mood-book

PROBABILITY

If an experiment is conducted any no. of times under essential and identical conditions, there is a set of possible outcomes associated with it. If the result is not certain in any one of the possible outcomes is called '<u>Random Experiment</u>' or '<u>Random trial</u>'.

<u>Ex</u>: Tossing of coins, throwing a dice and playing cards etc...

→ The outcomes of an random trial are known as '<u>elementary</u> <u>events</u>'.

→ The set of outcomes of a trial is an '<u>event</u>'.

→ every elementary event is an event.

Equally likely Events:

Events are said to be equally likely when there is no reason to expect any one of them rather than any one of the others.

<u>Ex</u>: When a card is drawn from a pack any card may be obtained. In this trial, all 52 elementary events are

equally likely.

Exhaustive Events:

All possible events in any trial is known as exhaustive events.

Ex: In throwing a dice getting 1 or 2 or 3 or 4 or 5 or 6. These 6 events are called exhaustive elementary events.

Mutually Exclusive events:

Events are said to be mutually exclusive if no two or more events can happen simultaneously in the same trial.

Classical Definition of probability:

In a random trial, let there be 'n' mutually exclusive and equally likely elementary events and 'E' be happening of an event with 'm' elementary events of an event E. Hence, the probability of an event E is defined as

$$P(E) = \frac{\text{No. of favourable events in } E}{\text{Total no. of events of an event } E}$$

$$\boxed{P(E) = \frac{m}{n}}$$

→ $\bar{E}$ denotes the event of non occurance of E.

The no. of elementary events in $\bar{E}$ is

$$\boxed{\bar{E} = n-m}$$

Therefore, $P(\bar{E}) = \frac{n-m}{n}$

$$P(\bar{E}) = 1 - \frac{m}{n}$$

$$P(\bar{E}) = 1 - P(E)$$

$$\boxed{P(\bar{E}) + P(E) = 1}$$

$\therefore 0 \leq P(E) \leq 1$

probability lies in between 0 & 1

P(E) is the probability of happening of an event E is known as probability of success.

$P(\bar{E})$ of non happening of the event is known as probability of failure.

P(E) = 1 is called certain Event.

P(E) = 0 is known as impossible event.

Sample space:

The set of all possible outcomes of a trial are known as sample space.

- It is denoted by 'S'
- Every element of a sample S is known as sample point
- Every subset of a sample space S is known as event

Ex: Tossing a coin the sample space

$S = \{H, T\}$

Throwing a dice, Then

$S = \{1, 2, 3, 4, 5, 6\}$

NOTE:

→ Two events A, B in a sample space S are said to be mutually exclusive or disjoint. If $A \cap B = \phi$

→ Two events A, B in a sample space S are said to be exhaustive events. If $A \cup B = S$

→ Two events A, B are said to be complementary events if $A \cap B = \phi$, $A \cup B = S$

→ If A, B are two events in S, if $A \subseteq B$ then
$$P(A) \leq P(B)$$

Simple event:
An event in a trial further cannot be split is known as simple event/ elementary event

A class consist of 6 girls and 10 boys. If a commitee of 3 is choosen at random from the class. Find the probability that
i) 3 boys are selected
ii) Exactly 2 girls are selected.

Total no. of students in a class $= 10+6 = 16$

$n(S) =$ no. of ways of choosing 3 from 16
$= 16C_3$

i) 3 boys are selected

$n(A) =$ no. of ways of choosing 3 boys out of 10
$= 10C_3$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{10C_3}{16C_3} = \frac{\frac{10 \times 9 \times 8}{3 \times 2 \times 1}}{\frac{16 \times 15 \times 14}{3 \times 2 \times 1}}$$

$$= \frac{3}{14}$$

iii) Exactly 2 girls are selected

$n(A) = 6C_2 \times 10C_1$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{6C_2 \times 10}{16C_3}$

$= \frac{\frac{6\times5}{2\times1} \times 10}{\frac{16\times15\times14}{3\times2\times1}}$

$= 0.26$

Two dice a thrown find the probability of getting the same number on both the faces.

$n(S)$ = Total no. of possible cases $= 6^2 = 36$

$n(A)$ = favourable cases of getting same number on both the faces.

$P(E) = \frac{n(E)}{n(S)}$

$n(E) = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$= 6$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{6}{36} = \frac{1}{6}$

Determine the probability for a non-defective bolt will be found out of 600 bolts already examine 12 are defective.

The probability of defective bolt $P(D) = \frac{n(D)}{n(S)}$

$= \frac{12}{600}$

The probability of finding a non-defective bolt $P(\bar{D}) = 1 - P(D)$

$= 1 - \frac{12}{600}$

$= \frac{600-12}{600}$

$= \frac{588}{600} = \frac{49}{50}$

$= 0.98$

A, B, and C in order toss a coin the 1st one to toss head wins the game. What are the probabilities of winning assuming that the game may continue indefinetly.

$P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

'A' may win in the 1st round with probability $1/2$

He may win in the 2nd rand after all of them are failed in 1st round with the probability $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$

In the 3rd round the probability is

$= (\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$

$= (\frac{1}{2})^6 (\frac{1}{2})$

The chance of A's success is

$= \frac{1}{2} + (\frac{1}{2})^3 (\frac{1}{2}) + (\frac{1}{2})^6 (\frac{1}{2}) + \dots$

$= \frac{1}{2}\left[1 + (\frac{1}{2})^3 + (\frac{1}{2})^6 + \dots\right]$ (the series in G.P)

$= \frac{1}{2}\left[\frac{1}{1-(\frac{1}{2})^3}\right]$ $a = 1$, $r = (\frac{1}{2})^3$

$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{8}}\right]$

$= \frac{1}{2}\left[\frac{1}{7/8}\right]$

$= \frac{1}{2} \times \frac{8}{7} = \frac{4}{7}$

B may win the first round after A's failure with probability $(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

second Round probability is

$(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = (\frac{1}{2})^3 (\frac{1}{4})$

3rd round probability is $(\frac{1}{2})^6 (\frac{1}{4})$

B chance of winning is

$\frac{1}{4}+\left(\frac{1}{2}\right)^3\left(\frac{1}{4}\right)+\left(\frac{1}{2}\right)^6\left(\frac{1}{4}\right)+\cdots$

$\frac{1}{4}\left[1+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^6+\cdots\right]$

$a=1,\ r=(1/2)^3$

$\frac{1}{4}\left[\frac{1}{1-\left(\frac{1}{2}\right)^3}\right]$

$\frac{1}{4}\left[\frac{1}{1-\frac{1}{8}}\right]$

$=\frac{1}{4}\left(\frac{8}{7}\right)=\frac{2}{7}$

C may win the first round after A's & B's failure with probability $=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{8}$

2nd round probability is $\left(\frac{1}{2}\right)^3\left(\frac{1}{8}\right)$

3rd round probability is $\left(\frac{1}{2}\right)^6\left(\frac{1}{8}\right)$

C chance of winning is

$\frac{1}{8}+\left(\frac{1}{2}\right)^3\left(\frac{1}{8}\right)+\left(\frac{1}{2}\right)^6\left(\frac{1}{8}\right)+\cdots$

$\frac{1}{8}\left[1+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^6+\cdots\right]$ (or)

$a=1,\ r=(1/2)^3$

$\frac{1}{8}\left[\frac{1}{1-\left(\frac{1}{2}\right)^3}\right]$

$\frac{1}{8}\left(\frac{8}{7}\right)=\frac{1}{7}$

$P(A)+P(B)+P(C)=1$

$\frac{4}{7}+\frac{2}{7}+P(C)=1$

$P(C)=1-\frac{6}{7}$

$=\frac{1}{7}$

∴ A's chance of winning is $P(A) = \frac{4}{7}$

B's chance of winning is $P(B) = \frac{2}{7}$

C's chance of winning is $P(C) = \frac{1}{7}$

A & B throw alternatively with a pair of ordinary dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is $\frac{30}{61}$.

(chance of A's winning) When two dice are thrown at a sametime $n(S) = 6^2 = 36$

The probability of A throwing 6

$\{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$$P(A) = \frac{5}{36}$$

The probability of A not throwing 6

$$P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{5}{36}$$

$$= \frac{31}{36}$$

The probability of B throwing 7

$\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$P(\bar{B}) = 1 - P(B)$
$= 1 - \frac{1}{6}$
$= \frac{5}{6}$

A's chance of winning

$P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots$

$\frac{5}{36} + \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{5}{36} + \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{5}{36} + \ldots$

$\frac{5}{36}\left[1 + \frac{31}{36} \cdot \frac{5}{6} + \left(\frac{31}{36} \cdot \frac{5}{2}\right)^2 + \ldots\right]$

$= \frac{5}{36}\left[\frac{1}{1-\left(\frac{31}{36} \cdot \frac{5}{6}\right)}\right] = \frac{5}{36} \times \frac{216}{61}$

$= \frac{30}{61}$

B's chance of winning

$P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(\bar{A})P(B) + \ldots$

$\frac{31}{36} \cdot \frac{1}{6} + \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{31}{36} \cdot \frac{1}{6} + \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{31}{36} \cdot \frac{5}{6} \cdot \frac{1}{6}$

$\frac{31}{36} \cdot \frac{1}{6}\left[1 + \frac{31}{36} \cdot \frac{5}{6} + \left(\frac{31}{36} \cdot \frac{5}{6}\right)^2 + \ldots\right]$

$\frac{31}{36} \cdot \frac{1}{6}\left[\frac{1}{1-\left(\frac{31}{36} \cdot \frac{5}{6}\right)}\right]$

$$\frac{31}{36}\cdot\frac{1}{6}\left[\frac{1}{1-\frac{155}{216}}\right]$$

$$\frac{31}{36}\cdot\frac{1}{6}\left[\frac{216}{61}\right]$$

$$=\frac{31}{61}$$

Axioms of probability:

- Axiom of positivity: $P(E)\geq 0$ for every subset $E(S)$
- Axiom of certainity $P(S)=1$
- Axiom of union: If E_1, E_2 are two disjoint subsets of S, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

What is the probability that a card drawn at random from the pack of playing cards may be either a queen or a king.

Let S be the sample space associated with a drawing of a card

$$\therefore n(S) = 52C_1 = 52$$

Let E_1 be the event of the card drawn being a queen

$$n(E_1) = 4C_1 = 4$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{52}$$

let E_2 be the event of the card drawn being a king.

$n(E_2) = 4C_1 = 4$

$P(E_2) = \frac{n(E_2)}{n(S)}$

$= \frac{4}{52}$

But E_1 & E_2 are mutually exclusive events.

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$= \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52}$

out of 15 items 4 are not in good condition 4 are selected at random. Find the probability that

i) all are not good

ii) 2 are not good.

Let S be the total no. of items

$n(S) = 15C_4$

i) Let E_1 be the event of picking 4 bad items in 4 ~~cards~~ items $= 4C_4$

$n(E_1) = 4C_4 = 1$

$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}}$

$= \frac{1}{1365}$

ii) Two are not good

let E_2 be the event containing 2 are not good items and other 2 are good

$$n(E_2) = 4C_2 \times 11C_2$$

$$P(E_2) = \frac{n(E_2)}{n(S)}$$

$$= \frac{4C_2 \times 11C_2}{15C_4}$$

$$= \frac{\frac{4\times3}{2\times1} \times \frac{11\times10}{2\times1}}{\frac{15\times14\times13\times12}{4\times3\times2\times1}}$$

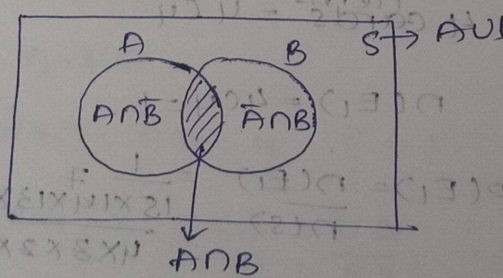
$$= \frac{6\times11\times5}{15\times7\times13} = \frac{22}{91}$$

Addition theorm of probability:

Statement: Let A, B be the events of a sample space S then

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

venn diagram:



Case 1: $A \cap B \neq \phi$

$A \cup B = (A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$

$A \cup B = A \cup (\bar{A} \cap B)$

Taking probability on both sides

$P(A \cup B) = P(A \cup (\bar{A} \cap B))$

$P(A \cup B) = P(A) + P(\bar{A} \cap B)$ — ①

Again from venn diagram

$B = (A \cap B) \cup (\bar{A} \cap B)$

Taking probability on both sides

$P(B) = P((A \cap B) \cup (\bar{A} \cap B))$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$P(B) - P(A \cap B) = P(\bar{A} \cap B)$ — ②

Substituting ② in ①

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Case 2: If $A \cap B = \phi$

$P(A \cup B) = P(A) + P(B) - 0$

$= P(A) + P(B) - P(\phi)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

A card is drawn from a well shuffled pack of cards. What is the probability that it is either a spade or an ace

Total number of cards in a pack = 52

$\therefore n(S) = 52$

Let E_1 be the event of getting a spade

$\therefore n(E_1) = 13C_1$

$n(S) = 52C_1$

probability of getting a spade $P(E_1) = \frac{n(E_1)}{n(S)}$

$= \frac{13C_1}{52C_1}$

$= \frac{13}{52}$.

E_2 be the event of getting an ace

$n(E_2) = 4C_1$

$\therefore$ Probability of getting an ace is $P(E_2) = \frac{n(E_2)}{n(S)}$

$= \frac{4C_1}{52C_1} = \frac{4}{52}$

The probability of event of getting both spade and an ace

$P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1C_1}{52C_1} = \frac{1}{52}$

By addition theorm of probability :

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$

3 students A, B, C are in running race A & B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins.

Given, A and B has same probability of winning $P(A) = P(B)$

each is twice as likely to win as C i.e

$P(A) = 2P(C)$

$P(B) = 2P(C)$

By axiom of certainity $P(S) = 1$

$\therefore P(A) + P(B) + P(C) = 1$

$2P(C) + 2P(C) + P(C) = 1$

$5P(C) = 1$

$P(C) = \frac{1}{5}$

$P(A) = 2P(C)$

$= 2\left(\frac{1}{5}\right) = \frac{2}{5}$

$P(B) = 2P(C)$

$= 2\left(\frac{1}{5}\right) = \frac{2}{5}$

$P(A) = P(B) = \frac{2}{5}$

$P(C) = 1/5$

By addition theorm.

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= \frac{2}{5} + \frac{1}{5} - 0$

$= \frac{3}{5}$

$\therefore$ P(either B or C wins) $= 3/5$

conditional event:

If E_1, E_2 are events of samples then E_2 occurs after the occurance of E_1 i.e event of occurance of E_2 after event E_1 is called conditional event of E_2 given E_1.

It is denoted by E_2/E_1

Ex: when a pair of coins are tossed the event of getting two tails given that there is atleast one tail is a conditional event

conditional probability:

If E_1, E_2 are two events in a sample space S with $P(E_1) \neq 0$. Then the probability of event E_2 after the event E_1 has occured is called conditional probability of E_2 given E_1

It is denoted & defined as $P\left(\frac{E_2}{E_1}\right)$ (or) $P(E_2/E_1)$

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

similarly, $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

$P(E_2) \neq 0$

counting sample point:

If an operation is performed in n_1 ways and if for each of these a 2nd operation can be performed n_2 ways. Then the two operations can be performed together in $n_1 \cdot n_2$ ways

Eg: when a pair of dice is thrown once the 1st dice can land in any one of n_1 ways $= 6$ ways

for each of these 6 ways the 2nd dice can also land in $n_2 = 6$ ways

$\therefore$ The pair of dice land in $n_1 n_2 = 6 \cdot 6 = 36$ ways

Multiplication theorm of probability:

In a random experiment if E_1, E_2 are two events of a sample space $S \ni P(E_1) \neq 0$, $P(E_2) \neq 0$ Then

i) $P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$

ii) $P(E_1 \cap E_2) = P(E_2) \cdot P\left(\frac{E_1}{E_2}\right)$

Proof: E_1, E_2 are two events of a sample space $S \ni P(E_1) \neq 0$, $P(E_2) \neq 0$

$P(E_1) \neq 0$, By definition of conditional probability of E_2 given E_1

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$\boxed{P(E_1) P\left(\frac{E_2}{E_1}\right) = P(E_1 \cap E_2)}$$

Similarly $P(E_2) \neq 0$ then by def of conditional probability of E_1 given E_2

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$P(E_2) \cdot P\left(\frac{E_1}{E_2}\right) = P(E_1 \cap E_2)$$

$$\boxed{\therefore P(E_1 \cap E_2) = P(E_2) P\left(\frac{E_1}{E_2}\right)}$$

Independent Events:

Two events are said to be independent if the result of the 2nd event is not affected by the result of the 1st event i.e. A and B are said to be independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

dependent Events:

Two events A and B are said to be dependent if the result of the 1st event affects the result of the 2nd event i.e

$P(A \cap B) \neq P(A) \cdot P(B)$

compound Event:

An event having more than one outcome is known as compound event

Find the probability of drawing two red balls in succession ~~one~~ from a bag containing 4 red, 5 black balls. when the ball that is drawn ~~first is~~

(i) not replaced (ii) replaced.

Let E_1 be the event of drawing a red ball in the 1st draw and E_2 be the event of drawing a red ball in the 2nd draw.

i) After the 1st draw the ball is not replaced

$P(E_1) = \frac{4C_1}{9C_1} = \frac{4}{9}$

$P(E_2/E_1) = \frac{3C_1}{8C_1} = \frac{3}{8}$

$\therefore P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$

$= \frac{4}{9} \cdot \frac{3}{8} = \frac{1}{6}$

ii) suppose the ball is replaced after 1st draw

Then

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$$

$$= \frac{4C_1}{9C_1} \times \frac{4C_1}{9C_1}$$

$$= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

Determine $P(B/A)$

ii) $P(A/B^c)$, If A and B are events with $P(A) = 1/3$, $P(B) = 1/4$, $P(A \cup B) = 1/2$

i) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

By addition theorm

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{2}$$

$$= \frac{4+3}{12} - \frac{1}{2}$$

$$= \frac{7}{12} - \frac{1}{2} = \frac{1}{12}$$

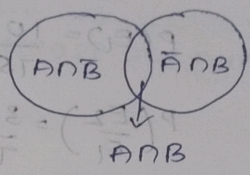
$$P(B/A) = \frac{1}{12} \times \frac{3}{1}$$

$$= \frac{1}{4}$$

ii) $P\left(\frac{A}{\bar{B}}\right) = \frac{P(A\cap\bar{B})}{P(\bar{B})}$

$P(\bar{B}) = 1 - P(B)$

$P(\bar{B}) = 1 - \frac{1}{4} = \frac{3}{4}$



$P(A\cap\bar{B}) = P(A) - P(A\cap B)$

$= \frac{1}{3} - \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$

$P\left(\frac{A}{\bar{B}}\right) = \frac{1/4}{3/4} = \frac{1}{3}$

2) Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles with replacement being made after each draw. Find the probability that

i) Both are white.

ii) 1st is red and 2nd is white.

Let E_1 be the event of drawing a white ball in the 1st draw and E_2 be the event of drawing a white ball in the 2nd draw.

i) Both are white

$P(E_1) = \frac{30C_1}{75C_1} = \frac{30}{75}$

$P\left(\frac{E_2}{E_1}\right) = P(E_1)\ \frac{30C_1}{75C_1} = \frac{30}{75}$

$P(E_1\cap E_2) = P(E_1)\cdot P\left(\frac{E_2}{E_1}\right)$

$= \frac{30}{75}\times\frac{30}{75} = \frac{4}{25}$

ii) 1st is red and 2nd is white. Then

$P(E_1) = \frac{104}{754} = \frac{10}{75}$

$P\left(\frac{E_2}{E_1}\right) = \frac{304}{754} = \frac{30}{75}$

$P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$

$= \frac{10}{75} \cdot \frac{30}{75}$

$= \frac{4}{75}$

Two dice are thrown let A be the event that the sum of the points on the faces is 9. let B be the event that atleast one number is 6. Find

i) $P(A \cap B)$

ii) $P(A \cup B)$

iii) $P(\bar{A} \cup \bar{B})$

When two dice are thrown at a same time The total no. of possible outcomes is $n(S) = 6^2$
$= 36$

Let A be the event of getting sum of the coins on faces is 9

∴ no. of possibilities are $A = \{(3,6), (6,3), (5,4), (4,5)\} = 4$

$P(A) = \frac{m}{n} = \frac{4}{36}$

B be the event of getting atleast one number is 6.

∴ no. of possibilities are

$B = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,1), (6,2), (6,3), (6,4), (6,5)\}$

$P(B) = \frac{11}{36}$

$A \cap B = \{(3,6), (6,3)\}$

i) $P(A \cap B) = \frac{2}{36}$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{36} + \frac{11}{36} - \frac{2}{36}$

$= \frac{15-2}{36} = \frac{13}{36}$

iii) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

$= 1 - P(A \cap B)$

$= 1 - \frac{2}{36}$

$= \frac{36-2}{36} = \frac{34}{36}$

Baye's theorm:-

$E_1, E_2, E_3 \ldots E_n$ are 'n' mutually exclusive and exhaustive events $\ni P(E_i) > 0$ in a sample space S 'A' is any event of S intersecting every $E_i \ni P(A) > 0$. If E_i is any of events $E_1, E_2 \ldots E_n$ where $P(E_1), P(E_2) \ldots, P(E_n), P(A/E_1), P(A/E_2) \ldots P(A/E_n)$ are known then

$$P(E_k/A) = \frac{P(E_k)P(A/E_k)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

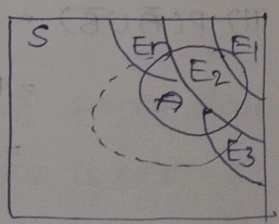
<u>proof</u>:-

$E_1, E_2 \ldots E_n$ are mutually exclusive events of a sample space S.

$$E_i \cap E_j = \phi \text{ for } i \neq j$$

$E_1, E_2 \ldots E_n$ are exhaustive events in S

$E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$



$A = A \cap S$

$A = A \cap (E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n)$

$A = (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \ldots \cup (A \cap E_n)$

(Distributive law)

Taking probability on both sides

$P(A) = P[(A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cdots \cup (A \cap E_n)]$

$P(A) = P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + \cdots + P(A \cap E_n)$

[$\because A \cap E_1, A \cap E_2, A \cap E_3, \cdots A \cap E_n$ are mutually exclusive]

$P(A) = P(E_1) P(A/E_1) + P(E_2) P(A/E_2) + P(E_3) P(A/E_3) + \cdots + P(E_n) P(A/E_n)$

[By Multiplication theorm]

By definition of conditional probability

$$P(E_k/A) = \frac{P(E_k \cap A)}{P(A)}$$

$$= \frac{P(E_k) P(A/E_k)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2) + \cdots + P(E_n) P(A/E_n)}$$

$$P(E_k/A) = \frac{P(E_k) P(A/E_k)}{\sum_{i=1}^{n} P(E_i) P(A/E_i)}$$

In a bolt factory machines A, B, C manufactu.. 20%, 30% and 50% of the total of their output and 6%, 3%, 2% are defective. A bolt is drawn at random found to be defective. Find the probabilities that it is manufactured from

i) Machine A

ii) Machine B

iii) Machine C

P(A), P(B), P(C) be the probabilities of events that bolts are manufactured by machines by A, B, C respectively. Then

$$P(A) = 20\% = \frac{20}{100} = \frac{1}{5}$$

$$P(B) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P(C) = 50\% = \frac{50}{100} = \frac{1}{2}$$

let D denote the bolt is defective then

$$P(D/A) = 6\% = \frac{6}{100}$$

$$P(D/B) = 3\% = \frac{3}{100}$$

$$P(D/C) = 2\% = \frac{2}{100}$$

i) P(Machine A)

(The bolt is defective, the probability that is from machine A)

$$P(A/D) = \frac{P(A)\cdot P(D/A)}{P(A)P(\frac{D}{A}) + P(B)P(\frac{D}{B}) + P(C)P(\frac{D}{C})}$$

$$= \frac{\frac{1}{5}\times\frac{6}{100}}{\frac{1}{5}\times\frac{6}{100} + \frac{3}{10}\times\frac{3}{100} + \frac{1}{2}\times\frac{2}{100}}$$

$$= \frac{12}{31}$$

ii) P(machine B)

The bolt is defective, the probability that is from machine B

$$P(B/D) = \frac{P(B)\cdot P(D/B)}{P(A)\cdot P(D/A) + P(B)\cdot P(D/B) + P(C)\cdot P(D/C)}$$

$$= \frac{\frac{3}{10}\times\frac{3}{100}}{\frac{1}{5}\times\frac{6}{100} + \frac{3}{10}\times\frac{3}{100} + \frac{1}{2}\times\frac{2}{100}}$$

$$= \frac{9}{31}$$

iii) P(machine C)

(The bolt is defective, the probability that is from machine C)

$$P(C/D) = \frac{P(C)\cdot P(D/C)}{P(A)\cdot P(D/A) + P(B)\cdot P(D/B) + P(C)\cdot P(D/C)}$$

$$= \frac{\frac{1}{2} \times \frac{2}{100}}{\frac{1}{5} \times \frac{6}{100} + \frac{3}{10} \times \frac{3}{100} + \frac{1}{2} \times \frac{2}{100}}$$

$$= \frac{10}{31}$$

A bussinessman goes to hotels X, Y, Z 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbings. what is the probability that businessman room having faulty plumbing is assigned to hotel Z.

P(X), P(Y), P(Z) be the probabilities of the event that busenessman goes to hotel by X, Y, Z respectively.

$$P(X) = \frac{20}{100} = \frac{1}{5}$$

$$P(Y) = \frac{50}{100} = \frac{1}{2}$$

$$P(Z) = \frac{30}{100} = \frac{3}{10}$$

Let 'E' be the faulty plumbings then

$$P(E/X) = \frac{5}{100} = \frac{1}{20}$$

$$P(E/Y) = \frac{4}{100} = \frac{1}{25}$$

$$P(E/Z) = \frac{8}{100}$$

probability of faulty plumbing assigned to hotel Z is

$$P(Z/E) = \frac{P(Z)\cdot P(\frac{E}{Z})}{P(X)\cdot P(\frac{E}{X}) + P(Y)\cdot P(\frac{E}{Y}) + P(Z)\cdot P(\frac{E}{Z})}$$

$$= \frac{\frac{3}{10} \times \frac{8}{100}}{\frac{1}{5}\times\frac{1}{20} + \frac{1}{2}\times\frac{1}{25} + \frac{3}{10}\times\frac{8}{100}}$$

$$P(Z/E) = \frac{8}{18} \quad \frac{4}{9}$$

Three machines I, II, III produce 40%, 30%, 30% of the total no. of items of the factory. The percentage of defective items of these machines are 4%, 2%, 3%. If an item is selected at random. Find the probability that the item is defective.

P(I), P(II), P(III) be the probabilities of items of the factory produced from machines I, II, III

$$P(I) = \frac{40}{100} = \frac{4}{10}$$

$$P(II) = \frac{30}{100} = \frac{3}{10}$$

$$P(III) = \frac{30}{100} = \frac{3}{10}$$

Let D denote the defective items then

$P(D/I) = \frac{4}{100} = \frac{1}{25}$

$P(D/II) = \frac{2}{100} = \frac{1}{50}$

$P(D/III) = \frac{3}{100}$

$P(D) = P(I).P(D/I) + P(II).P(D/II) + P(III).P(D/III)$

$= \frac{4}{10} \times \frac{1}{25} + \frac{3}{10} \times \frac{1}{50} + \frac{3}{10} \times \frac{3}{100}$

$= \frac{31}{1000}$

In a group consisting of equal no. of men and women; 10% of men and 45% of women are unemployed. If a person is selected at random from the group. Find the probability that the person is an employee

We have $P(M) = \frac{1}{2}$; $P(W) = \frac{1}{2}$

Let A be an event of unemployed, E be an event of employed

$\therefore P(A/M) = \frac{10}{100} = \frac{1}{10}$; $P(E/M) = 1 - P(A/M)$

$= 1 - \frac{1}{10} = \frac{9}{10}$

$P(A/W) = 45\% = \frac{45}{100} = \frac{9}{20}$; $P(E/W) = 1 - P(A/W)$

$= 1 - \frac{9}{20}$

$= \frac{11}{20}$

probability that a person is employed

$P(E) = P(M) \cdot P(E/M) + P(W) P(E/W)$

$= \frac{1}{2} \times \frac{9}{10} + \frac{1}{2} \times \frac{11}{20}$

$= \frac{9}{20} + \frac{11}{40}$

$= \frac{29}{40}$

(v) of the 3 men the chances that a politician, a businessman, an academician will be appointed as a vice chancellor of a university are 0.5, 0.3, 0.2 respectively. probability that research is promoted by these persons If they are appointed as V.C are 0.3, 0.7, 0.8 respectively.

i) Determine the probability that research is promoted.

ii) If research is promoted what is the probability that V.C is an academecian

P(P), P(B), P(A) be the probabilities of the men will be appointed as vice chancellor.

$P(P) = 0.5 = \frac{5}{10}$

$P(B) = 0.3 = \frac{3}{10}$

$P(A) = 0.2 = \frac{2}{10}$

let R be the probability of research

$P(R/P) = \frac{3}{10}$

$P(R/B) = \frac{7}{10}$

$P(R/A) = \frac{8}{10}$

$P(R) = P(P) \cdot P(R/P) + P(B) \cdot P(R/B) + P(A) \cdot P(R/A)$

$= \frac{5}{10} \times \frac{3}{10} + \frac{3}{10} \times \frac{7}{10} + \frac{2}{10} \times \frac{8}{10}$

$= \frac{13}{25} = 0.52$

(ii) $P(A/R) = \frac{P(R/A) \cdot P(A)}{P(P) \cdot P(R/P) + P(B) \cdot P(R/B) + P(A) \cdot P(R/A)}$

$= \frac{\frac{2}{10} \times \frac{8}{10}}{\frac{5}{10} \times \frac{3}{10} + \frac{3}{10} \times \frac{7}{10} + \frac{2}{10} \times \frac{8}{10}}$

$= \frac{4}{13}$

$= 0.3076$

Naive's Bayes classifier Algorithm:

It is a supervised learning algorithm based on Bayes theorm used for solving classification problems.

- It is mainly used in text classification that include a high dimensional training dataset.
- It is one of the simple and most effective classification algorithm which helps in building the fast machine learning models that can make quick predictions.

Advantages:

- It is one of the fast and easy machine learning algorithm to predict a class of datasets.
- It is the most popular choice for test classification problems.