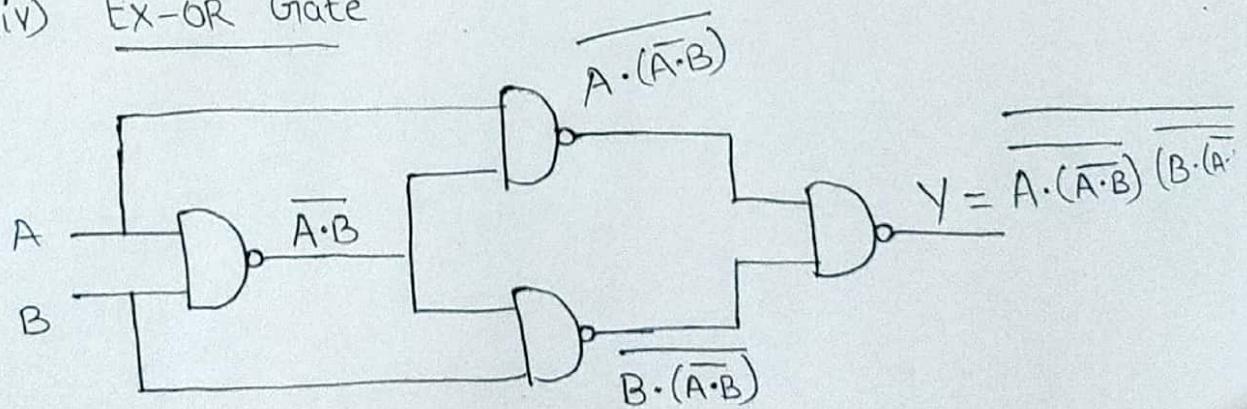
(ii) AND Gate



(iii) OR Gate



(iv) Ex-OR Gate



$$\begin{aligned} Y &= \overline{A \cdot (\bar{A} \cdot B)} \cdot \overline{B \cdot (\bar{A} \cdot B)} \\ &= \overline{\overline{A \cdot (\bar{A} \cdot B)}} + \overline{B \cdot (\bar{A} \cdot B)} \end{aligned}$$

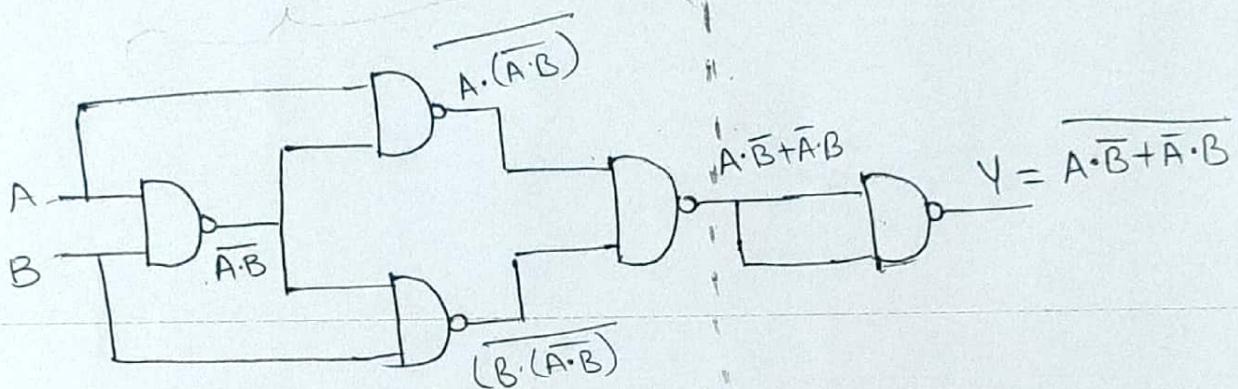
(Q)

$$\begin{aligned}
 Y &= A \cdot (\overline{A} \cdot \overline{B}) + B (\overline{A} \cdot \overline{B}) \\
 &= A \cdot (\overline{A} + \overline{B}) + B (\overline{A} + \overline{B}) \\
 &= A \cdot \overline{A} + A \cdot \overline{B} + \overline{A} \cdot B + B \cdot \overline{B} \quad (\because A \cdot \overline{A} = 0) \\
 &= 0 + A \cdot \overline{B} + \overline{A} \cdot B + 0
 \end{aligned}$$

$$Y = A \cdot \overline{B} + \overline{A} \cdot B \equiv A \oplus B$$

(iv) Ex-NOR Gate

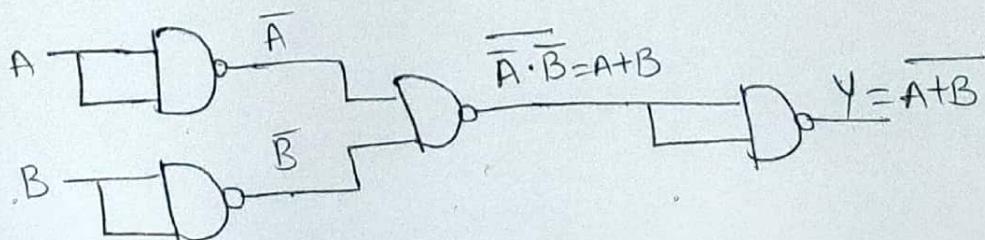
similar to Ex-OR



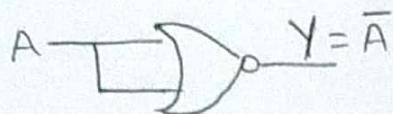
$$\begin{aligned}
 Y &= \overline{A \cdot \overline{B} + \overline{A} \cdot B} \\
 &= (\overline{A \cdot \overline{B}}) (\overline{\overline{A} \cdot B}) \\
 &= (\overline{A} + \overline{B}) (\overline{\overline{A}} + \overline{B}) = (\overline{A} + B) (A + \overline{B}) \\
 &= \overline{\overline{A} \cdot A + \overline{A} \cdot \overline{B} + A \cdot B + B \cdot \overline{B}} \\
 &= 0 + \overline{A} \cdot \overline{B} + A \cdot B + B \cdot \overline{B}
 \end{aligned}$$

$$Y = \overline{\overline{A} \cdot \overline{B} + A \cdot B} = \overline{A \oplus B}$$

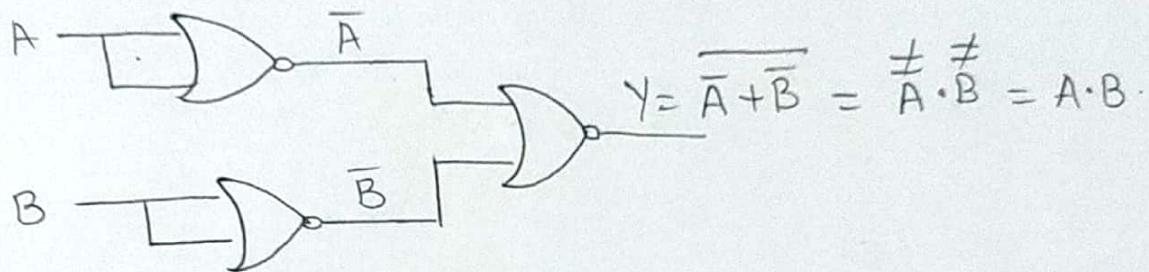
(v) NOR Gate



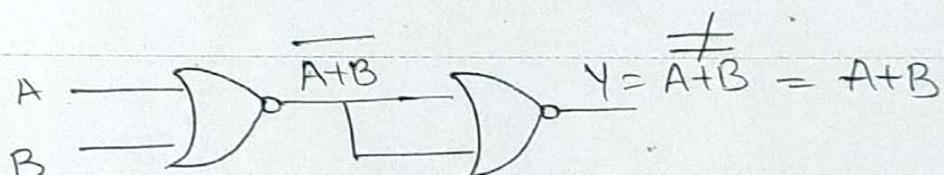
6 (i) NOT Gate



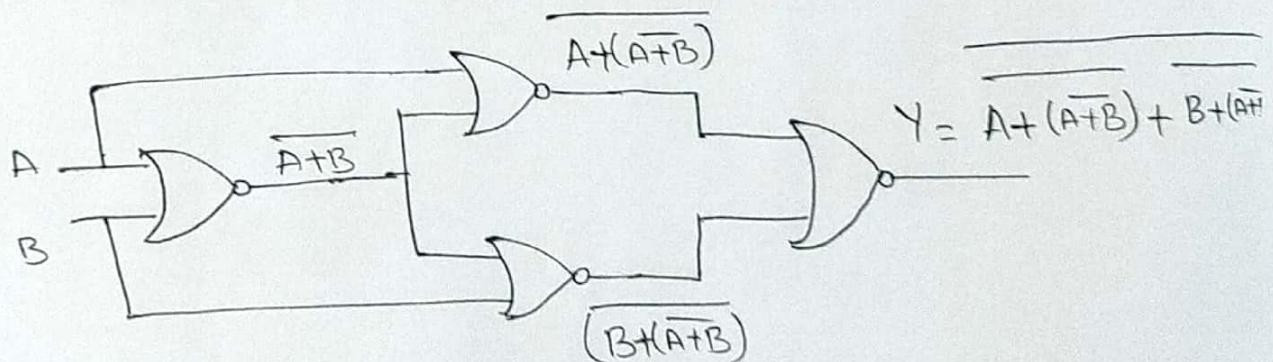
(ii) AND Gate



(iii) OR Gate



(iv) Ex-NOR Gate



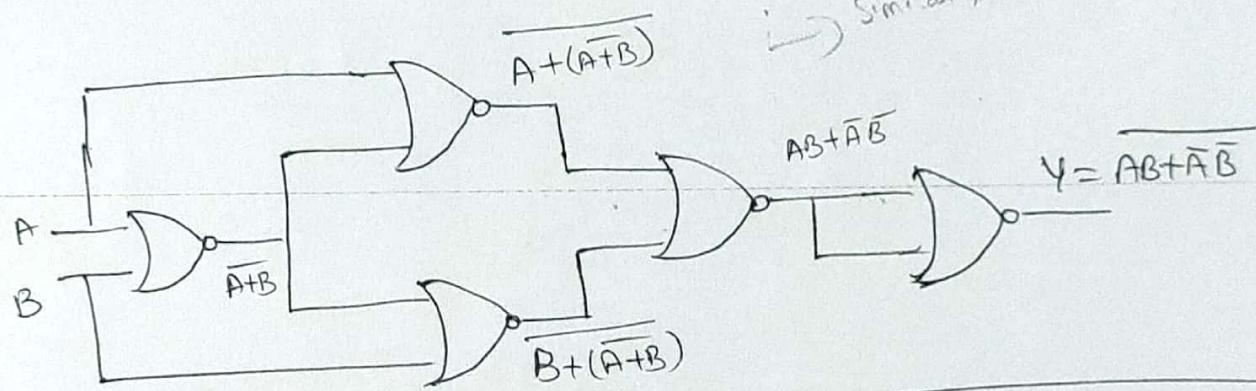
$$Y = \overline{\overline{A + (\bar{A} \cdot B)} + \overline{\bar{B} + (\bar{A} \cdot B)}}$$

$$Y = \left(\overline{\overline{A + (\bar{A} \cdot B)}} \right) \cdot \left(\overline{\overline{\bar{B} + (\bar{A} \cdot B)}} \right)$$

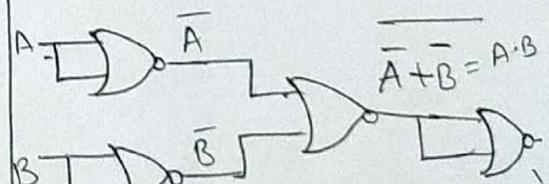
$$= (A + (\bar{A} \cdot B)) \cdot (B + (\bar{A} \cdot B))$$

$$= (A + (\bar{A} \cdot B)) \cdot (B + \bar{A} \cdot \bar{B})$$

$$\begin{aligned}
 &= (A + \bar{A}) (A + \bar{B}) \cdot (B + \bar{A}) \cdot (B + \bar{B}) \quad \stackrel{(4)}{\therefore} (A + \bar{A} = 1) \\
 &= 1 (A + \bar{B}) \cdot (B + \bar{A}) \cdot 1 \\
 &= (A + \bar{B}) (B + \bar{A}) \\
 &= AB + A\bar{A} + B\bar{B} + \bar{A}\bar{B} \\
 &= AB + 0 + 0 + \bar{A}\bar{B} \\
 Y &= AB + \bar{A}\bar{B} = \overline{A \oplus B}
 \end{aligned}$$

(v) Ex-NOR Gate:

$$\begin{aligned}
 Y &= \overline{AB + \bar{A}\bar{B}} \\
 &= (\overline{AB}) (\overline{\bar{A}\bar{B}}) \\
 &= (\bar{A} + \bar{B}) (\bar{A} + B) \\
 &= (\bar{A} + \bar{B}) (A + B) \\
 &= \bar{A} \cdot A + \bar{A}B + A\bar{B} + B \cdot \bar{B} \\
 &= 0 + \bar{A}B + A\bar{B} + 0 \\
 Y &= \bar{A}B + A\bar{B} = A \oplus B
 \end{aligned}$$

(vi) NAND Gate:

$$Y = \overline{A \cdot B}$$

② Explain and design the logic diagram of a circuit for 4 bit parallel adder / subtractor?

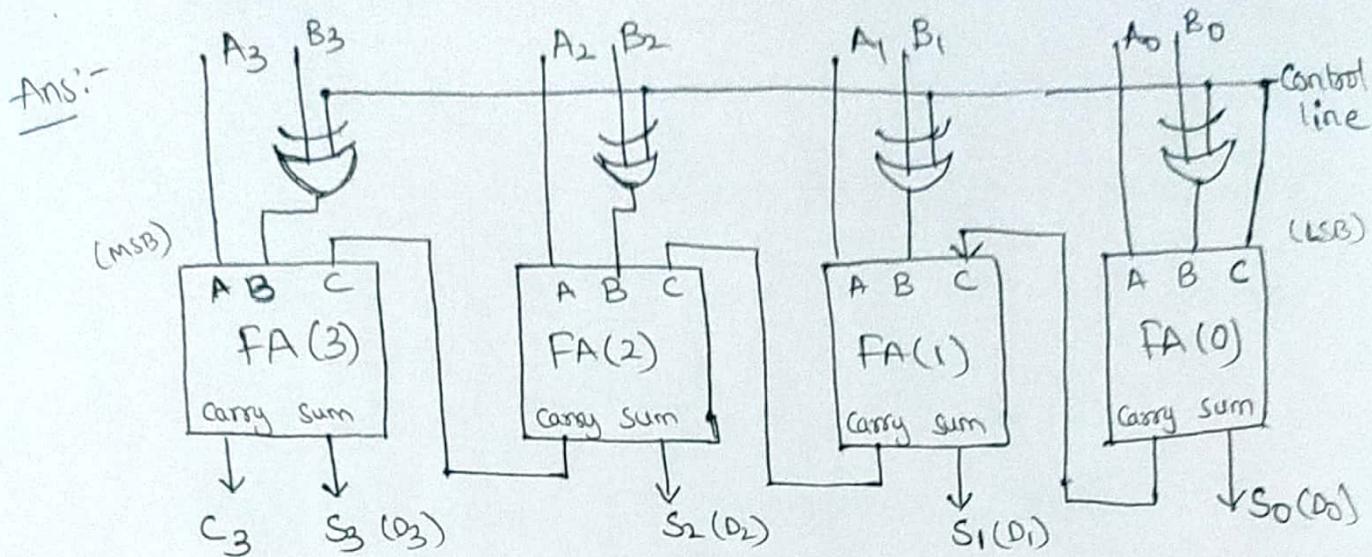


fig: 4 bit parallel adder / subtractor.

→ The 4 bit parallel binary adder / subtractor circuit is shown in above figure.

→ To perform the operation of both addition and subtraction. It required 4 full adder's (FA). Each full adder has three inputs (ie., A, B & c) and two outputs (ie., sum and carry).

→ for 4 bit parallel adder / subtractor, it has two 4 bit inputs $A_3\ A_2\ A_1\ A_0$ and $B_3\ B_2\ B_1\ B_0$. The control line, connected with C of the least significant bit (LSB) of the full adder, is used to perform the operations of addition and subtraction.

→ The Ex-OR gates are used as controlled inverters.

→ In a computer, one has to carry out the addition as well as subtraction. Hence it is more convenient to have a single circuit out both these arithmetic functions instead of two different circuits.

→ The binary subtraction can be converted into the addition by considering the 2's complement method.

→ In the circuit, when control line = 0, the circuit acts as an adder, the number $B_3 B_2 B_1 B_0$ is added to $A_3 A_2 A_1 A_0$ and gives sum = $S_3 S_2 S_1 S_0$.

When control line = 1, the circuit acts as subtractor, the number $B_3 B_2 B_1 B_0$ is subtracted from $A_3 A_2 A_1 A_0$ and gives the difference = $D_3 D_2 D_1 D_0$.

Operations:

① Adder :- When control line = 0, the output of the Ex-OR gates will be same as the B data bits. The carry input to the first full adder is 0 (i.e., Control line = 0). Hence A and B bits are applied as the two inputs to the parallel adder. Thus the output would correspond to the sum of A & B.

(+) ②

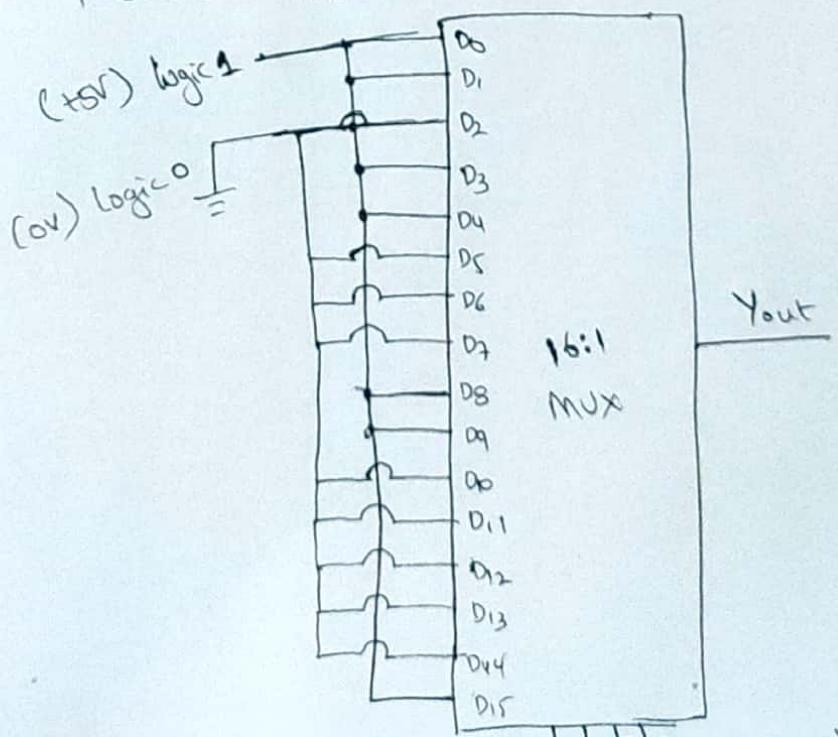
Subtractor: when control line = 1, then the output of the Ex-OR gates will be the complement of the ~~B data~~ B data bit. The carry input to the first full adder is now 1 (i.e., control line = 1) - hence addition of 1. to the 1's Complement of B bits forms the 2's Complement B bits are applied as the inputs to the parallel adder. This is the process of subtracting a binary number from another using 2's complement method.

③ Implement the logic function $f(A, B, C, D) = \sum(0, 1, 3, 4, 8, 9)$ using (i) 16:1 MUX (ii) 8:1 MUX (iii) 4:1 MUX

for (i) 16:1 MUX

$$16 = 2^4$$

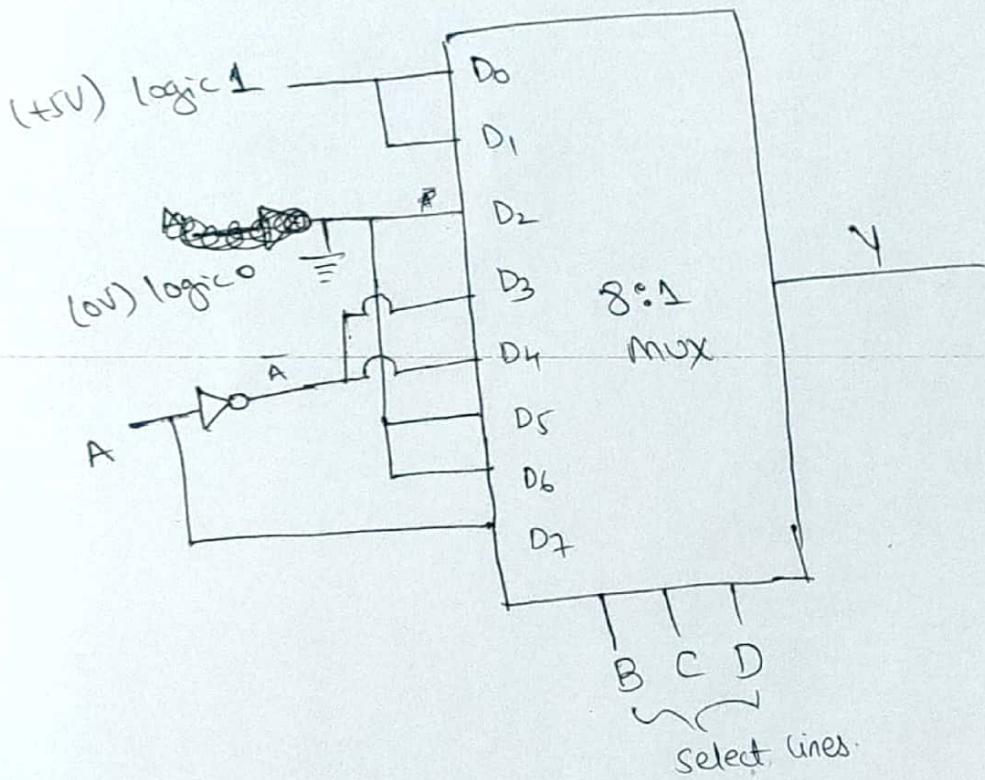
for 16:1 MUX, it has 16 inputs, one output & 4 select lines.



8:1 MUX

$BCD \rightarrow$ inputs are used for selection lines i.e.

	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
\bar{A}	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	1	1	0	\bar{A}	\bar{A}	0	0	A

4:1 MUX

$CD \rightarrow$ inputs are used for select lines.

	D_0	D_1	D_2	D_3
$\bar{A}\bar{B}$	0	1	2	3
$A\bar{B}$	4	5	6	7
$\bar{A}B$	8	9	10	11
AB	12	13	14	15
	F_1	F_2	$F_3=0$	F_4

$$\begin{aligned}
 F_1 &= \bar{A}\bar{B} + A\bar{B} + \bar{A}B \\
 &= \bar{A}(\bar{B} + B) + A\bar{B} \\
 &= \bar{A} + A\bar{B} \\
 &= (\bar{A} + A)(\bar{A} + \bar{B})
 \end{aligned}$$

$$\Rightarrow F_1 = \bar{A} + \bar{B} \equiv D_0$$

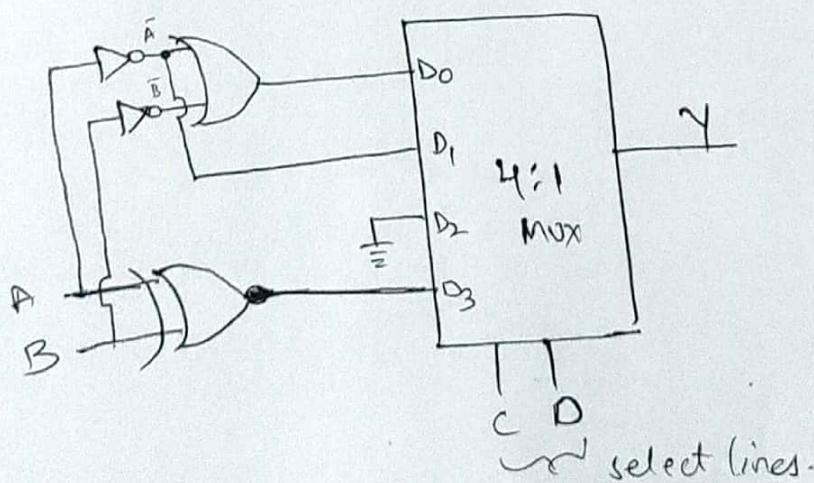
$$\begin{aligned}
 F_2 &= \bar{A}\bar{B} + \bar{A}B \\
 &= \bar{A}(\bar{B} + B)
 \end{aligned}$$

$$\Rightarrow F_2 = \bar{A} \equiv D_1$$

$$\Rightarrow F_3 = 0 \equiv D_2$$

$$F_4 = \bar{A}\bar{B} + AB$$

$$\Rightarrow F_4 = A \oplus B \equiv D_3$$



5) Explain the working of a bit Comparator.

Ans:- Comparator:-

A Comparator is a logic circuit that compares the value of two numbers. Let us assume that the bits A & B are compared. The truth table for the Comparator action is shown below.

Inputs		Outputs		
A	B	$A > B$	$A = B$	$A < B$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

Truth table for one bit Comparator.

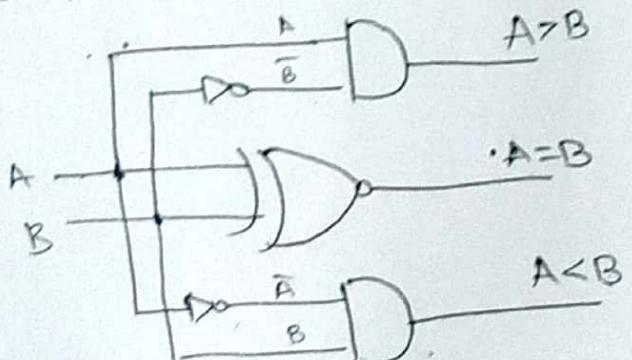
From the truth table, the expression for the outputs $A > B$, $A = B$ and $A < B$ can be written as

$$A > B \Rightarrow A \cdot \bar{B}$$

$$A = B \Rightarrow \overline{A \oplus B}$$

$$A < B \Rightarrow \bar{A} \cdot B$$

The single bit comparator can be realized by using logic gates,



Two bit Comparator:-

Let the numbers to be compared be A (A_1, A_0) and B (B_1, B_0). 

→ A will be greater than B when any one of the following conditions is satisfied.

- ① when $A_1=1, B_1=0$ irrespective of other bits
- ② If $A_1=B_1$ and $A_0=1, B_0=0$.

→ A will be equal to B when the following conditions satisfied.

- ③ $A_1=B_1, A_0=B_0$.

→ A will be less than B when any one of the following conditions is satisfied.

- ④ when $A_1=0, B_1=1$ irrespective of other bits
- ⑤ If $A_1=B_1$ and $A_0=0, B_0=1$

→ To represent these conditions in the form of a truth table, it requires 16 combinations & hence a function table is given as:

Inputs		Outputs		
$A_1 \neq B_1$	$A_0 \neq B_0$	$A > B$	$A = B$	$A < B$
$A_1 > B_1$	X	1	0	0
$A_1 < B_1$	X	0	0	1
$A_1 = B_1$	$A_0 > B_0$	1	0	0
$A_1 = B_1$	$A_0 < B_0$	0	0	1
$A_1 = B_1$	$A_0 = B_0$	0	1	0

Function table of 2 bit Comparator.

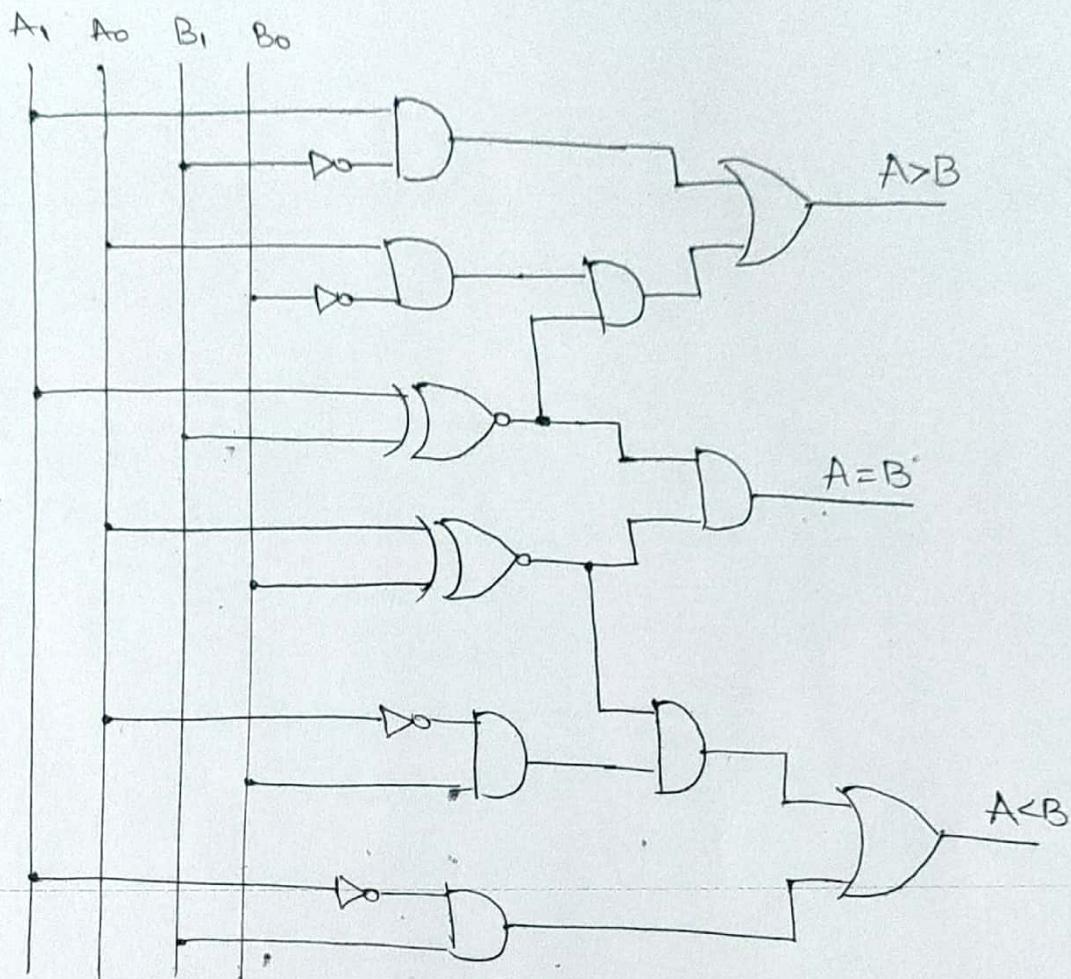
→ In the function table , a X indicates the don't care condition. From the function table , the Expressions for $A > B$, $A = B$ & $A < B$.

$$A > B \Rightarrow A_1 \bar{B}_1 + \overline{A_1 \oplus B_1} \cdot \overline{A_0 B_0}$$

$$A = B \Rightarrow \overline{A_1 \oplus B_1} \cdot \overline{A_0 \oplus B_0}$$

$$A < B \Rightarrow \overline{A_1 B_1} + \overline{A_1 \oplus B_1} \cdot \overline{A_0 B_0}$$

→ The two bit comparator can be realized by using logic gates,



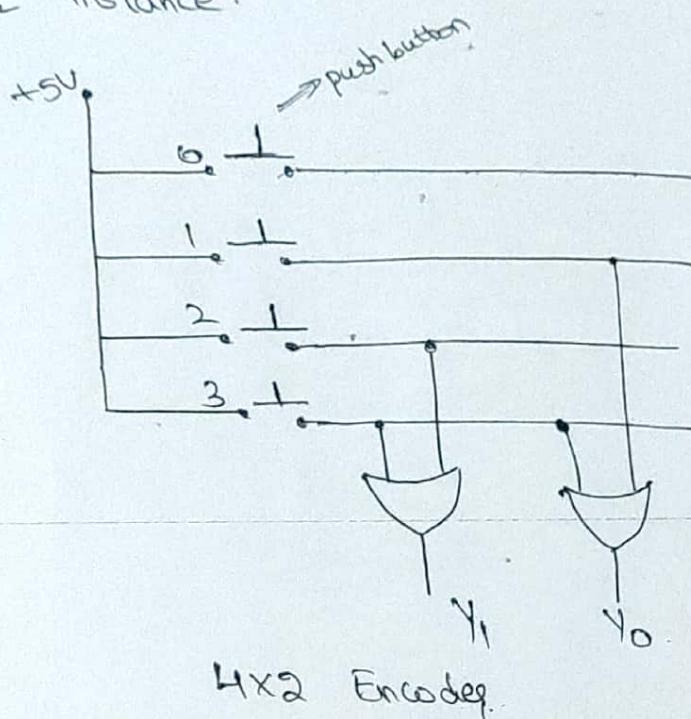
⑤ Explain about Encoders and Decoders with Suitable Examples?

Ans:- Encoder:

- The encoder circuit is used to convert one form of data into binary form.
- The input of data may be in form other than binary i.e., decimal, octal, hexadecimal etc.
- Encoders are required at the interface of man-machine communication and for transmission of data between the various digital subsystem that do not employ the same code.

→ The encoders can also be designed using combinational logic circuits.

→ The circuit diagram of 4x2 Encoder, this circuit converts decimal numbers 0, 1, 2 and 3 into binary for instance.



position of the push button (Input)	Outputs	
	Y ₁	Y ₀
0	0	0
1	0	1
2	1	0
3	1	1

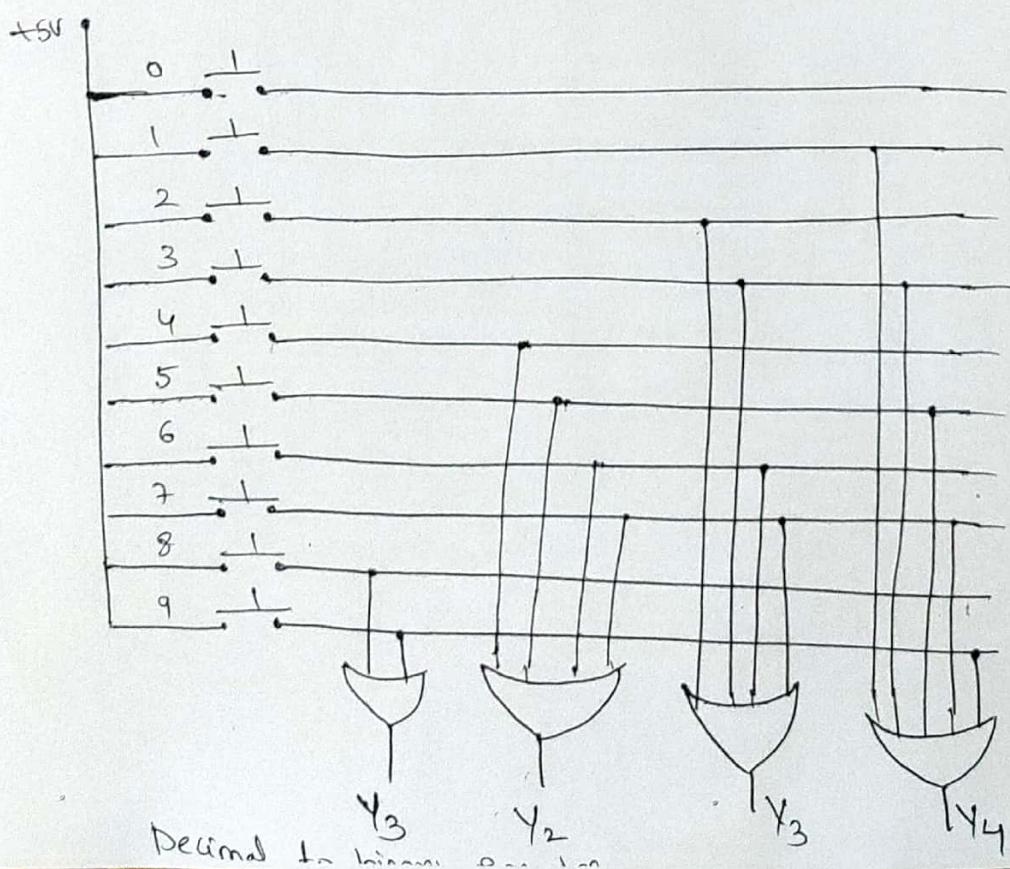
Table of 4x2 Encoder

- When push button 0 is pressed, the Y₁ and Y₀ OR gates have low outputs, therefore output word is Y₁, Y₀ = 0.
- If the button 1 is keyed, the Y₁ and Y₀ OR gates have low and high outputs, therefore the output word is Y₁, Y₀ = 01.
- Similarly when the button 2 is keyed the output word is Y₁, Y₀ = 10.
- When the button 3 is keyed the output word is Y₁, Y₀ = 11.

Decimal to binary Encoder:-

Position of the push button (input)	outputs			
	y_3	y_2	y_1	y_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

Table of Decimal to binary Encoder.



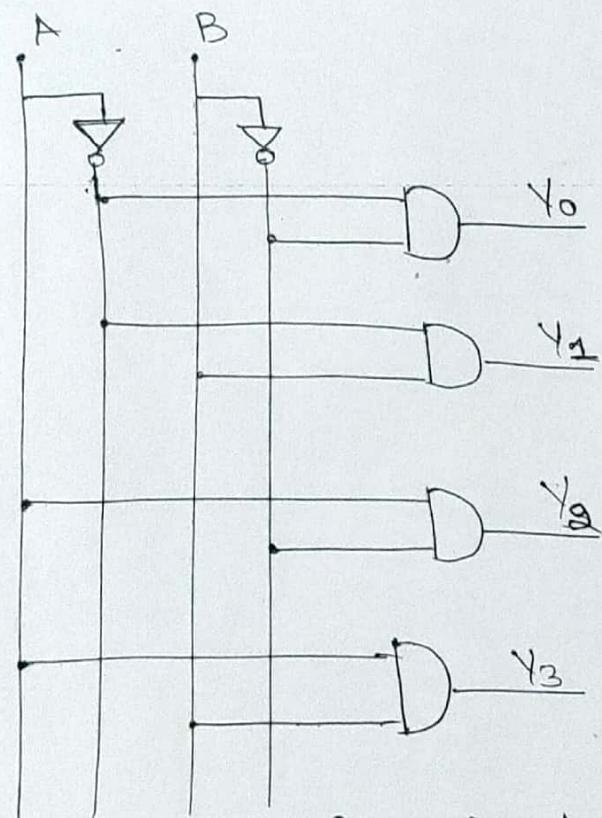
Decoders:

- The decoder is the opposite of the encoder.
- This circuit converts binary data in to other form i.e., decimal, octal, hexadecimal etc.
- Decoder is a combinational logic circuit which accepts the states from various inputs and delivers a logic 1 at any one of the 2^n outputs.
- Thus the decoder circuits of 2 input lines gives $2^2 = 4$ outputs, similarly the 3 input decoder gives $2^3 = 8$ outputs.

 2×4 Decoder: [2 inputs, 4 outputs]

- When the inputs A & B are 0 then the output of Y_0 AND gate is 1 because all inputs of the Y_0 gate is high & outputs of other AND gates are low, because atleast one input is low for these.
- for inputs A=0 & B=1, then $Y_1=1$.
- for inputs A=1 & B=0, then $Y_2=1$
- for inputs A=1 & B=1, then $Y_3=1$
- Therefore in an decoder only one output is high for one combination and other outputs low.

Inputs		Outputs			
A	B	y_3	y_2	y_1	y_0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

Table of 2×4 DecoderFig: 2×4 Decoder3x8 Decoder:

→ For instance when $A=0$, $B=0$ & $C=0$, the y_0 AND gate has all high inputs, therefore the y_0 is high. Furthermore, all other AND gates have atleast one low input as a result all other AND gates have low outputs.

Inputs			Outputs							
A	B	C	y_7	y_6	y_5	y_4	y_3	y_2	y_1	y_0
0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	1	0	0	0	0	0	0	1	0	0
0	1	1	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0
1	0	1	0	0	1	0	0	0	0	0
1	1	0	0	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

Table of 3x8 Decoder.

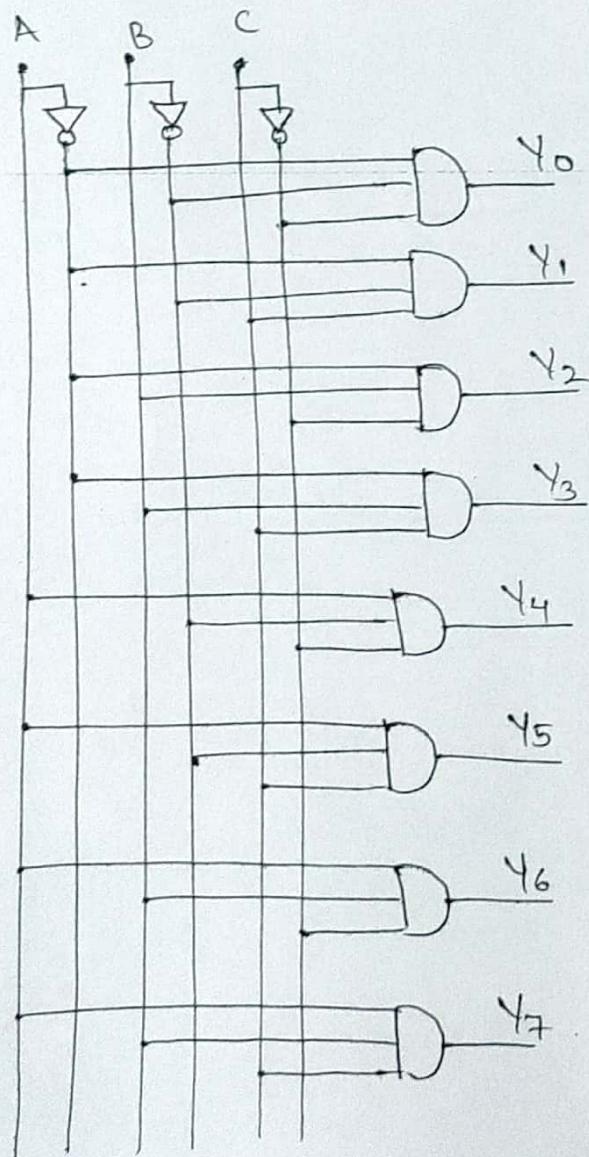


Fig: 3x8 Decoder.

⑥ @ Simplify the following function using K-map method

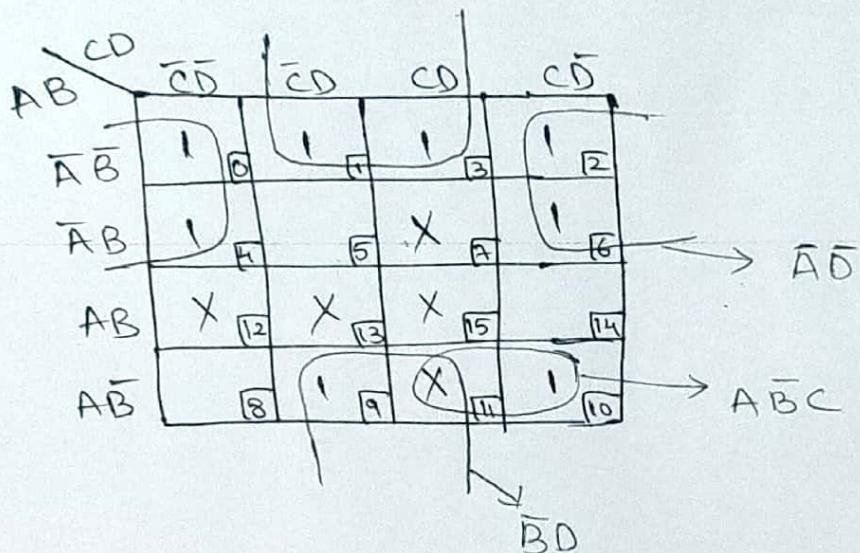
$$F(A, B, C, D) = \sum(0, 1, 2, 3, 4, 6, 9, 10) + d(7, 11, 12, 13, 15)$$

(b) Simplify the following function using K-map method

Obtain (i) minimal SOP & (ii) minimal POS Expression.

$$F(w_1, x_1, y_1, z_1) = \sum(0, 2, 3, 6, 7) + d(8, 10, 11, 15)$$

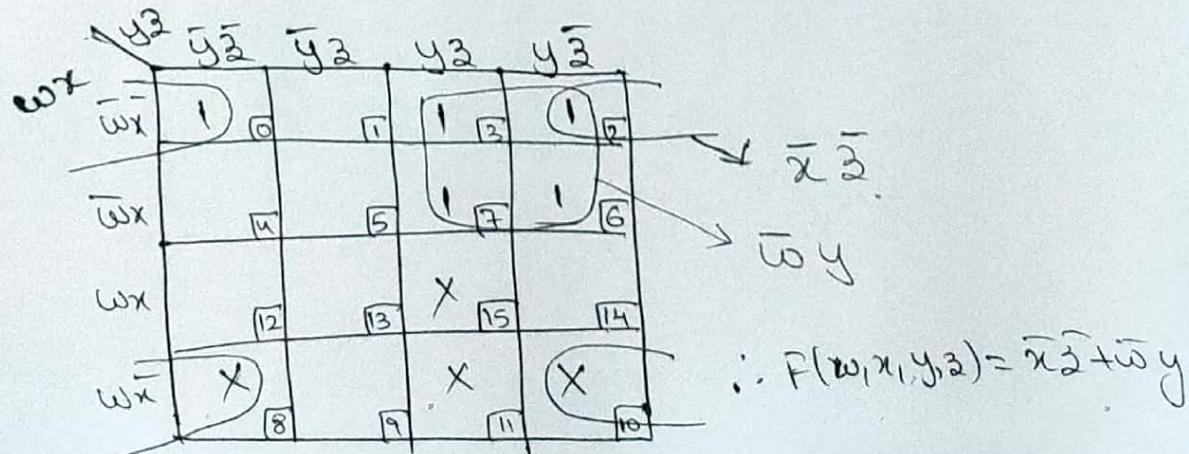
Ans: @ $F(A, B, C, D) = \sum(0, 1, 2, 3, 4, 6, 9, 10) + d(7, 11, 12, 13, 15)$



$$F(A, B, C, D) = \bar{A}\bar{D} + A\bar{B}C + \bar{B}D.$$

(b) (i) minimal SOP.

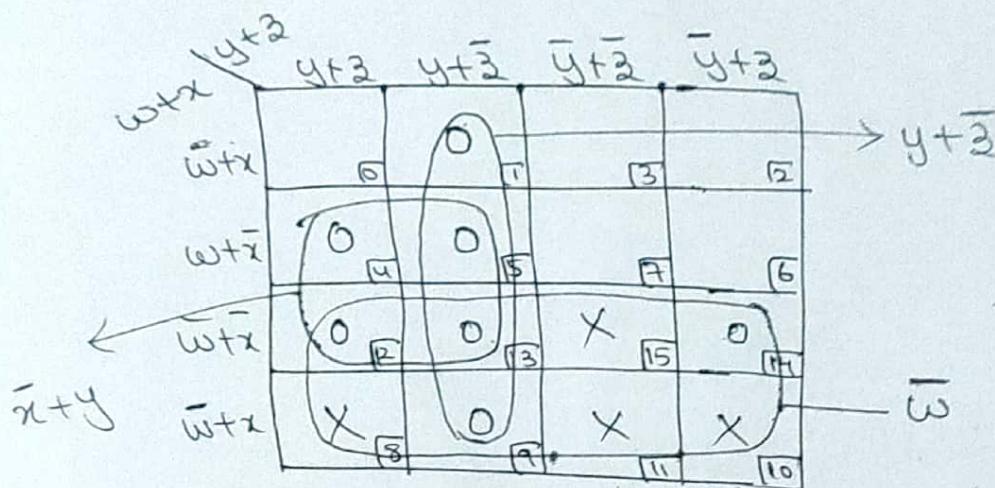
$$F(w_1, x_1, y_1, z_1) = \sum(0, 2, 3, 6, 7) + d(8, 10, 11, 15)$$



(ii) Minimal POS Expression: (Q5)

given $F(w, x, y, z) = \sum(0, 2, 3, 6, 7) + \sum(8, 10, 11, 15)$

POS $\overline{F(w, x, y, z)} = \prod(1, 4, 5, 9, 12, 13, 14) +$
 $\sum(8, 10, 11, 15).$



minimal POS $f(w, x, y, z) = (\overline{w})(\overline{y} + \overline{z})(\overline{x} + y)$

Short Answers: -

- ① @ list the properties of boolean Algebra?
- ② State Duality & DeMorgan's theorem?

Ans: @ properties of Boolean Algebra: -

→ Boolean algebra is a mathematical system consisting of a set of two (or) more distinct elements, "two binary" operators denoted by the symbols (+) & (·) and one "unary" operator denoted by the symbol either bar (-) or prime (').

① Commutative property:

$$A+B = B+A$$

$$A \cdot B = B \cdot A$$

② Associative property:

$$A+(B+C) = (A+B)+C$$

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

③ Distributive property:

$$A+BC = (A+B)(A+C)$$

$$A \cdot (B+C) = (A \cdot B) + (A \cdot C)$$

④ Complementary property:

$$A + \bar{A} = 1$$

$$A \cdot \bar{A} = 0$$

⑤ Involution property:-

$$(A')' = A \text{ (or)} \quad \bar{\bar{A}} = A$$

⑥ Idempotency property:-

$$A+A = A$$

$$A \cdot A = A$$

⑦ Absorption law:

$$A+\bar{A}B = A+B$$

⑧ Consensus law:

$$xy + \bar{x}z + yz = xy + \bar{x}z$$

↳ This law is used to eliminate redundant term from the boolean expression.

⑥ Duality Theorem:-

The duality of the function can be obtained by interchanging $\text{OR}(+)$ $\leftrightarrow \text{AND}(\cdot)$ & $\text{AND}(\cdot)$ $\leftrightarrow \text{OR}(+)$

$$\text{ex: } 0Y = x + y$$

$$Y_d \text{ (duality function)} = x \cdot y$$

$$② F = \bar{A}B + C\bar{D}$$

$$F_d = (\bar{A} + B) \cdot (C + D)$$

Demorgan's Theorem:-

→ Demorgan's 1st Theorem:

It stated as , the complement of a product of variables is equal to the sum of complement of individual variables

the variables

$$\overline{AB} = \bar{A} + \bar{B}$$

A	B	$\bar{A}\bar{B}$	$\bar{A} + \bar{B}$
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	0

(Q3)

\rightarrow Demorgan's 2nd Theorem:-

It stated as, the complement of a sum of variables is equal to the product of individual complement of the variables

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

AB	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0 0	1	1
0 1	0	0
1 0	0	0
1 1	0	0

(Q2) a) Simplify the given expression

$$Y = \overline{A}\overline{B}\overline{C} + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC$$

b) Find the complement of the expression

$$Y = ABC + ABC + \overline{A}\overline{B}C + \overline{ABC}$$

Ans: (a) $Y = \overline{A}\overline{B}\overline{C} + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC$

$$= \overline{B}\overline{C}(\overline{A}+A) + A\overline{B}C + AB(C+\overline{C})$$

$$= \overline{B}\overline{C} \cdot 1 + \overline{ABC} + AB \cdot 1$$

$$= \overline{B}\overline{C} + A\overline{B}C + AB$$

$$= \overline{B}(\overline{C} + \overline{A}C) + AB$$

$$= \overline{B}((\overline{C} + \overline{A})(\overline{C} + C)) + AB$$

$$= \overline{B}((\overline{C} + \overline{A}) \cdot 1) + AB$$

$$= \overline{B}(\overline{C} + \overline{A}) + AB$$

$$= \overline{B}\overline{C} + \overline{A}\overline{B} + AB$$

$$= \overline{B}\overline{C} + A(B + \overline{B})$$

$$Y = A + \overline{B}\overline{C}$$

(24)

$$\textcircled{b} \quad Y = ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C$$

$$\bar{Y} = \overline{ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C}$$

$$= \overline{AB(C+\bar{C}) + \bar{A}C(\bar{B}+B)}$$

$$= \overline{AB \cdot 1 + \bar{A}C \cdot 1}$$

$$= \overline{AB + \bar{A}C}$$

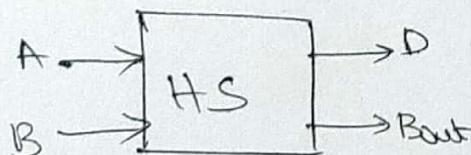
$$= (\overline{AB})(\overline{\bar{A}C})$$

$$\bar{Y} = (\bar{A} + \bar{B})(A + \bar{C}) //$$

\textcircled{3} what is a half-subtractor? write its truth table?

Ans: The half subtractor is a combinational circuit which is used to perform subtraction of two bits.

→ It has two inputs, A (minuend) and B (subtrahend) and two outputs D (difference) & Bout (Borrow out).



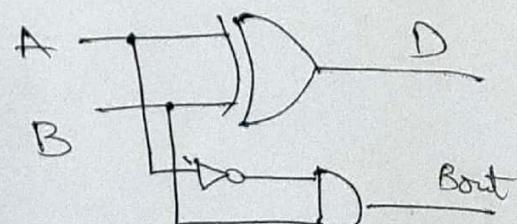
@ logic symbol

Inputs		outputs	
A	B	D	Bout
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Truth table

$$D = A \oplus B$$

$$\text{Bout} = \bar{A}B$$



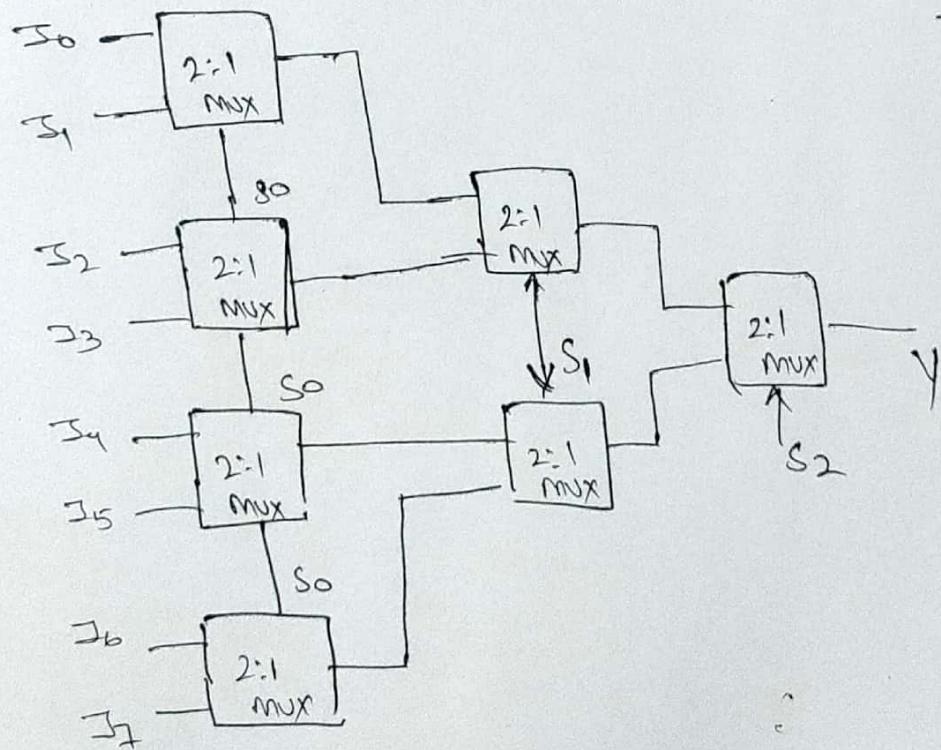
logic diagram

④ Difference between Serial and parallel Adder?

Ans:-

Serial Adder	parallel Adder
① It is slower	① It is faster
② only one bit can add at a time	② All the bits can be added simultaneously
③ only one full adder is required irrespective of number of bits	③ Number of full adders required is equal to number of bits
④ Economical	④ Costly.

⑤ Design 8:1 MUX by using 2:1 MUX?



$$\frac{8}{2} = 4 \rightarrow 1^{\text{st}} \text{ stage}$$

$$\frac{4}{2} = 2 \rightarrow 2^{\text{nd}} \text{ stage}$$

$$\frac{2}{2} = 1 \rightarrow 3^{\text{rd}} \text{ stage}$$

Q) List the applications of Encoders & decoders.

Ans: Applications:

Encoder: These are used to convert one form of data into binary. These are used as

- ① Decimal to Binary converters (BCD)
- ② Octal to Binary converters
- ③ Hexadecimal to Binary converters
- ④ used to drive 7 segment displays.

Decoders: These are used to convert binary data in to other form. These are used as

- ① BCD to decimal conversion
- ② Binary to octal conversion
- ③ Hexadecimal to Binary conversion
- ④ BCD to gray (or) gray to BCD conversion.