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UNIT-2

Expectation theory makes place a very imporlant role in decision making because of the most of the time we take decision based on, which is expected to happen.

Expectation of discrete variable:

suppose a random variable x assumes the values $x_1, x_2 --- x_n$ with a respective probabilites $p_1, p_2 ----- p_n$

The mathematical expectation are mean or expected value of x is defined and denoted by $E(x)$

It is defined as sum of product of different values of x and the corresponding probabilities

$$E(x) = x_1p_1 + x_2p_2 + x_3p_3 + ---- + x_np_n$$

$$\boxed{E(x) = \sum_{i=1}^{n} p_i x_i}$$

Imporiant results on expectation:

If x is a random variable and k is a constant then

$$E(x+k) = E(x) + k$$

<u>proof</u>: By definition, we have,

$$E(x) = \sum_{i=1}^{n} p_i x_i$$

$E(x+k) = \sum_{i=1}^{n} P_i(x_i+k)$

$= \sum_{i=1}^{n} P_i x_i + P_i k$

$= \sum_{i=1}^{n} P_i x_i + \sum_{i=1}^{n} P_i k$

$= E(x) + k \sum_{i=1}^{n} P_i$

$= E(x) + k(1) \quad [\because \sum P_i = 1]$

$= E(x) + k$

NOTE:

1. Expected value of any constant is constant i.e $E(k) = k$.
2. If k is a constant then $E(kx) = kE(x)$
3. If x is a random variable and a, b are constants, then $E(ax \pm b) = aE(x) \pm b$

2) If x and Y are two random variables then $E(x+Y) = E(x) + E(Y)$

Proof: Let x be a random variable assumes $x_1, x_2, \ldots x_n$

y be a random variable which assumes $y_1, y_2 \ldots, y_n$

By def

$$E(x) = \sum_{i=1}^{n} P_i x_i, \quad E(y) = \sum_{j=1}^{m} P_j y_j$$

By joint ~~point~~ probability function, we have

$P_{ij} = P(X = x_i \cap Y = y_j) = P(x_i, y_j)$

The sum $x+y$ is a random variable which can take $m \times n$ values of $(x_i + y_j)$ where $i = 1, 2, \ldots, n$

$j = 1, 2, \ldots, m$

$$E(X+Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij}(x_i + y_j)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} x_i + \sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} y_j$$

$$= \sum_{i=1}^{n} x_i \left[\sum_{j=1}^{m} P_{ij} \right] + \sum_{j=1}^{m} y_j \left[\sum_{i=1}^{n} P_{ij} \right]$$

$$= \sum_{i=1}^{n} x_i P_i + \sum_{j=1}^{m} y_j P_j$$

$$= E(X) + E(Y)$$

$$\therefore E(X+Y) = E(X) + E(Y)$$

NOTE: $E(ax \pm by) = aE(X) \pm bE(Y)$

3) If X and Y are two independent random variables then

$$E(XY) = E(X) \cdot E(Y)$$

Proof: Let X be a random variable assumes $x_1, x_2, \ldots, x_n$

Let y be a random variable which assumes $y_1, y_2, \ldots, y_m$.

By def

$$E(X) = \sum_{i=1}^{n} p_i x_i \; ; \quad E(Y) = \sum_{j=1}^{m} p_j y_j$$

By joint probability function, we have

$$p_{ij} = P(X = x_i \cap Y = y_j)$$

$$= P(X = x_i)\, P(Y = y_j) \quad [\because X, Y \text{ are Independent variables}]$$

$$= p_i p_j$$

$$E(XY) = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}(x_i y_j)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} p_i p_j x_i y_j$$

$$= \sum_{i=1}^{n} p_i x_i \sum_{j=1}^{m} p_j y_j$$

$$= E(X) E(Y)$$

$$\therefore E(XY) = E(X) E(Y)$$

Mean:

The mean of the probability distribution is defined and denoted as

$$\mu = \frac{\sum p_i x_i}{\sum p_i}$$

$$\mu = \frac{E(X)}{1} \quad [\sum p_i = 1]$$

$$\boxed{\mu = E(X)}$$

Variance:-

The variance of a probability distribution is denoted with σ^2 and it is defined as expectation of exp $[X-E(X)]^2$

$$\therefore \sigma^2 = var(X) = E[X-E(X)]^2$$

$$\sigma^2 = E(X-\mu)^2$$

$$= \sum_{i=1}^{n} P_i(x_i-\mu)^2$$

$$= \sum P_i((x_i^2) + \mu^2 - 2x_i\mu)$$

$$= \sum P_i x_i^2 + \sum P_i \mu^2 - 2\mu \sum P_i x_i$$

$$= \sum P_i x_i^2 + \mu^2 \sum P_i - 2\mu(\mu)$$

$$= E(X^2) + \mu^2(1) - 2\mu^2$$

$$\left[\because \sum P_i x_i = \mu, \ \sum P_i = 1\right]$$

$$= E(X^2) - \mu^2$$

$$= E(X^2) - [E(X)]^2$$

$$var(X) = \sigma^2 = E(X^2) - \mu^2$$

Standard deviation:

positive square root of variance. It is given by $\sigma = \sqrt{E(X^2)-\mu^2}$

NOTE:

1) Variance of a constant is 0 i.e $var(K) = 0$

Then $var(KX) = K^2 var(X)$

2) If x is a Random variable, k is a constant the $var(x+k) = var(x)$.

3) If x is a discrete random variable then $var(ax+b) = a^2 var(x)$

4) If x and y are two independent random variables then $var(x \pm y) = var(x) \pm var(y)$

A fair die is tossed let the random variable x denotes the twice the number appearing on the die.

i) write the probability distribution of x

ii) Mean

iii) variance.

When a die is thrown the total possibilities are $n(s) = 6$

x denote the twice the number appearing on a face when die is thrown.

$\therefore x$ can take the values. 2, 4, 6, 8, 10, 12

As appearance of a number on a fair die is equally likely

$\therefore P(x=1) = P(x=2) = 1/6$

$P(x=2) = P(x=4) = 1/6$

$P(x=3) = P(x=6) = 1/6$

$P(x=4) = P(x=8) = 1/6$

$P(x=5) = P(x=10) = 1/6$

$P(X=6) = P(X=12) = 1/6$

i) The required probability distribution is

X	2	4	6	8	10	12
$P(X=x_i)$	1/6	1/6	1/6	1/6	1/6	1/6

ii) Mean$(\mu) = E(X)$

$= \sum x_i p_i$

$= 2\times\frac{1}{6} + 4\times\frac{1}{6} + 6\times\frac{1}{6} + 8\times\frac{1}{6} + 10\times\frac{1}{6} + 12\times\frac{1}{6}$

$= 7$

iii) Variance $V(X) = E(X^2) - \mu^2$

$= \left(2^2\times\frac{1}{6} + 4^2\times\frac{1}{6} + 6^2\times\frac{1}{6} + 8^2\times\frac{1}{6} + 10^2\times\frac{1}{6} + 12^2\times\frac{1}{6}\right) - (7)^2$

$= 11.667$

For the discrete probability distribution

x	0	1	2	3	4	5	6
$P(X=x_i)$	0	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find i) k

ii) Expectation

iii) variance.

$\Sigma P_i = 1$

$0+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$8k+10k^2=1$

$k = \frac{-4+\sqrt{26}}{10}$

$k = 0.1099$

(i) Expectation $= E(X)$

$= \Sigma p_i x_i$

$= 0\times 0 + 1\times 2k + 2\times 2k + 3\times 3k + 4\times k^2 + 5(2k^2) + 6(7k^2+k)$

$= 2k+4k+9k+4k^2+10k^2+42k^2+6k$

$= 21k+56k^2$

$= 21(0.1099)+56(0.1099)^2$

$= 2.9842$

(ii) variance $V(X) = E(X^2) - \mu^2$

$= 0\times 0 + 1^2\times 2k + 2^2\times 2k + 3^2\times 3k + 4^2\times k^2 + 5^2\times 2k^2 + 36(7k^2+k) - (2.9842)^2$

$= 2k+8k+27k+16k^2+50k^2+252k^2+36k - (2.9842)^2$

$= 73k+318k^2-(2.9842)^2$

$= 2.9580$

Let x denotes the minimum of the two numbers that appear on a pair of dice is thrown once. Determine

1) Discrete probability distribution

2) Expectation

3) variance

When two dice are thrown the total no. of cases $n(S) = 36$

Here $X(S) = \min(a,b) = X(a,b)$

The minimum numbers with the numbers 1,2,3,4,5,6 and their probability are

$P(X=1) = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$

$= \frac{11}{36}$

$P(X=2) = \{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$

$= \frac{9}{36}$

$P(X=3) = \{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$

$= \frac{7}{36}$

$P(X=4) = \{(4,4),(4,5),(4,6),(5,4),(6,4)\}$

$= \frac{5}{36}$

$P(X=5) = \{(5,5),(5,6),(6,5)\}$

$= \frac{3}{36}$

$P(X=6) = \{(6,6)$

$= \frac{1}{36}$

∴ The required probability distribution is

X	1	2	3	4	5	6
$P(X=x_i)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(ii) Expectation $= E(X)$

$= \Sigma p_i x_i$

$= 1\times\frac{11}{36} + 2\times\frac{9}{36} + 3\times\frac{7}{36} + 4\times\frac{5}{36} +$

$5\times\frac{3}{36} + 6\times\frac{1}{36}$

$= 2.5277$

(iii) Variance $V(X) = E(X^2) - \mu^2$

$= 1\times\frac{11}{36} + 4\times\frac{9}{36} + 9\times\frac{7}{36} + 16\times\frac{5}{36} + 25\times\frac{3}{36}$

$+ 36\times\frac{1}{36} - (2.5277)^2$

$= 1.9718$

A Random variable X has the following distribution

X	-3	-2	-1	0	1	2	3
P(X)	k	0.1	k	0.2	2k	0.4	2k

i) Find k

ii) Mean

iii) Variance.

from a lot of

i) $\Sigma p_i = 1$

$k + 0.1 + k + 0.2 + 2k + 0.4 + 2k = 1$

$6k + 0.7 = 1$

$6k = 1 - 0.7$

$\boxed{k = 0.05}$

ii) Mean $\mu = \Sigma p_i x_i$

$-3(k) + (-2)0.1 + (-1)k + 0 + 1(2k) +$
$2(0.4) + 3(2k)$

$4k + 0.6$

$4(0.05) + 0.6 = 0.80$

iii) variance $V(x) = \Sigma P(x_i)^2 - \mu^2$

$= 9 \times k + 4 \times 0.1 + 1 \times k + 1 \times 2k + 4 \times 0.4 +$
$9 \times 2k - (0.80)^2$

$= 9k + 0.4 + k + 2k + 1.6 + 18k - (0.8)^2$

$= 30k + 2 - 0.64$

$= 30(0.\overset{5}{0}) + 2 - 0.64$

$= 2.8600$

From a lot of 10 items containing 3 defectives A sample of 4 items drawn at random let the random variable X denote the no. of defective items in the sample find the probability distribution of X and Expectation.

X denotes the no. of defective items among 4 times from 10

Total items = 10.

Good items = 7.

defective items = 3

$X = \{0, 1, 2, 3\}$

$P(X=0) = P(\text{No defective})$

$= \dfrac{7C_4}{10C_4} = \dfrac{7}{30} = \dfrac{1}{6}$

$P(X=1) = P(1 \text{ defective}, 3 \text{ good item})$

$= \dfrac{3C_1 \times 7C_3}{10C_4} = \dfrac{\dfrac{3 \times 7 \times 6 \times 5 \times 4}{3 \times 2 \times 1}}{\dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}} = \dfrac{1}{2}$

$P(X=2) = P(2 \text{ defective}, 2 \text{ good item})$

$= \dfrac{3C_2 \times 7C_2}{10C_4} = \dfrac{\dfrac{3 \times 2 \times 7 \times 6}{1 \times 2 \quad 5 \times 4}}{\dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}} = \dfrac{3}{10}$

$P(X=3) = P(3 \text{ defective})$

$= \dfrac{3C_3 \times 7C_1}{10C_4} = \dfrac{\dfrac{3 \times 2 \times 1}{1 \times 2 \times 3} \times 7}{\dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}} = \dfrac{1}{30}$

X	0	1	2	3
$P(X=x_i)$	$1/6$	$1/2$	$3/10$	$\frac{1}{30}$

Expectation $E(X)$

$= 0 \times \frac{1}{6} + 1 \times \frac{1}{2} + 2 \times \frac{3}{10} + 3 \times \frac{1}{30}$

$= \frac{8}{5}$

A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. Find the Expected number E of defective items.

X denotes the no. of defective items among 4 items from 12.

Total items = 12

Defective items = 5

Good items = 7

$X = \{0, 1, 2, 3, 4\}$

$P(X=0) = P(\text{No defective item})$

$$= \frac{7C4}{12C4} = \frac{\frac{7\times 6\times 5\times 4}{1\times 2\times 3\times 4}}{\frac{12\times 11\times 10\times 9}{1\times 2\times 3\times 4}} = \frac{7}{99}$$

$P(X=1) = P(1 \text{ defective item, } 3 \text{ good items})$

$$= \frac{5C1\times 7C3}{12C4} = \frac{5\times\frac{7\times 6\times 5}{1\times 2\times 3}}{\frac{12\times 10\times 9\times 8}{4\times 3\times 2\times 1}} = \frac{35}{99}$$

$P(X=2) = P(2 \text{ defective items, } 2 \text{ good items})$

$$= \frac{5C2\times 7C2}{12C4} = \frac{42}{99}$$

$P(X=3) = P(3 \text{ defective items}, 1 \text{ good item})$

$= \frac{5C_3 \times 7C_1}{12C_4} = \frac{14}{99}$

$P(X=4) = P(\text{all are defective items})$

$= \frac{5C_4}{12C_4} = \frac{1}{99}$

∴ The required probability distribution is

X	0	1	2	3	4
$P(X=x_i)$	$\frac{7}{99}$	$\frac{35}{99}$	$\frac{42}{99}$	$\frac{14}{99}$	$\frac{1}{99}$

$E(X) = \sum p_i x_i$

$= 0 \times \frac{7}{99} + 1 \times \frac{35}{99} + 2 \times \frac{42}{99} + 3 \times \frac{14}{99} + 4 \times \frac{1}{99}$

$= \frac{35}{99} + \frac{84}{99} + \frac{42}{99} + \frac{4}{99}$

$= \frac{165}{99}$

Find the mean and variance of the uniform probability distribution given by $f(x) = \frac{1}{n}$ for $x = 1, 2, 3 \ldots, n$

Given,

$f(x) = \frac{1}{n} \; \forall x = 1, 2, \ldots n$

The probability distribution is

x	1	2	3	4	----	n
f(x)	1/n	1/n	1/n	1/n	----	1/n

i) Mean $\mu = \sum p_i x_i$

$= 1\times\frac{1}{n} + 2\times\frac{1}{n} + 3\times\frac{1}{n} + \dots + n\times\frac{1}{n}$

$= \frac{1}{n}[1+2+3+\dots+n]$

$= \frac{1}{n}\left[\frac{n(n+1)}{2}\right]$

$\mu = \frac{n+1}{2}$

ii) variance $V(X) = E(X^2) - \mu^2$

$= \sum p_i x_i^2 - \mu^2$

$= 1^2\times\frac{1}{n} + 2^2\times\frac{1}{n} + 3^2\times\frac{1}{n} + \dots + n^2\times\frac{1}{n} - \left[\frac{n+1}{2}\right]^2$

$= \frac{1}{n}[1^2+2^2+3^2+\dots+n^2] - \frac{(n+1)^2}{4}$

$= \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{(n+1)^2}{4}$

$= \left(\frac{n+1}{2}\right)\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$

$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$

$= \frac{n+1}{2}\left[\frac{n-1}{6}\right] = \frac{n^2-1}{12}$

A random sample of with replacement of size 2 is taken from $S=\{1,2,3\}$. Let the random variable x denote the sum of the 2 numbers taken. Write the probability distribution of x

ii) Mean

iii) Variance

The numbers formed with the size 2 is

$S=\{1,2,3\} = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

Let x denote the sum of 2 numbers

$p(x=2)=\{(1,1)\} = \frac{1}{9}$

$p(x=3)=\{(1,2),(2,1)\} = \frac{2}{9}$

$p(x=4)=\{(1,3),(2,2),(3,1)\} = \frac{3}{9}$

$p(x=5)=\{(2,3),(3,2)\} = \frac{2}{9}$

$p(x=6)=\{(3,3)\} = \frac{1}{9}$

∴ The required probability distribution is

x	2	3	4	5	6
$p(x=xi)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(ii) Mean $= E(x)$

$= \sum P(x_i)$

$= 2\times\frac{1}{9} + 3\times\frac{2}{9} + 4\times\frac{3}{9} + 5\times\frac{2}{9} + 6\times\frac{1}{9}$

$= \frac{2}{9} + \frac{6}{9} + \frac{12}{9} + \frac{10}{9} + \frac{6}{9}$

$\mu = \frac{36}{9} = 4$

(iii) variance $V(x) = E(x^2) - \mu^2$

$= 4\times\frac{1}{9} + 9\times\frac{2}{9} + 16\times\frac{3}{9} + 25\times\frac{2}{9} + 36\times\frac{1}{9}$

$= \frac{4}{9} + \frac{18}{9} + \frac{48}{9} + \frac{50}{9} + \frac{36}{9} - 16$

$= \frac{156}{9} - 16$

$= \frac{52}{3} - 16$

$= \frac{4}{3}$

A discrete random variable x has following distribution function.

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ 1/3 & \text{for } 1 \le x < 4 \\ 1/2 & \text{for } 4 \le x < 6 \\ 5/6 & \text{for } 6 \le x < 10 \\ 1 & \text{for } x \ge 10 \end{cases}$$

Find i) $P(2 < x \le 6)$ ii) $P(x=5)$ iii) $P(x=4)$

iv) $P(x \le 6)$ v) $P(x=6)$

i) $P(2 < x \le 6) = F(6) - F(2)$ $[\because F(x) = P(x \le x)]$

$= P(x \le 6) - P(x < 2)$

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

ii) $P(x=5) = P(x \le 5) - P(x < 5)$

$= F(5) - \frac{1}{2}$

$= \frac{1}{2} - \frac{1}{2} = 0$

iii) $P(x=4) = P(x \le 4) - P(x < 4)$

$= F(4) - \frac{1}{3}$

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

iv) $P(x \le 6) = F(6)$

$= 5/6$

v) $P(x=6) = P(x \le 6) - P(x < 6)$

$= F(6) - P(x < 6)$

$= \frac{5}{6} - \frac{1}{2}$

$= \frac{5-3}{6} = \frac{2}{6}$

$= \frac{1}{3}$

Mean, Median, Mode, Mean deviation and Variance of a continous random variable:

Mean:

The Mean of a distribution is given by

$$E(X) = \mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

If x is defined from a to b Then

$$\mu = E(X) = \int_{a}^{b} x f(x)\,dx$$

Median:

Median is the point which divides the entire area into two equal parts.

Thus, If x is defined from a to b and M is the median Then

$$\int_{a}^{M} f(x)\,dx = \int_{M}^{b} f(x)\,dx = \frac{1}{2}$$

By solving above integrals we get the median.

Mode:

Mode is the value of x for which $f(x)$ is maximum i.e $f'(x) = 0$

$f''(x) < 0;\ a < x < b$

Variance:

Variance of distribution is given by

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx \text{ (or) } \sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

Suppose x is defined from a to b Then

$$\sigma^2 = \int_a^b x^2 f(x)\,dx - \mu^2$$

Mean deviation:

Mean deviation about mean μ is given by $\int_{-\infty}^{\infty} |x-\mu| f(x)\,dx$

1. If a random variable has the probability density $f(x)$ as

$$f(x) = \begin{cases} 2e^{-2x} \text{ for } x > 0 \\ 0 \text{ for } x \le 0 \end{cases}$$ Find the probability that is

takes on a value

i) between 1 & 3 ii) Greater than 0.5

i) $P(1 < x < 3) = \int_1^3 f(x)\,dx$

$$= \int_1^3 2e^{-2x}\,dx$$

$$= 2\int_1^3 e^{-2x}\,dx$$

$= 2\left[\frac{e^{-2x}}{2}\right]_1^3$

$= \frac{2}{-2}\left[e^{-2x}\right]_1^3$

$= -[e^{-6} - e^{-2}]$

$= e^{-2} + e^{-6}$

(ii) $P(x > 0.5) = \int_{0.5}^{\infty} f(x)\,dx$

$= \int_{0.5}^{\infty} 2e^{-2x}\,dx$ $\quad [\because e^{-\infty} = 0]$

$= 2\left[\frac{e^{-2x}}{-2}\right]_{0.5}^{\infty}$

$= -[e^{-2x}]_{0.5}^{\infty}$

$= -[e^{-2(\infty)} - e^{-2(0.5)}]$

$= -[0 - e^{-1}]$

$= e^{-1} = \frac{1}{e}$

A continuous random variable has the probability density function

$$f(x) = \begin{cases} k x e^{-\lambda x} & \text{for } x \geq 0, \lambda > 0 \\ 0 & \text{for otherwise} \end{cases}$$

Determine i) k ii) Mean iii) variable

We know that

i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$

$0 + \int_{0}^{\infty} kx\,e^{-\lambda x}\,dx = 1$

$k \int_{0}^{\infty} x\,e^{-\lambda x}\,dx = 1$

$k\left[\frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]_0^{\infty} = 1$

$k\left[-\frac{1}{\lambda}(x e^{-\lambda x})_0^{\infty} - \frac{1}{\lambda^2}(e^{-\lambda x})_0^{\infty}\right] = 1$

$k\left[-\frac{1}{\lambda}[0-0] - \frac{1}{\lambda^2}\left[e^{-\infty} - e^{0}\right]\right] = 1$

$k\left[-\frac{1}{\lambda^2}[0-1]\right] = 1$

$k\left(\frac{1}{\lambda^2}\right) = 1$

$\boxed{k = \lambda^2}$

$\int x\,e^{-\lambda x}\,dx$

$u = x$	$v = e^{-\lambda x}$
D.O.S	I.O.B.S
$u' = 1$	$v_1 = \frac{e^{-\lambda x}}{-\lambda}$
$u'' = 0$	$v_2 = \frac{e^{-\lambda x}}{\lambda^2}$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 \ldots$

$\int x e^{-\lambda x} dx = x\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 1\left(\frac{e^{-\lambda x}}{\lambda^2}\right)$

$\int x e^{-\lambda x} dx = \frac{x e^{-\lambda x}}{-\lambda} - \frac{e^{-\lambda x}}{\lambda^2}$

ii) We know

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{\infty} x f(x)\,dx$$

$$= 0 + \int_{0}^{\infty} x\left[\lambda^2 x e^{-\lambda x}\right] dx$$

$$= \int_{0}^{\infty} x^2 e^{-\lambda x} \lambda^2\,dx$$

$$= \lambda^2\left[\int_{0}^{\infty} x^2 e^{-\lambda x}\,dx\right]$$

$$= \lambda^2\left[\frac{-1}{\lambda}\left[x^2 e^{-\lambda x}\right]_0^{\infty} - \frac{2}{\lambda^2}\left[x e^{-\lambda x}\right]_0^{\infty} - \frac{2}{\lambda^3}\left[2 e^{-\lambda x}\right]_0^{\infty}\right]$$

$$= \lambda^2\left[\frac{-1}{\lambda}(0-0) - \frac{2}{\lambda^2}(0-0) - \frac{2}{\lambda^3}(e^{-\infty} - e^{0})\right]$$

$$= \lambda^2\left[0 - 0 - 2/\lambda^3(0-1)\right]$$

$$= \lambda^2\left[\frac{2}{\lambda^3}\right]$$

$$= \frac{2}{\lambda}$$

$$\boxed{\text{Mean}(\mu) = \frac{2}{\lambda}}$$

(iii) variance $= \sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$$\int_{-\infty}^{0} x^2 f(x)\,dx - \mu^2 + \int_{0}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$0 + \int_0^{\infty} x^2 [\lambda^2 x e^{-\lambda x}] - \left(\frac{2}{\lambda}\right)^2$$

$$= \int_0^{\infty} x^3 \lambda^2 e^{-\lambda x}\,dx - \left(\frac{2}{\lambda}\right)^2$$

$$= \lambda^2 \left[\int_0^{\infty} x^3 e^{-\lambda x}\right] dx - \left(\frac{2}{\lambda}\right)^2$$

proof:

$$\int x^2 e^{-\lambda x}\,dx$$

$u = x^2$ $\quad$ $v = e^{-\lambda x}$

$u' = 2x$ $\quad$ $v' = \frac{e^{-\lambda x}}{-\lambda}$

$u'' = 2$ $\quad$ $v'' = \frac{e^{-\lambda x}}{\lambda^2}$

$u''' = 0$ $\quad$ $v''' = \frac{e^{-\lambda x}}{-\lambda^3}$

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$$

$$\int x^2 e^{-\lambda x}\,dx = x^2\left[\frac{e^{-\lambda x}}{-\lambda}\right] - 2x\left[\frac{e^{-\lambda x}}{\lambda^2}\right] + 2\left[\frac{e^{-\lambda x}}{-\lambda^3}\right]$$

$$= \frac{x^2 e^{-\lambda x}}{-\lambda} - \frac{2x e^{-\lambda x}}{\lambda^2} - \frac{2 e^{-\lambda x}}{\lambda^3}$$

$\int x^3 e^{-\lambda x}$

$u = x^3$ $\quad v = e^{-\lambda x}$

$u' = 3x^2$ $\quad v_1 = \frac{e^{-\lambda x}}{-\lambda}$

$u'' = 6x$ $\quad v_2 = \frac{e^{-\lambda x}}{\lambda^2}$

$u''' = 6$ $\quad v_3 = \frac{e^{-\lambda x}}{-\lambda^3}$

$u'''' = 0$ $\quad v_4 = \frac{e^{-\lambda x}}{\lambda^4}$

$$\sigma^2 = \lambda^2\left[\frac{-1}{\lambda}\left[x^3 e^{-\lambda x}\right]_0^\infty - \frac{3}{\lambda^2}\left[x e^{-\lambda x}\right]_0^\infty - \frac{6}{\lambda^3}\left[x e^{-\lambda x}\right]_0^\infty - \frac{6}{\lambda^4}\left[e^{-\lambda x}\right]_0^\infty - \frac{4}{\lambda^2}\right.$$

$$\sigma^2 = \lambda^2\left[\frac{-1}{\lambda}[0-0] - \frac{3}{\lambda^2}[0-0] - \frac{6}{\lambda^3}[0-0] - \frac{6}{\lambda^4}[0-1]\right] - \frac{4}{\lambda^2}$$

$$\sigma^2 = \lambda^2\left[\frac{6}{\lambda^4}\right] - \frac{4}{\lambda^2} \Rightarrow \frac{6}{\lambda^2} - \frac{4}{\lambda^2}$$

$$\sigma^2 = \frac{6-4}{\lambda^2} = \frac{2}{\lambda^2}$$

probability density function of a random variable x is given by

$$f(x) = \begin{cases} \frac{1}{2}\sin x & \text{for } 0 \le x \le \pi \\ 0 & \text{elsewhere} \end{cases}$$

find Mean, Median, Mode of the distribution & also find the probability between 0 & $\pi/2$

If the probability density function of a random variable x is given by

$$f(x) = \begin{cases} kx^3 & \text{for } 0 \le x \le 3 \\ 0 & \text{eleswhere} \end{cases}$$

find the value of k and also find the probabilty between $x = \frac{1}{2}$ & $x = \frac{3}{2}$

We know $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1$$

$$= 0 + \int_{0}^{3} kx^3\,dx + 0 = 1$$

$$= k\left[\frac{x^4}{4}\right]_0^3 = 1$$

$$K\left[\frac{81}{4}\right]=1$$

$$K=\frac{4}{81}$$

$$P\left(\frac{1}{2}<x<\frac{3}{2}\right)=\int_{1/2}^{3/2} f(x)\,dx$$

$$=\int_{1/2}^{3/2}\frac{4}{81}x^3\,dx$$

$$=\frac{4}{81}\int_{1/2}^{3/2}x^3\,dx$$

$$=\frac{4}{81}\left[\frac{x^4}{4}\right]_{1/2}^{3/2}$$

$$=\frac{4}{81}\left[\frac{81}{16(4)}-\frac{1}{16(4)}\right]$$

$$=\frac{4}{81}\left[\frac{\cancel{80}^{10}}{\cancel{16(4)}_{2}}\right]$$

$$=\frac{1\cancel{0}5}{1\cancel{62}81}=\frac{5}{81}$$

The density function of a random variable x is $f(x)$

$$f(x)=\begin{cases} e^{-x} & \text{for } x\geq 0 \\ 0; & \text{eleswhere}\end{cases}$$

Find $E(X)$, $E(X^2)$, $V(X)$

$$E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{\infty} x f(x)\, dx$$

$$= 0 + \int_{0}^{\infty} x e^{-x} dx$$

$$= \left[x(-e^{-x}) - 1(e^{-x})\right]_0^{\infty}$$

$$= \left[-(x e^{-x})_0^{\infty} - (e^{-x})_0^{\infty}\right]$$

$$= \left[-(0-0) - (e^{-\infty} - e^{0})\right]$$

$$= [0 - (0-1)]$$

$$\therefore \boxed{E(X) = 1}$$

$$E(X^2) = \int_{0}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{0} x^2 f(x)\, dx + \int_{0}^{\infty} x^2 f(x)\, dx$$

$$= 0 + \int_{0}^{\infty} x^2 e^{-x} dx$$

$$= \left[x^2(e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right]_0^{\infty}$$

$$= \left[-(x^2 e^{-x})_0^{\infty} - 2(x e^{-x})_0^{\infty} - 2(e^{-x})_0^{\infty}\right]$$

$$= \left[0 - 2(0) - 2(e^{-\infty} - e^{0})\right]$$

$$= [0 - 0 - 2(0-1)] = 2$$

$\boxed{E(x^2)=2}$

$V(x) = E(x^2) - [E(x)]^2$

$= 2-(1)^2$

$= 2-1$

$= 1$

If x is a continous random varcable and $Y = ax+b$ Then prove that

i) $E(Y) = aE(x)+b$

ii) $V(Y) = a^2V(x)$

Given: $y = ax+b$ — ①

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} x f(x)\,dx = E(x)$$

$E(y) = E(ax+b)$

$= \int_{-\infty}^{\infty} (ax+b) f(x)\,dx$

$= \int_{-\infty}^{\infty} ax f(x)\,dx + b\int_{-\infty}^{\infty} f(x)\,dx$

$= aE(x) + b(1)$

$= aE(x)+b$

ii) $V(Y) = E(Y^2) - [E(Y)]^2$

$= E((ax+b)^2) - [E(ax+b)]^2$

$$= \int_{-\infty}^{\infty} (ax+b)^2 f(x)\,dx - \left[\int_{-\infty}^{\infty} (ax+b) f(x)\,dx \right]^2$$

$$= \int_{-\infty}^{\infty} (a^2x^2 + b^2 + 2axb) f(x)\,dx - \left[\int_{-\infty}^{\infty} ax f(x)\,dx + \int_{-\infty}^{\infty} b f(x)\,dx \right]^2$$

$$= \int_{-\infty}^{\infty} a^2x^2 f(x)\,dx + \int_{-\infty}^{\infty} b^2 f(x)\,dx + \int_{-\infty}^{\infty} 2xab\, f(x)\,dx - \left[a\int_{-\infty}^{\infty} x f(x)\,dx + b\int_{-\infty}^{\infty} f(x)\,dx \right]^2$$

$$= a^2 \int_{-\infty}^{\infty} x^2 f(x)\,dx + b^2 \int_{-\infty}^{\infty} f(x)\,dx + 2ab\int_{-\infty}^{\infty} x f(x)\,dx - \left[aE(X) + b \right]^2$$

$$= a^2E(X^2) + b^2(1) + 2abE(X) - \left[a^2(E(X))^2 + b^2 + 2abE(X) \right]$$

$$= a^2E(X^2) + b^2 + 2abE(X) - a^2(E(X))^2 - b^2 - 2abE(X)$$

$$= a^2E(X^2) - a^2(E(X))^2$$

$$= a^2\left(E(X^2) - (E(X))^2\right)$$

$$= a^2 V(X)$$

$$\therefore \boxed{var(Y) = a^2 var(X)}$$

If x is a continous random variable whose density function is given by

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 2-x & \text{if } 1 \le x \le 2 \\ 0, & \text{eleswhere} \end{cases}$$

Then find $E(25x^2+30x-5)$

$$E(25x^2) + E(30x) - E(5)$$

$$25E(x^2) + 30E(x) - 5$$

$$25\int_{-\infty}^{\infty} x^2 E(x)\,dx + 30\int_{-\infty}^{\infty} x\,E(x)\,dx - 5$$

$$25\left[\int_{-\infty}^{0} x^2 E(x)\,dx + \int_{0}^{1} x^2 E(x)\,dx + \int_{1}^{2} x^2 E(x)\right.$$

probability density function of a random variable x is given by

$$f(x) = \begin{cases} \tfrac{1}{2}\sin x, & \text{for } 0 \le x \le \pi \\ 0, & \text{elsewhere} \end{cases}$$

find mean, Median, Mode of the distribution & also find the probability between 0 & $\pi/2$

$$\text{Mean} = \mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{\pi} x f(x)\,dx + \int_{\pi}^{\infty} x f(x)\,dx$$

$= 0 + \int_0^{\pi} x \cdot \frac{1}{2} \sin x \, dx + 0$

$= \frac{1}{2} \int_0^{\pi} x \sin x \, dx$

$= \frac{1}{2} \Big[x(-\cos x) - 1(-\sin x) \Big]_0^{\pi}$

$= \frac{1}{2} [-x\cos x + \sin x]_0^{\pi}$

$\mu = \frac{1}{2} \left[(-\pi \cos \pi + \sin \pi) - (-0\cos(0) + \sin(0)) \right]$

$\mu = \frac{1}{2} \left[(-\pi(-1) + 0) + (0 - 0) \right]$

$\mu = \frac{1}{2} [\pi]$

$\boxed{\mu = \frac{\pi}{2}}$

Median:-

M is the Median of the distribution in $(0, \pi)$ is given by

$\int_0^M f(x)\,dx = \int_M^{\pi} f(x)\,dx = \frac{1}{2}$

$\int_0^M \frac{1}{2} \sin x \, dx = \int_M^{\pi} \frac{1}{2} \sin x \, dx = \frac{1}{2}$

$\Rightarrow \int_0^M \frac{1}{2} \sin x \, dx = \frac{1}{2}$

$\frac{1}{\cancel{2}} \int_0^M \sin x \, dx = \frac{1}{\cancel{2}}$

$[-\cos x]_0^m = 1$

$-\cos m + \cos 0 = 1$

$-\cos m + 1 = 1$

$-\cos m = 0$

$\cos m = 0 = \cos \pi/2$

$\boxed{m = \pi/2}$

$$P(0 \le x \le \pi/2) = \int_0^{\pi/2} f(x)\,dx$$

$$= \int_0^{\pi/2} \frac{1}{2} \sin x\,dx$$

$$= \frac{1}{2}\int_0^{\pi/2} [\sin x]\,dx$$

$$= \frac{1}{2}[-\cos x]_0^{\pi/2}$$

$$= \frac{-1}{2}[\cos \pi/2 - \cos 0]$$

$$= \frac{-1}{2}[-1]$$

$$= \frac{1}{2}$$

Mode:

Mode is the value of x for which $f(x)$ has maximum value

$$f'(x) = 0$$

$$\frac{d}{dx}[f(x)] = \frac{d}{dx}\left[\frac{1}{2}\sin x\right]$$

$$= \frac{1}{2}\left[\frac{d}{dx}\sin x\right]$$

$$= \frac{1}{2}\cos x$$

$\therefore f'(x) = 0$

$\frac{1}{2}\cos x = 0$

$\cos x = 0 = \cos \pi/2$

$\boxed{x = \pi/2}$

Now, $[f''(x)]_{x=\pi/2}$

$\left[\frac{d}{dx}\left(\frac{1}{2}\cos x\right)\right]_{x=\pi/2}$

$= \left[\frac{-1}{2}\sin x\right]_{x=\frac{\pi}{2}}$

$= \frac{-1}{2}\sin(\pi/2)$

$= \frac{-1}{2}(1)$

$= \frac{-1}{2} < 0$

$f''(x) < 0$

Hence $x = \pi/2$ is mode of the distribution

$\therefore f(x)$ has maximum value of $x = \pi/2$

$E(25x^2) + E(30x) - E(5)$

$25E(x^2) + 30E(x) - 5$

$25\int_{-\infty}^{\infty} x^2 E(x)\,dx + 30\int_{-\infty}^{\infty} x E(x)\,dx - 5$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$= \int_0^1 x^2 f(x)\,dx + \int_1^2 x^2 f(x)\,dx$

$= \int_0^1 x^2 (x)\,dx + \int_1^2 x^2(2-x)\,dx$

$= \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_1^2$

$= \frac{1}{4} + \left(\frac{16}{3} - \frac{16}{4} - \left(\frac{2}{3} - \frac{1}{4}\right)\right)$

$= \frac{1}{4} + \left(\frac{64-48-8+3}{12}\right)$

$= \frac{1}{4} + \frac{5}{12} + \frac{9}{12}$

$E(x^2) = \frac{8}{12}\ \frac{7}{6}$

$E(x) = 1$

$E(x^2) = \frac{7}{6}$

$25E(x^2) + 30E(x) - 5$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{0}^{1} x f(x)\,dx + \int_{1}^{2} x f(x)\,dx$$

$$= \int_{0}^{1} x(x)\,dx + \int_{1}^{2} x(2-x)\,dx$$

$$= \frac{x^3}{3}\Big|_0^1 + \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{3} + \left(4 - \frac{8}{3} - \frac{1}{1} + \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{7}{3} + 3$$

$$= -2 + 3 = 1$$

$$25\left(\frac{7}{6}\right) + 30(1) - 5$$

$$= 54.16$$

covariance of Random variables:

covariance is a measure of relationship between two random variables (or) it is a measure of the variance between the random variables i.e variance of one variable is equal to variance of other variable

- It is also used to find the coefficient of the correlation.
- covariance and correlation both assess the relationship between variables.
- The covariance formula is similar to the formula of correlation and deals with the calculation of data points from the average value of data set
- The covariance between two random variables x and y can be calculated by using the following formula

$$cov(x,y) = \frac{\sum (x_i-\bar{x})(y_j-\bar{y})}{n} \text{ (or) } E(XY) - \bar{X}\bar{Y}$$

where x_i be the values of x variable

y_j be the values of y variable

$\bar{x}$ be the mean of x variables

$\bar{y}$ be the mean of y variables

n be the no. of data points.

Types:-

covariance are of 2 types:

1. positive covariance:

When there is a positive relationship between x and y is said to be positive covariance i.e x and Y tend to move in same direction.

Ex: Income and education, height and weight

2. Negative covariance:

when there is a negative relationship between x and Y is said to be negative covariance i.e x and Y tend to move in opposite direction.

Ex: Education and poverty, House hold income and no.of children.

when cov(x,Y) is 0. Then the two random variables x and Y are uncorrelated.

when the random variables are uncorrelated the correlation is equal to the product of their means.

The correlation coefficient between x and Y is given by:

$$r = \frac{cov(x,Y)}{\sigma_x \sigma_y}$$

where

$$cov(x,Y) = \frac{\Sigma(x_i-\bar{x})(y_j-\bar{Y})}{n}$$

or

$$\frac{1}{n}\Sigma xy - \bar{x}\bar{Y}$$

$\sigma_x = \sqrt{\frac{1}{n}\Sigma x^2 - (\bar{x})^2}$

$\sigma_y = \sqrt{\frac{1}{n}\Sigma y^2 - (\bar{y})^2}$

The coefficient of correlation lies between -1 to 1

$-1 \leq r \leq 1$

$|r| \leq 1$

Find the coefficient of correlation between x and y for the following data.

X	10	12	18	24	23	27
Y	13	18	12	25	30	10

To calculate coefficient of correlation, we need to find $cov(X,Y)$, σx, σy using following table

X	Y	X^2	Y^2	XY
10	13	100	169	130
12	18	144	324	216
18	12	324	144	216
24	25	576	625	600
23	30	529	900	690
27	10	729	100	270
$\Sigma x = 114$	$\Sigma y = 108$	$\Sigma x^2 = 2402$	$\Sigma y^2 = 2262$	$\Sigma xy = 2122$

coefficient of correlation

$$r = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$cov(x,y) = \frac{1}{n}\sum xy - \bar{x}\bar{y}$$

$$\sigma_x = \sqrt{\frac{1}{n}\sum x^2 - (\bar{x})^2}, \quad \sigma_y = \sqrt{\frac{1}{n}\sum y^2 - (\bar{y})^2}$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{114}{6} = 19, \quad \bar{y} = \frac{\sum y_i}{n} = \frac{108}{6} = 18$$

$$cov(x,y) = \frac{1}{6}(2122) - 19(18)$$

$$= 11.66$$

$$\sigma_x = \sqrt{\frac{1}{6}(2402) - (19)^2} = 6.2716$$

$$\sigma_y = \sqrt{\frac{1}{6}(2262) - (18)^2} = 7.2801$$

$$r = \frac{11.66}{(6.2716)(7.2801)}$$

$$\boxed{r = 0.2553}$$

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9

To calculate coeffieient of correlation, we need to find cov(x,y), σ_x, σ_y by using the following table

x	y	x^2	y^2	xy
9	15	81	225	135
8	16	64	256	128
7	14	49	196	98
6	13	36	169	78
5	11	25	121	55
4	12	16	144	48
3	10	9	100	30
2	8	4	64	16
1	9	1	81	9
$\Sigma x=45$	$\Sigma y=108$	$\Sigma x^2=285$	$\Sigma y^2=1356$	$\Sigma xy=597$

coefficient of correlation

$$r=\frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$cov(x,y)=\frac{1}{n}\Sigma xy-\bar{x}\bar{y}$$

$$\sigma_x=\sqrt{\frac{1}{n}\Sigma x^2-(\bar{x})^2} \quad \sigma_y=\sqrt{\frac{1}{n}\Sigma y^2-(\bar{y})^2}$$

$\bar{x} = \frac{\sum x_i}{n} = \frac{45}{9} = 5$

$\bar{y} = \frac{\sum y_i}{n} = \frac{108}{9} = 12$

$cov(x, y) = \frac{1}{9}(597) - 5(12) = 6.333$

$\sigma_x = \sqrt{\frac{1}{9}(285) - 25} = 2.5819$

$\sigma_y = \sqrt{\frac{1}{9}(1356) - (12)^2} = 2.5819$

$r = \frac{6.3333}{(2.5819)(2.5819)}$

$= 0.95001$

X	1	2	3	4	5	6	7	8	9
Y	12	11	13	15	14	17	16	19	18

X	Y	X^2	Y^2	XY
1	12	1	144	12
2	11	4	121	22
3	13	9	169	39
4	15	16	225	60
5	14	25	196	78
6	17	36	289	102
7	16	49	256	112

8	19	64	361	152
9	18	81	324	162
$\Sigma x=45$	$\Sigma y=135$	$\Sigma x^2=285$	$\Sigma y^2=2085$	$\Sigma xy=731$

$$r=\frac{cov(x,y)}{\sigma_x\sigma_y}$$

$$cov(x,y)=\frac{1}{n}\Sigma xy-\bar{x}\bar{y}=\frac{1}{9}\Sigma 731-5(15)$$

$$=6.22$$

$$\bar{x}=\frac{45}{9}=5 \quad \bar{y}=\frac{135}{9}=15$$

$$\sigma_x=\sqrt{\frac{1}{n}\Sigma x^2-(\bar{x})^2}$$

$$=\sqrt{\frac{1}{9}(285)-25}=2.5819$$

$$\sigma_y=\sqrt{\frac{1}{n}\Sigma y^2-(\bar{y})^2}$$

$$=\sqrt{\frac{1}{9}(2085)-(15)^2}=2.5819$$

$$r=\frac{cov(x,y)}{\sigma_x\sigma_y}$$

$$=\frac{6.22}{(2.5819)(2.5819)}$$

$$=0.9330$$

Mean and variance of linear combination of random variables:

suppose $x_1, x_2 \ldots x_n$ are n independent random variables with means $\mu_1, \mu_2 \ldots \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots \sigma_n^2$.

Then mean and variance of the linear combination of Y is given by

$$\mu_Y = \sum_{i=1}^{n} a_i \mu_i$$

$$\sigma_Y^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2$$

Here $\mu_Y = E(Y)$

$$= E\left(\sum_{i=1}^{n} a_i x_i\right)$$

$$= E[a_1x_1 + a_2x_2 + \ldots + a_nx_n]$$

$$= E(a_1x_1) + E(a_2x_2) + \ldots + E(a_nx_n)$$

$$= a_1E(x_1) + a_2E(x_2) + \ldots + a_nE(x_n)$$

$$= a_1\mu_1 + a_2\mu_2 + \ldots + a_n\mu_n$$

$$\mu_Y = \sum_{i=1}^{n} a_i \mu_i$$

$$\sigma_Y^2 = E[(Y - \mu_Y)^2]$$

$$= E\left[\left(\sum_{i=1}^{n} a_i x_i - \sum_{i=1}^{n} a_i \mu_i\right)^2\right]$$

$$= E\left(\sum_{i=1}^{n} a_i^2 (x_i - \mu_i)^2\right)$$

$= E[a_1^2(x_1-\mu_1)^2 + a_2^2(x_2-\mu_2)^2 + \cdots + a_n^2(x_n-\mu_n)^2]$

$= a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \cdots + a_n^2\sigma_n^2$

$$\sigma_Y^2 = \sum_{i=1}^{n} a_i^2\sigma_i^2$$

A random variable x has mean 10 and variance 2. Find the mean and variance of

i) $3x+1$

ii) $\frac{1}{2}x-2$

Given, $\mu = E(x) = 10$

$\sigma^2 = V(x) = 2$

i) $E(3x+1) = 3E(x)+1$

$= 3(10)+1$

$= 31$

$V(3x+1) = 3^2 V(x)$

$= 9(2) = 18$

ii) $E\left(\frac{1}{2}x-2\right) = \frac{1}{2}E(x) - 2$

$= \frac{1}{2}(10) - 2$

$= 3$

$V\left(\frac{1}{2}x-2\right) = \left(\frac{1}{2}\right)^2 V(x)$

$= \frac{1}{4}(2) = \frac{1}{2}$

A factory produces chocolate bunnies. The weights of the bunnies have mean 150gm and standard deviation is 3gm. They are packed in crates containing 12 bunnies each. The crates having mean weight 200gm and standard deviation 5gm.

i) Find the mean and variance of the weight of a full crate.

ii) The factory made a special edition bunny for Eister that was 5 times heavier. Find the mean and variance of the weight of bigger chocolate bunnies.

The mean weight of bunnies is $E(B)=150$gm

The variance weight of bunnies is $V(B)=9$gm

The mean weight of crates is $E(C)=200$gm

The variance weight of crates is $V(C)=25$gm

i) Mean of full crate:

$$E(F.C)=12\,E(B)+E(C)$$

$$=12\times150+200$$

$$=2000\text{gm}=2\text{kg}$$

Variance of full crate:

$$V(F.C)=12V(B)+V(C)$$

$$=12\times9+25$$

$$=133\text{ gms}$$

ii) $E(5B) = 5E(B)$

$= 5(150)$

$= 750 gms$

$V(5B) = 5^2 V(B)$

$= 25 \times 9$

$= 225 gms$

Chebyshev's theorm:

chebyshev's theorm states that the proportion or percentage of any dataset that lies with in k standard deviation of the mean.

where k is any positive real number ≥ 1

atleast $1 - \frac{1}{k^2}$ (or) chebyshev's inequality states that atleast $1 - \frac{1}{k^2}$ of data fro[m] a sample must fall with in the k stan[dard] deviation from the mean.

where k is positive real number ≥ 1

NOTE:

The distance between μ and $\mu + k\sigma$ is $k\sigma$

i.e. $\mu + k\sigma - \mu = k\sigma$

The distance between μ and $\mu - k\sigma$ is $k\sigma$

i.e $\mu - (\mu - k\sigma) = k\sigma$

The answer to $1-\frac{1}{k^2}$ is usually expressed as percentage after doing the computation.
The mean score of an insurance comission licensure examination is 75 with a standard deviation of 5. what percentage of the dataset lies between 50 and 100.

$\mu = 75$

$\sigma = 5$

lower value

$\mu - k\sigma = 50$

$75 - k(5) = 50$

$75 - 50 = k(5)$

$\frac{25}{5} = k$

$\boxed{k = 5}$

By chebyshev's inequality

$1-\frac{1}{k^2} = 1-\frac{1}{5^2}$

$= \frac{25-1}{25}$

$= \frac{24}{25} = 0.96$

$= 96\%$

$\therefore$ 96% of the dataset lies between 50 and 100.

The mean age of a flight attendant of a pal is 40 years old with standard deviation of 8. What percentage of dataset lies between 20 and 60.

$\mu = 40$

$\sigma = 8$

$\mu - k\sigma = 20$

$40 - k(8) = 20$

$20 = 8k$

$k = \frac{20}{8} = \frac{5}{2}$

$\boxed{k = 5/2}$

By using chebyshev's inequality

$1 - \frac{1}{k^2} = 1 - \frac{1}{\frac{25}{4}}$

$= 1 - \frac{4}{25}$

$= \frac{25-4}{25} = \frac{21}{25}$

$= 0.84$

$= 84\%$

The mean age of sales ladies in an ABC department store is 30 with a standard deviation of 6. between which two age limits must 75% of the dataset lie.

$\mu = 30$

$\sigma = 6$

$1 - \frac{1}{k^2} = 75\%$

$1 - \frac{1}{k^2} = \frac{3}{4}$

$1 - \frac{3}{4} = \frac{1}{k^2}$

$\frac{4-3}{4} = \frac{1}{k^2}$

$\frac{1}{4} = \frac{1}{k^2}$

$k^2 = 4$

$k = \pm 2$

$\therefore \boxed{k = 2}$

lower age limit $\mu - k\sigma = 30 - 2(6)$
$= 30 - 12 = 18$

upper age limit $\mu + k\sigma = 30 + 2(6)$
$= 30 + 12 = 42$

$\therefore$ The mean age of 30 with standard deviation 6 must lie between the age 18 and 42 to represent 75% of the dataset

The mean score on an accounting test is 80 with standard deviation of 10 betw which two scores must this mean line to represent 8/9 of the dataset

$\mu = 80$

$\sigma = 10$

$1 - \frac{1}{k^2} = \frac{8}{9}$

$1 - \frac{8}{9} = \frac{1}{k^2}$

$\frac{9-8}{9} = \frac{1}{k^2}$

$\frac{1}{9} = \frac{1}{k^2}$

$k^2 = 9$

$k = \pm 3$

$\boxed{k = 3}$

lower limit score $\mu - k\sigma = 80 - 3(10)$
$= 80 - 30 = 50$

upper limit score $\mu + k\sigma = 80 + 3(10)$
$= 80 + 30 = 110$

∴ The mean score on accounting test of 80 with standard deviation 10 must lie between the scores 50 and 110 to represent the dataset of 8/9.

Binomial distribution:-

A random variable x has a binomial distribution. If it assumes only non-negative values and its probability function is given by

$$p(X=r)=P(r)=\begin{cases} nC_r p^r q^{n-r}; & r=0,1,\dots n \quad q=1-p \\ 0 & ; \text{elsewhere} \end{cases}$$

Binomial probability distribution function

$$F_X(r)=P(X\le r)=\sum_{r=0}^{n} nC_r p^r q^{n-r}$$

i.e The probability that it will occur exactly r times out of n independent tries.

Mean of the binomial distribution:

The Binomial probability distribution is given by $p(r)=nC_r p^r q^{n-r}$; $r=0,1,\dots,n$
$q=1-p$

$$\mu=\sum_{r=0}^{n} r\,p(r)$$

$$=\sum_{r=0}^{n} r\cdot nC_r p^r q^{n-r}$$

$$=[0+1\cdot nC_1 p^1 q^{n-1}+2\cdot nC_2 p^2 q^{n-2}+\dots+nC_n p^n q^0]$$

$$=[np\,q^{n-1}+\frac{2n(n-1)}{2}p^2q^{n-2}+\dots+np^n]$$

$= np[q^{n-1} + (n-1)pq^{n-2} + \ldots + p^{n-1}]$

$= np[(n-1)c_0 p^0 q^{n-1} + (n-1)c_1 p^1 q^{(n-1)-1} + \ldots + (n-1)c_{n-1} p^{n-1} q^0]$

$= np[q+p]^{n-1}$ $\quad [(a+x)^n = nc_0 a^0 x^n + nc_1 a^1 x^{n-1} + \ldots + nc_n a^n]$

$\mu = np(1) = np$

$\boxed{\text{Mean} = \mu = np}$

Variance of the Binomial probability distribution:

$$\text{Variance } (\sigma^2) = E(x^2) - \mu^2$$

$$= \sum_{r=0}^{n} r^2 p(r) - \mu^2$$

$$= \sum_{r=0}^{n} (r^2 - r + r) p(r) - \mu^2$$

$$= \sum_{r=0}^{n} (r(r-1) + r) p(r) - \mu^2$$

$$= \sum_{r=0}^{n} r(r-1)p(r) + \sum_{r=0}^{n} r p(r) - \mu^2$$

$$= \sum_{r=0}^{n} r(r-1) nc_r p^r q^{n-r} + \mu - \mu^2$$

$$\sigma^2 = [0 + 0 + 2(2-1)nc_2 p^2 q^{n-2} + 3(3-1)nc_3 p^3 q^{n-3} + \ldots + n(n-1)nc_n p^n q^0] + \mu - \mu^2$$

$$= [2nc_2 p^2 q^{n-2} + 6nc_3 p^3 q^{n-3} + \ldots + n(n-1)p^n] + \mu - \mu^2$$

$$= \left[\frac{2n(n-1)}{2}p^2q^{n-2} + \frac{6n(n-1)(n-2)}{6}p^3q^{n-3} + \dots + n(n-1)p^n\right] + \mu - \mu^2$$

$$= n(n-1)p^2\left[q^{n-2} + (n-2)pq^{n-3} + \dots + p^{n-2}\right] + \mu - \mu^2$$

$$= n(n-1)p^2\left[(n-2)C_0 p^0 q^{n-2} + (n-2)C_1 p^{(n-2)-1} + \dots + p^{n-2}\right] + \mu - \mu^2$$

$$= n(n-1)p^2[q+p]^{n-2} + \mu - \mu^2$$

$$= n(n-1)p^2 + np - n^2p^2$$

$$\sigma^2 = n(np^2 - p^2) + np - n^2p^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p)$$

$$\sigma^2 = npq$$

$$\sigma = \sqrt{npq}$$

Bernoulli's distribution:-

A random variable x which takes two values 0 and 1. with probability q and p. i.e $P(x=0) = q$, $P(x=1) = p$. Then $q = 1-p$ is called Bernoulli's distribution and its probability function is given by

$P(x) = p^x q^{1-x}$, $x = 0, 1$

Mean:

Mean of the Bernoulli distribution is $\mu = p$

variance:

variance of the Bernoulli distribution is $\sigma^2 = pq$

$\sigma = \sqrt{pq}$

NOTE: let the probability for success be p and probability for failure be q are called Bernoulli tries.

Binomial distribution Eg:

- No. of defective bolts in a box containing n bolts
- No. of postgraduates in a group of n men.

1. A fair coin is tossed 6 times. Find the probability of getting 4 heads.

Here the event "to getting head",

$n = 6$

$r = 4$

p = probability of getting head

$p = 1/2$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

By Binomial distribution function

$p(r) = nc_r p^r q^{n-r}$

$P(4) = 6C_4 (1/2)^4 (\frac{1}{2})^{6-4}$

$= \frac{\overset{3}{\cancel{6}} \times 5 \times 4 \times 3}{\cancel{4 \times 3 \times 2} \times 1} (\frac{1}{2})^4 (\frac{1}{2})^2$

$= 15 \times \frac{1}{2^6}$

$\therefore P(4) = \frac{15}{64} = 0.23438$

10 coins thrown simultaneously. Find the probability of getting atleast

i) 7 heads

ii) 6 heads

iii) 1 head.

Here the event "to getting a head"

$n = 10$

p = probability to getting head

$p = 1/2$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

i) P(atleast 7 heads) $= P(x \geq 7) + P(x=8) + P(x=9) + P(x=10)$

$= 10C_7 (\frac{1}{2})^7 (\frac{1}{2})^3 + 10C_8 (\frac{1}{2})^8 (\frac{1}{2})^2 +$

$10C_9 (\frac{1}{2})^9 (\frac{1}{2}) + 10C_{10} (\frac{1}{2})^{10}$

$\left(\frac{1}{2}\right)^{10}\left[10C_7+10C_8+10C_9+10C_{10}\right]$

$\left(\frac{1}{2}\right)^{10}\left[120+45+10+1\right]$

$= \frac{176}{2^{10}} = 0.1718$

ii) $P(X\geq 6) = P(X=6)+P(X=7)+P(X=8)+P(X=9) + P(X=10)$

$= 10C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 + 0.1718$

$= 0.2050+0.1718$

$= 0.3768$

iii) $P(X\geq 1) = 1-P(X<1)$

$= 1-P(X=0)$

$= 1-10C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10}$

$= 1-\left[1.\frac{1}{2^{10}}\right]$

$= 1-(0.00098)$

$= 0.99902$

Determine the Binomial distribution for which mean is 4 and variance 3

Mean $= \mu = 4$

variance $(\sigma^2) = 3$

$np = 4$ — ①

$npq = 3$ — ②

$\frac{②}{①} = \frac{npq}{np} = \frac{3}{4}$

$q = \frac{3}{4}$

$p = 1 - q$

$= 1 - \frac{3}{4}$

$p = 1/4, q = 3/4$

substitute p in eqn ①

$np = 4$

$n(1/4) = 4$

$\boxed{n = 16}$

∴ The Binomial distribution is

$p(r) = nCr\, p^r q^{n-r}$

$= 16Cr \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{16-r}$

The mean and variance of a binomial distribution are 4 and 4/3 find $P(X \geq 1)$

$\mu = 4$

$\sigma^2 = 4/3$

$np = 4$ — ①

$npq = 4/3$ — ②

$\frac{②}{①} = \frac{npq}{np} = \frac{4}{3(4)}$

$q = \frac{1}{3}$

$p = 1 - q$

$1 - \frac{1}{3} = \frac{2}{3}$

$\therefore p = 2/3,\ q = 1/3$

Substitute p in eqn ①

$n(2/3) = 4^2$

$n = 6$

$P(X \geq 1) = \cancel{P}\ 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - 6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^{6-0}$

$= 1 - 6 \times \frac{1}{3^6}$

$= 1 - 0.00823$

$= 0.9986$

Two dice are thrown 5 times. find the probability of getting 7 as sum

i) atleast once

ii) exactly two times.

iii) $P(1 < x < 5)$

i) P = probability of getting sum 7 when 2 dice are thrown at a time

$$P = \frac{6}{36} = \frac{1}{6}$$

$$q = 1 - P = 1 - \frac{1}{6} = \frac{5}{6}$$

$$n = 5$$

i) $P(\text{atleast once}) = P(r \geq 1)$

$$= 1 - P(r < 1)$$

$$= 1 - P(r = 0)$$

$$= 1 - 5C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5$$

$$= 1 - \frac{5^5}{6^5} = 0.5981$$

ii) $P(\text{exactly 2 times}) = P(r = 2)$

$$= 5C_2\left(\frac{1}{2}\right)^2\left(\frac{5}{6}\right)^3$$

$$= 10 \cdot \frac{1}{36} + \frac{125}{216}$$

$$= 0.1607$$

iii) $P(1<x<5) = P(1<r<5)$

$= P(r=2) + P(r=3) + P(r=4)$

$= 5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 + 5C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2 + 5C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^1$

$= 10 \cdot \frac{1}{36} \cdot \frac{125}{218} + 10 \cdot \frac{1}{216} \cdot \frac{25}{36} + 5 \cdot \frac{1}{1296} \cdot \frac{5}{6}$

$= 0.1946$

If the probability of defective bolt is 1/8 find mean and variance for the distribution of the defective bolts of 640

Let P be the probability of a defective bolt

is $P = 1/8$

$1-p = 1-\frac{1}{8} = \frac{7}{8}$

$n = 640$

i) Mean $\mu = np$

$= 640 \times \frac{7}{8} = 80$

ii) variance $\sigma^2 = npq$

$= 640 \times \frac{1}{8} \times \frac{7}{8}$

$= 70$

Out of 800 families with 5 children each how many would you expect to have

i) three boys

ii) five girls

iii) either 2 or 3 boys

iv) atleast one boy

assume equal probabilities for boys and girls

Here the event is "a boy"

p = probability of a boy

$p = 1/2$

$q = 1 - p \Rightarrow 1 - \frac{1}{2} = \frac{1}{2}, n = 5$

i) $p(3 boys) = p(r=3)$

$= 5C_3 (\frac{1}{2})^3 (\frac{1}{2})^2$

$= 10 \cdot \frac{1}{25}$

$= 0.3125$

ii) No. of families having 3 boys are

$= 800 \times 0.3125$

$= 250$

ii) $p(5\text{ girls}) = p(r=0)$

$= 5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5$

$\therefore$ No. of families having 5 girls i.e no boys are $800 \times \frac{1}{2^5} = 25$

iii) $p(\text{either } 2 \text{ boys or } 3 \text{ boys})$

$p(r=2) + p(r=3)$

$5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$10 \times \frac{1}{2^5} + 10 \times \frac{1}{2^5}$

$= \frac{10}{2^4}$

$\therefore$ No. of families having 2 boys or 3 boys

$800 \times \frac{10}{2^4} = 500$

iv) $p(\text{atleast one boy})$

$= p(r \geq 1)$

$= 1 - p(r=0)$

$= 1 - 5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

$= 1 - \frac{1}{2^5} = \frac{31}{32}$

$\therefore$ No. of families having atleast one boy is $500 \times \frac{31}{32} = 775$

20% of items produced from a factory are defective find the probability that in a sample of 5 choosen at random.

i) None is defective

ii) 1 is defective

iii) $p(1<x<4)$

By binomial distribution we have

$p(x) = nC_x p^x q^{n-x}$

p = probability of defective bolts = 20%

$= 0.2$

$q = 1 - p$

$= 1 - 0.2 = 0.8, \; n = 5$

i) None is defective $= p(x=0)$

$= 5C_0 (0.2)^0 (0.8)^5$

$= 0.3276$

ii) p(None is defective) $= p(x=1)$

$= 5C_1 (0.2)^1 (0.8)^4$

$= 0.4096$

iii) $p(1<x<4) = p(x=2) + p(x=3)$

$= 5C_2 (0.2)^2 (0.8)^3 + 5C_3 (0.2)^3 (0.8$

$= 0.256$

Fit a binomial distribution of x value

x	0	1	2	3	4	5
f	2	14	20	34	22	8

$n=5$

$N=\Sigma f_i=2+14+20+34+22+8$

$N=100$

$$\text{Mean } \mu=\frac{\Sigma x_i f_i}{\Sigma f_i}=\frac{0\times2+1\times14+2\times20+3\times34+4\times22+5\times8}{100}$$

$$=\frac{284}{100}=2.84$$

$\mu=np=2.84$

$\therefore np=2.84$

$5p=2.84$

$p=\frac{2.84}{5}=0.568$

$q=1-p=1-0.568=0.432$

$\therefore n=5,\ N=100$

$p=0.568$

$q=0.432$

By using binomial dist, we have

$N(q+p)^n=100(0.432+0.568)^5$

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$(q+p)^n = nC_0 q^n + nC_1 q^{n-1} p + nC_2 q^{n-2} p^2 + \cdots + p^n$

$= 100[5C_0(0.432)^5 + 5C_1(0.432)^4(0.568) + 5C_2(0.432)^3(0.568)^2 + 5C_3(0.432)^2(0.568)^3$
$+ 5C_4(0.432)(0.568)^4 + 5C_5(0.568)^5]$

$= 100[0.0152 + 0.098 + 0.2601 + 0.3419 + 0.2248 + 0.0591]$

$= 1.52 + 9.8 + 26.0 + 34.19 + 22.48 + 5.91$

The above respective terms are expected (or) theoritical frequencies but frequencies should be round off values

x	0	1	2	3	4	5
f	2	14	20	34	22	8
Expected frequencies	2	10	26	34	22	6

x	0	1	2	3	4
F	38	144	342	287	164

Poisson Distribution:-

The poisson distribution can be derived as a limiting case of binomial distribution under the conditions that

i) n is very large

ii) p is very small

iii) $np = \lambda$ (say) is finite

Def: A random variable x is said to be a poisson distribution if it assumes only non negative values and its probability density function is given by

$$P(X=x) = p(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & ; x = 0,1,2 \ldots \\ 0 & ; \text{elsewhere} \end{cases}$$

Here $\lambda > 0$ is the parameter of the distribution.

NOTE:

$$\sum_{x=0}^{\infty} P(X=x) = \sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \ldots \infty\right]$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots \infty\right]$$

$$= e^{-\lambda}[e^{\lambda}] = e^0 = 1$$

Mean of the poisson distribution:

$$\mu = E(X)$$

$$= \sum_{x=0}^{\infty} x\,p(x)$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x(x-1)!}$$

put $x-1=y$

$x = y+1$

$x=1 \Rightarrow y=0$

$x=\infty \Rightarrow y=\infty$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x}}{(x-1)!}$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y+1}}{y!}$$

$$\mu = e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^{y}\lambda}{y!}$$

$$= e^{-\lambda}\lambda \sum_{y=0}^{\infty} \frac{\lambda^{y}}{y!}$$

$$= \lambda e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \infty\right]$$

$$= \lambda e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda(1) = \lambda$$

$$\mu = \lambda = np$$

Variance of the poisson distribution:

$$\sigma^2 = E(X^2) - (E(X))^2$$

$$= E(X^2) - \mu^2$$

$$= \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

$$= \sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!} - \mu^2$$

$$= \sum_{x=0}^{\infty} \frac{x^2 e^{-\lambda}\lambda^x}{x(x-1)!} - \mu^2$$

$$= e^{-\lambda}\sum_{x=1}^{\infty} \frac{x\lambda^x}{(x-1)!} - \mu^2$$

$$= e^{-\lambda}\sum_{x=1}^{\infty} \frac{(x-1+1)\lambda^x}{(x-1)!} - \mu^2$$

$$\sigma^2 = e^{-\lambda}\left[\sum_{x=1}^{\infty} \frac{(x-1)\lambda^x}{(x-1)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}\right] - \mu^2$$

$$= e^{-\lambda}\left[\sum_{x=2}^{\infty} \frac{(x-1)\lambda^x}{(x-1)(x-2)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}\right] - \mu^2$$

put $x-2=y$; $x-1=z$

$x = y+2$; $x = z+1$

$x=2 \Rightarrow y=0$; $x=1 \Rightarrow z=0$

$x=\infty \Rightarrow y=\infty$; $x=\infty \Rightarrow z=\infty$

$$e^{-\lambda}\left[\sum_{y=0}^{\infty}\frac{\lambda^{y+2}}{y!}+\sum_{z=0}^{\infty}\frac{\lambda^{z+1}}{z!}\right]-\mu^2$$

$$\sigma^2=e^{-\lambda}\left[\lambda^2\sum_{y=0}^{\infty}\frac{\lambda^y}{y!}+\lambda\sum_{z=0}^{\infty}\frac{\lambda^z}{z!}\right]-\mu^2$$

$$=e^{-\lambda}\left[\lambda^2\left[\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\cdots+\infty\right]+\lambda\left[\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\cdots+\infty\right]\right]-\mu^2$$

$$=e^{-\lambda}[\lambda^2e^{\lambda}+\lambda e^{\lambda}]-\mu^2$$

$$\sigma^2=e^{-\lambda}e^{\lambda}[\lambda^2+\lambda]-\mu^2$$

$$=\lambda^2+\lambda-\lambda^2$$

$$\boxed{\sigma^2=\lambda}$$

Recurrence relation for the poisson distribution:

$$P(x)=\sum_{x=0}^{\infty}\frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(x+1)=\sum_{x=0}^{\infty}\frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$$

$$=\sum_{x=0}^{\infty}\frac{e^{-\lambda}\lambda^x\lambda}{(x+1)\,x!}$$

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} \cdot \frac{\lambda}{x+1}$$

$$= P(x) \cdot \frac{\lambda}{x+1}$$

$$\therefore P(x+1) = \frac{\lambda}{x+1} P(x)$$

If a random variable has a poisson distribution such that $P(x=1) = P(x=2)$. find

i) Mean of the distribution

ii) $P(4)$

iii) $P(x \geq 1)$

iv) $P(1 < x < 4)$

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(x=1) = P(x=2)$$

$$\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\boxed{\lambda = 2}$$

i) Mean $= \mu = \lambda = 2$

ii) $P(4) = \dfrac{e^{-2}2^4}{4!} = \dfrac{e^{-2}(16)}{4 \times 3 \times 2}$

$$= e^{-2}\left(\frac{2}{3}\right)$$

$$= 0.0902$$

iii) $P(X \geq 1) = 1 - P(X=0)$

$= 1 - \frac{e^{-2} 2^0}{0!}$

$= 1 - e^{-2}$

$= 0.86466$

iv) $P(1 < x < 4) = P(x=2) + P(x=3)$

$= \frac{e^{-2} 2^2}{2!} + \frac{e^{-2} 2^3}{3!}$

$= e^{-2}\left[2 + \frac{8}{3}\right]$

$= e^{-2}\left[\frac{10}{3}\right] = 0.4511$

If 2 cards are drawn from a pack of 52 cards which are diamonds. By using poisson distribution. Find the probability of getting two diamonds atleast 3 times in 51 consecutive trials of 2 cards drawing each time

P = probability of getting 2 diamond cards from a pack of 52 cards

$= \frac{13C_2}{52C_2} = \frac{13 \times 12}{52 \times 51} = \frac{3}{51}$

$P = \frac{3}{51}$

$n =$ no. of trails $= 51$

$\lambda = np = 51 \times \frac{3}{51} = 3$

$\boxed{\lambda = 3}$

$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-3}3^x}{x!}$

i) $P(X \geq 3) = 1 - P(X < 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!}\right]$

$= 1 - [e^{-3} + e^{-3}3 + e^{-3}\frac{9}{2}]$

$= 1 - e^{-3}\left[1 + 3 + \frac{9}{2}\right]$

$= 1 - e^{-3}\left[\frac{17}{2}\right]$

$= 0.5769$

1. 2% of items of a factory are defective the items are packed in boxes. What is probability that their will be i) Two defective items ii) atleast 3 defective items in a box of 100 items

Given $n = 100$

p = probability of defective items = 2%

$= \frac{2}{100}$

$\lambda = np = 100 \times \frac{2}{100}$

$\boxed{\lambda = 2}$

By definition of poisson distribution we have

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-2}2^{x}}{x!}$$

i) p(2 defective item) $= p(x=2)$

$$= \frac{e^{-2}e^{2}}{2!} = \frac{e^{-2}(4)}{2}$$

$$= 2e^{-2}$$

$$= 0.2706$$

ii) p(atleast 3 item) $= p(x \geq 3)$

$$= 1 - p(x < 3)$$

$$= 1 - [p(x=0) + p(x=1) + p(x=2)]$$

$$= 1 - \left[\frac{e^{-2}2^{0}}{0!} + \frac{e^{-2}2^{1}}{1!} + \frac{e^{-2}2^{2}}{2!}\right]$$

$$= 1 - e^{-2}[1+2+2]$$

$$= 1 - 5e^{-2} = 0.3233$$

2. Average no. of accidents on any day on national highway is 1.8. Determine the probability that no. of accidents are
i) atleast one ii) atmost one

Given $\lambda = 1.8$

By def of poisson distribution, we have

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.8} \times 1.8^x}{x!}$$

i) P(atleast one) $= P(X \geq 1)$

$$= 1 - P(X=0)$$

$$= 1 - \left[\frac{e^{-1.8}(1.8)^0}{0!}\right]$$

$$= 1 - e^{-1.8}$$

$$= 0.8347$$

ii) P(atmost one) $= P(X \leq 1)$

$$= P(X=0) + P(X=1)$$

$$= \frac{e^{-1.8}(1.8)^0}{0!} + \frac{e^{-1.8}(1.8)^1}{1!}$$

$$= e^{-1.8}[1 + (1.8)]$$

$$= e^{-1.8}(2.8)$$

$$= 0.4628$$

3. suppose 2% of the people on the average are left handed. find i) The probability of finding 3 or more left handed.

ii) The probability of finding none or one left handed.

Given $\lambda = \frac{2}{100}$

i) By def of poisson distribution, we have

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.02}(0.02)^{x}}{x!}$$

$P(3 \text{ or more}) = P(x \geq 3)$

$= 1 - P(x < 3)$

$= 1 - P(x=0) + (x=1) + P(x=2)$

$$= 1 - \frac{e^{-0.02}(0.02)^{0}}{0!} + \frac{e^{-0.02}(0.02)^{1}}{1!} + \frac{e^{-0.02}(0.02)^{2}}{2!}$$

$$= 1 - e^{-0.02}\left[1 + 0.02 + \frac{(0.02)^{2}}{2!}\right]$$

$$= 1 - e^{-0.02}[1.02 + 0.0002]$$

$$= 1 - e^{-0.02}[1.0202]$$

$$= 1.3134 \times 10^{-6}$$

ii) p(none or one) $= p(x \le 1)$

$= p(x=0) + p(x=1)$

$= \frac{\bar{e}^{0.02} \times 0.02^0}{0!} + \frac{\bar{e}^{0.02} \times 0.02^1}{1!}$

$= \bar{e}^{0.02} + \bar{e}^{0.02} \times 0.02$

$= \bar{e}^{0.02}[1 + 0.02]$

$= \bar{e}^{0.02}[1.02]$

$= 0.9998$

4. The average no. of phone calls per minute coming in to a switch board between 2pm and 4pm is 2.5 determine the probability that during one particular minute their will be

i) 4 or fewel ii) more than 6 calls (0 to 6)

Given $\lambda = 5$

i) By definition of poisson distribution we have

$P(x) = \frac{\bar{e}^{\lambda} \lambda^{x}}{x!} = \frac{\bar{e}^{(2.5)}(2.5)^x}{x!}$

$p(x \le 4) = p(x=0) + p(x=1) + p(x=2) + p(x=3) +$

$p(x=4)$

$$= \frac{e^{-2.5} \times 2.5^0}{0!} + \frac{e^{-2.5} \times (2.5)^1}{1!} + \frac{e^{-2.5} \times (2.5)^2}{2!} + \frac{e^{-2.5}(2.5)^3}{3!} + \frac{e^{-2.5} + (2.5)^4}{4!}$$

$$= e^{-2.5}\left[1 + 2.5 + \frac{(2.5)^2}{2} + \frac{(2.5)^3}{6} + \frac{(2.5)^4}{24}\right]$$

$$= e^{-2.5}(10.8567)$$

$$= 0.8911$$

(ii) $p(x>6) = 1 - p(x<6)$

$$= 1 - [P(x=0) + p(x=1) + p(x=2) + p(x=3) + P(x=4) + P(x=5) + p(x=6)]$$

$$= 1 - \left[\frac{e^{-2.5} \times (2.5)^0}{0!} + \frac{e^{-2.5} \times (2.5)^1}{1!} + \frac{e^{-2.5} \times (2.5)^2}{2!} + \frac{e^{-2.5}(2.5)^3}{3!} + \frac{e^{-2.5}(2.5)^4}{4!} + \frac{e^{-2.5}(2.5)^5}{5!} + \frac{e^{-2.5}(2.5)^6}{6!}\right]$$

$$= 1 - e^{-2.5}\left[1 + 2.5 + \frac{(2.5)^2}{2} + \frac{(2.5)^3}{6} + \frac{(2.5)^4}{24} + \frac{(2.5)^5}{120} + \frac{(2.6)^2}{720}\right]$$

$$= 1 - e^{-2.5}[10.8567 + 0.8138 + 0.3390]$$

$$= 1 - e^{-2.5}[12.0095]$$

$$= 1 - e^{-2.5}$$

$= 1 - 0.9857$

$= 0.0143$

If x is a poisson variate such that $3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$. Then find

i) Mean of x

ii) $P(x \le 2)$

Given,

$3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$

$\frac{3e^{-\lambda}\lambda^4}{4!} = \frac{1}{2} \cdot \frac{e^{-\lambda}\lambda^2}{2!} + \frac{e^{-\lambda}\lambda^0}{0!}$

$\frac{3e^{-\lambda}\lambda^4}{24 \; 8} = \frac{e^{-\lambda}\lambda^2}{4} + e^{-\lambda}$

$\frac{\lambda^4}{8} = \frac{\lambda^2}{4} + 1$

$\frac{\lambda^4}{8_{\,2}} = \frac{\lambda^2 + 4}{4}$

$\lambda^4 = 2\lambda^2 + 8$

$\lambda^4 - 2\lambda^2 - 8 = 0$

$\lambda^4 - 4\lambda^2 + 2\lambda^2 - 8 = 0$

$\lambda^2(\lambda^2 - 4) + 2(\lambda^2 - 4) = 0$

$(\lambda^2 - 4)(\lambda^2 + 2) = 0$

$\lambda^2 - 4 = 0 \qquad \lambda^2 + 2 = 0$

$\lambda^2 = 4 \qquad \times$

$\lambda = \pm 2$

$\therefore \boxed{\lambda = 2}$

i) Mean $= \mu = \lambda = 2$

ii) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$$= \frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!}$$

$$= e^{-2}[1+2+2]$$

$$= 5e^{-2}$$

$$= 0.67667$$

using recurrence formula find the probabilities when $x = 0, 1, 2, 3, 4$ and 5. If the mean of the poisson distribution is 3

$\lambda = 3$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-3} 3^x}{x!}$$

By using recurrence relation, we have

$$P(x+1) = \frac{\lambda}{x+1} P(x) \quad \text{--- (1)}$$

put $x = 0$ in (1)

$$P(0+1) = \frac{3}{0+1} P(0)$$

$$= \frac{3 e^{-3} 3^0}{0!}$$

$$P(1) = 3e^{-3}$$

$$\boxed{P(1) = 0.1493}$$

put $x=1$ in ①

$$P(1+1) = \frac{3}{1+1} P(1)$$

$$P(2) = \frac{3}{2} \frac{e^{-3} 3^1}{1!}$$

$$= \frac{9}{2} e^{-3} = 0.2240$$

$$\boxed{P(2) = 0.2240}$$

put $x=2$ in ①

$$P(2+1) = \frac{3}{2+1} P(2)$$

$$= 1 \frac{e^{-3} 3^2}{2!}$$

$$= 0.2240$$

$$\boxed{P(3) = 0.2240}$$

put $x=3$ in ①

$$P(3+1) = \frac{3}{3+1} P(3)$$

$$= \frac{3}{4} \frac{e^{-3} 3^3}{3!}$$

$$= 0.1680$$

$$\boxed{P(4) = 0.1680}$$

put $x=4$ in ①

$$P(4+1) = \frac{3}{4+1} P(4)$$

$$= \frac{3}{5}(0.1680)$$

$$\boxed{P(5) = 0.1008}$$

Fit a poisson distribution to the following data

x	0	1	2	3	4	5
f	142	156	69	27	5	1

$N = \Sigma f_i = 142+156+69+27+5+1$

$= 400$

$\boxed{N = 400}$

$Mean = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{0\times142+1\times156+2\times69+3\times27+4\times5+5\times1}{400}$

$= 1$

$\boxed{Mean = \lambda = 1}$

In poisson distribution the theoritical frequencies are obtained by $NP(x)$

put $x=0$

$N.P(0) = \frac{400 e^{-1} 1^0}{0!} = 147.151$

$x=1$

$NP(1) = \frac{400 e^{-1} 1^1}{1!} = 147.151$

$x=2$

$NP(2) = \frac{400 e^{-1} 1^2}{2!} = 73.575$

$x=3$

$N.P(3) = \frac{400 \times e^{-1}(1)^3}{3!} = 24.525$

$x = 4$

$$N \cdot P(4) = \frac{400 \cdot e^{-1} (1)^4}{4!}$$

$= 6.131$

$x = 5$

$$N \cdot P(5) = \frac{400 e^{-1} 1^5}{5!}$$

$= 1.226$

x	0	1	2	3	4	5
f	142	156	69	27	5	1
Excepted frequencies	147	147	74	25	6	1

If $x = 0$