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UNIT-3

Normal distribution.

A random variable x is said to have normal distribution. if its density function is given by

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$ where

$-\infty < x < \infty$

$-\infty < \mu < \infty$

$\sigma > 0$

Find the normally distributed variate its mean 1 standard deviation 3 Find the probability that

i) $3.43 \leq 6.19$

ii) $-1.43 \leq x \leq 6.19$

i) $\mu = 1$, $\sigma = 3$

$z = \frac{x - \mu}{\sigma}$

$3.43 \leq x \leq 6.19$

put $x = 3.43$

$z = \frac{3.43 - 1}{3} = \frac{2.43}{3} = 0.81 = z_1$ (say)

put $x = 6.19$

$z = \frac{6.19 - 1}{3} = \frac{5.19}{3} = 1.73 = z_2$ (say)

clearly here z_1, z_2 are +ve

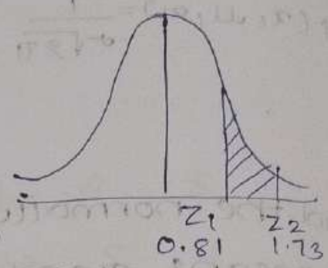
$P(x_1 \le X \le x_2) = P(z_1 \le z \le z_2)$

$= |A(z_2) - A(z_1)|$

$= |A(1.73) - A(0.81)|$

$= |0.4582 - 0.2910|$

$= 0.1672$



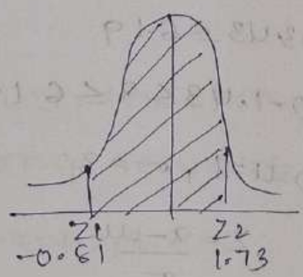
(ii) $-1.43 \le X \le 6.19$

let $x_1 = -1.43$

$z = \frac{x_1 - \mu}{\sigma} = \frac{-1.43 - 1}{3} = -0.81$

$x_2 = 6.19$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{6.19 - 1}{3} = 1.73$



when $z_1 < 0$ & $z_2 > 0$

then $p(z_1 \le z \le z_2) = A(z_2) + A(z_1)$ $[A(-z) = A(z)]$

$= A(1.73) + A(-0.81)$

$= 0.4582 + 0.2910$

$= 0.7492$

Let x is a normal variate with mean 30 and standard deviation 5.

Find, i) $P(x \geq 45)$ $P(Z > z_1)$

ii) $P(26 \leq x \leq 40)$ case I: If $z_1 > 0$

Given, $\mu = 30$, $\sigma = 5$ $P(Z > z_1) = 0.5 - A(z_1)$.

$P(X \geq 45)$

$X = 45$

$$Z = \frac{X-\mu}{\sigma} = \frac{45-30}{5} = \frac{15}{5} = 3 = z_1 (say)$$

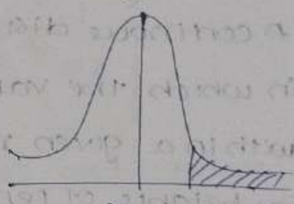
$P(X \geq 45) = P(Z \geq 3)$

$= 0.5 - A(z_1)$

$= 0.5 - A(3)$

$= 0.5 - 0.4987$

$= 0.0013$



ii) $P(26 \leq X \leq 40)$

Let $x_1 = 26$

$$z_1 = \frac{x_1-\mu}{\sigma} = \frac{26-30}{5} = \frac{-4}{5} = -0.8$$

$x_2 = 40$

$$z_2 = \frac{x_2-\mu}{\sigma} = \frac{40-30}{5} = \frac{10}{5} = 2$$

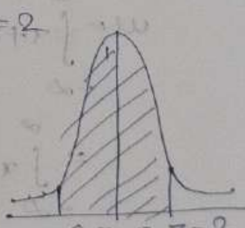
$P(X_1 \leq X \leq X_2) = P(z_1 \leq Z \leq z_2)$

$= A(z_2) + A(z_1)$

$= A(2) + A(-0.8)$

$= A(2) + A(0.8)$

$= 0.4772 + 0.2881 = 0.7653$



Continous uniform distribution:

It is the probability distribution of a random number selection from the continous interval between a and b.

→ It's density function is (determined) defined by

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{for } x < a \text{ or } x > b \end{cases}$$

A continous distribution is the distribution in which the variate takes all the values with in a given range

Ex: heights of persons, speed of vehicle.

Mean of the Normal distribution:

consider the normal distribution with b, σ is given by

$$f(x, b, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2}$$

The mean density function is

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_{-\infty}^{\infty} x f(x, b, \sigma) dx$$

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$$\mu = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2} dx$$

put

$\frac{x-b}{\sigma} = z$

$x - b = z\sigma$

$x = z\sigma + b$

$dx = \sigma dz$

$x = -\infty, z = -\infty$

$x = \infty, z = \infty$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (z\sigma + b)\, e^{-\frac{1}{2}(z)^2} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} z\sigma e^{-\frac{z^2}{2}} dz + \int_{-\infty}^{\infty} b e^{-\frac{z^2}{2}} dz \right.$$

$$\mu = \frac{1}{\sqrt{2\pi}} \left[\sigma \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}} dz + b \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz \right]$$

$\left[\begin{array}{l} z e^{-\frac{z^2}{2}} \text{ is an odd function} \\ e^{-\frac{z^2}{2}} \text{ is an even function} \end{array} \right]$

$$\mu = \frac{1}{\sqrt{2\pi}} \left[\sigma(0) + 2b \int_{0}^{\infty} e^{-\frac{z^2}{2}} dx \right.$$

$$\left[\int_{0}^{\infty} e^{-\frac{x^2}{2}} dx = \sqrt{\frac{\pi}{2}} \right]$$

$$\mu = \frac{1}{\sqrt{2\pi}} \left(2b \frac{\sqrt{\pi}}{\sqrt{2}} \right)$$

$$\mu = b$$

$$\therefore \boxed{\text{Mean} = \mu = b}$$

Variance of the Normal distribution

variance is given by

$\sigma^2 = E(x-\mu)^2$

$\sigma^2 = E(x-b)^2$

variance $\sigma^2 = \int_{-\infty}^{\infty} \cancel{x^2 f(x)} \; (x-b)^2 f(x)\,dx$

$= \int_{-\infty}^{\infty} (x-b)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2} dx$

$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-b)^2 e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2} dx$

put $\frac{x-b}{\sigma} = z$

$\Rightarrow x-b = z\sigma$

$x = z\sigma + b$

$dx = \sigma\,dz$

$x = -\infty$ to ∞

$z = -\infty$ to ∞

variance $= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (z\sigma)^2 e^{-\frac{z^2}{2}} \sigma\,dz$

$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} z^2 e^{-\frac{z^2}{2}} dz$ [$\because z^2 e^{-\frac{z^2}{2}}$ is an even fun)

put $\frac{z^2}{2} = t$

$z^2 = 2t$

$z = \sqrt{2t}$

$\frac{2zdz}{2} = dt$

$dz = \frac{dt}{z}$

$zdz = dt$

$z \to -\infty$ to ∞

$t \to -\infty$ to ∞

variance $= \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} 2t\, e^{-t} \frac{dt}{\sqrt{2t}}$

$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} t\, e^{-t} \frac{dt}{\sqrt{t}}$

$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} t\, e^{-t} \frac{dt}{\sqrt{t}}$

$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t}\, t\, t^{-1/2}\, dt$

$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t}\, t^{1-1/2}\, dt$

$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t}\, t^{1/2}\, dt$

$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t}\, t^{\frac{3}{2}-1}\, dt$

$= \frac{2\sigma^2}{\sqrt{\pi}} \Gamma\left(\frac{3}{2}\right)$

$= \frac{2\sigma^2}{\sqrt{\pi}} \left[\Gamma\left(\frac{1}{2}+1\right)\right]$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left[\frac{1}{2} \Gamma\left(\frac{1}{2}\right) \right]$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left[\frac{1}{2} \sqrt{\pi} \right]$$

$$var = \sigma^2$$

Standard deviation of normal distribution is

$\sqrt{variance} = \sqrt{\sigma^2} = \sigma$

Mode of the normal distribution:

Mode is the value of x for which $f(x, \mu, \sigma)$ is maximum i.e the mode is the solution of x $f'(x) = 0$, $f''(x) > 0$

Therefore, $x = \mu$ is the mode of normal distribution.

Median of the Normal distribution:

The Median is the point which can be divide the entire area into two equal parts.

If M is the medium of normal distribution then.
$$\int_{-\infty}^{M} f(x)\,dx = \int_{M}^{\infty} f(x)\,dx = \frac{1}{2}$$

$$Median\,(M) = \mu$$

In Normal distribution Mean = Median = Mode

Mean deviation from the median for a normal distribution is equal to $\frac{4}{5}$ times of S.D $\frac{4}{5} \times \sigma$.

Given that the mean heights of students in a class is 158 cm with standard deviation of 20 cm. find how many students heights lies b/w 150 cm and 170 cm if there are 100 students

$\mu = 158$ cms

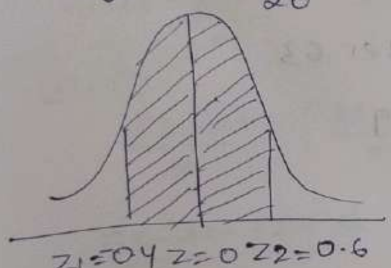
$\sigma = 20$ cms

$x = 150$ cms

$$z = \frac{x-\mu}{\sigma} = \frac{150-158}{20} = -0.40 = z_1 (\text{say})$$

$x = 170$ cms

$$z = \frac{x-\mu}{\sigma} = \frac{170-158}{20} = 0.60 = z_2 (\text{say})$$



Here $z_1 < 0$ and $z_2 > 0$ then

$$P(150 < X < 170) = P(z_1 < z < z_2)$$

$$= P(-0.4 < z < 0.6)$$

$$= A(z_2) + A(z_1)$$

$$= A(0.6) + A(-0.4)$$

$$= A(0.6) + A(0.4)$$

$$= 0.2258 + 0.1554$$

$$= 0.3812$$

∴ Number of students whose heights lies in between 150cm to 170cms are

$100 \times 0.3812 = 38.12$

$\simeq 38$ (app)

In a Normal distribution 7% of items are under 35, 89% are under 63. Determine the mean and variance of distribution.

: Let μ be the mean and σ be the standard deviation of the normal curve

7% of items are under 35

$P(X<35) = 7\% = 0.07$

89% of items are under 63

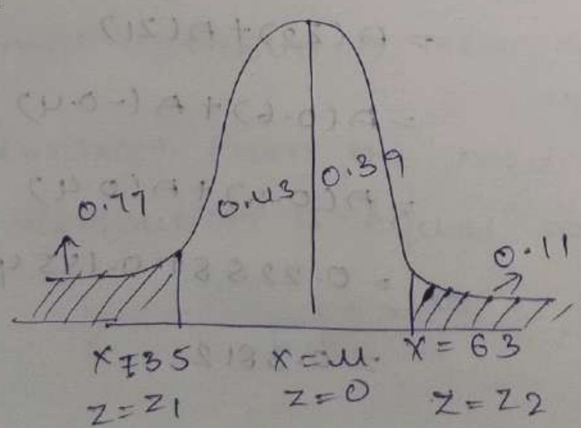
$P(X<63) = 89\% = 0.89$

$P(X>63) = 1 - P(X<63)$

$= 1 - 0.89$

$P(X>63) = 0.11$

The points $x=35$ & $x=63$ are as shown in figure



From the figure,

$P(0<Z<Z_1) = 0.43 = -Z_1$ (say) $= 1.48$ (from table)

$P(0<Z<Z_2) = 0.39 = Z_2$ (say) $= 1.23$

when $x = 35$

$$Z = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma} = -Z_1 \text{ (say)}$$

$$-1.48 = \frac{35-\mu}{\sigma}$$

when $x = 63$

$$Z = \frac{x-\mu}{\sigma} = \frac{63-\mu}{\sigma} = Z_2 \text{ (say)}$$

$$1.23 = \frac{63-\mu}{\sigma}$$

$$-1.48 = \frac{35-\mu}{\sigma} \quad -(1) \qquad 1.23 = \frac{63-\mu}{\sigma} \quad -(2)$$

(2) − (1)

$$\frac{63-\mu}{\sigma} - \left(\frac{35-\mu}{\sigma}\right) = 1.23 - (-1.48)$$

$$\frac{63-\mu-35+\mu}{\sigma} = 2.71$$

$$\frac{28}{\sigma} = 2.71$$

$$\frac{28}{2.71} = \sigma$$

$$10.332 = \sigma$$

$$\sigma^2 = 106.7502$$

Sub σ in eqn (2)

$$1.23 = \frac{63-\mu}{10.332}$$

$$1.23 \times 10.332 = 63 - \mu$$

$$\mu = 63 - 12.7083$$

$$\mu = 50.29$$

Mean $\mu = 50.29 \simeq 50.3$

$$\sigma^2 = 106.7502$$

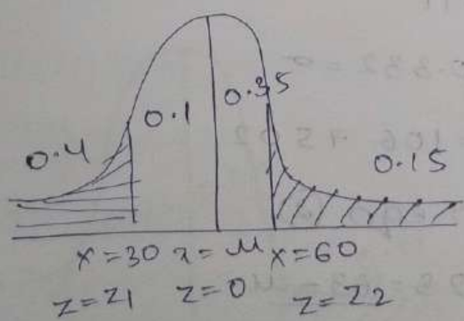
The Marks obtained in statistics in a certain examination found to be normally distributed if 15% of the students greater than or equal to 60 marks and 40% of the students less than 30 marks. Find Mean and standard deviation.

Let μ be the mean and σ be the standard deviation of the normal curve.

$$P(X<30) = 40\% = 0.4$$

$$P(X \geq 60) = 15\% = 0.15$$

The $x=30$ + $x=60$ are shown in below fig



from the fig

$p(0 < z < z_1) = 0.1 = -z_1 (say)$

$p(0 < z < z_2) = 0.35 = z_2 (say)$

$\therefore -z_1 = -0.25$

$z_2 = 1.04$ from tables

when $x = 30$

$$z = \frac{x-\mu}{\sigma} = \frac{30-\mu}{\sigma} = -z_1 (say)$$

$$\therefore -0.25 = \frac{30-\mu}{\sigma} \quad -(1)$$

$x = 60$

$$z = \frac{x-\mu}{\sigma} = \frac{60-\mu}{\sigma} = z_2 (say)$$

$$1.04 = \frac{60-\mu}{\sigma} \quad -(2)$$

$\therefore (2)-(1)$

$$\frac{60-\mu}{\sigma} - \left(\frac{30-\mu}{\sigma}\right) = 1.04-(-0.25)$$

$$\frac{60-\mu-30+\mu}{\sigma} = 1.29$$

$$\frac{30}{\sigma} = 1.29$$

$$\frac{30}{1.29} = \sigma$$

$$\sigma = 23.25$$

$$\sigma^2 = 540.5625$$

sub σ in eqn (2)

$$1.04 = \frac{60-\mu}{23.25}$$

$60-(1.04)(23.25)=\mu$

$\mu=35.82$

$\sigma=23.25$

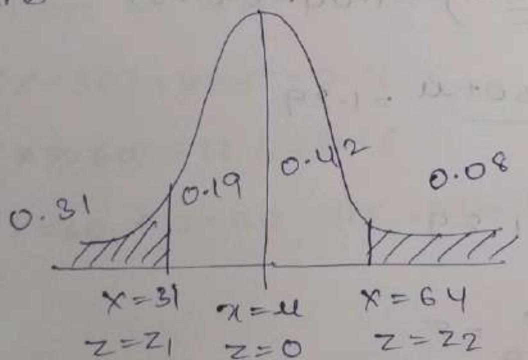
In a normal distribution 31% of items are under 45 and 8% are over 64. Find mean and variance of distribution

Let μ be the mean and σ be the standard deviation of the normal curve

$P(X<45)=31\%=0.31$

$P(X>64)=8\%=0.08$

The $x=31$ & $x=64$ are shown in below figure



from the figure

$P(0<z<z_1)=0.19 - z_1$ (say)

$P(0<z<z_2)=0.42=z_2$ (say)

$\therefore -z_1 = -0.50$

$z_2 = 1.40$ from tables

when $x = 31$

$$z = \frac{x-\mu}{\sigma} = \frac{31-\mu}{\sigma} = z_1 \text{ (say)}$$

$$-0.5 = \frac{31-\mu}{\sigma} \quad -①$$

$x = 64$

$$z = \frac{x-\mu}{\sigma} = \frac{64-\mu}{\sigma} = z_2 \text{ (say)}$$

$$1.40 = \frac{64-\mu}{\sigma} \quad -②$$

$\therefore$ ② − ①

$$\frac{64-\mu}{\sigma} - \left(\frac{31-\mu}{\sigma}\right) = 1.40-(-0.5)$$

$$\frac{64-\mu-31+\mu}{\sigma} = 1.90$$

$$\frac{33}{\sigma} = 1.90$$

$$\frac{33}{1.90} = \sigma$$

$$\sigma = 17.368$$

$$\sigma^2 = 301.6474$$

Sub σ in eqn ②

$$1.40 = \frac{64-\mu}{17.368}$$

$$1.40(17.368) = 64-\mu$$

$$\mu = 64 - 1.40(17.368)$$

$$\mu = 39.684$$

The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%. Determine i) how many students got marks above 90%.

ii) what was the highest mark obtained by the lowest 10% of the students.

iii) with in which limits did the middle of 90% of the students lie

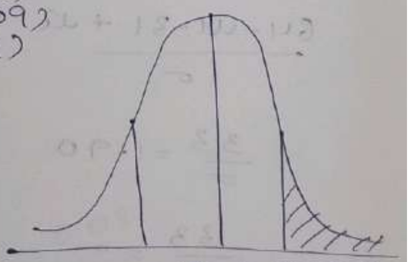
Given mean $\mu = 78\% = 0.78$

$\sigma = 11\% = 0.11$

i) $P(X > 90\%) = P(X > 0.9)$

$P(Z > 1.09)$

$= 0.5 - A(z_1)$



$z_1 = \frac{x - \mu}{\sigma}$

$= \frac{0.9 - 0.78}{0.11}$

$= 1.09$

$= 0.5 - A(1.09)$

$= 0.5 - 0.3621$

$= 0.1379$

Hence the no. of students marks > 9[0%] are $= 0.1379 \times 1000$

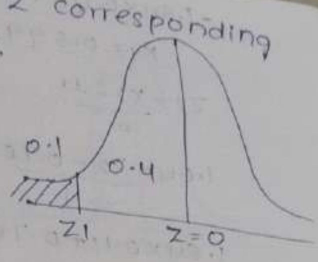
$= 137.9$

≈ 138

(ii) The area of 0.1 left to z corresponding to highest of lowest 10%.

$P(0 < Z < Z_1) = 0.4$

$-Z_1 = -1.28$



$\therefore Z_1 = \frac{x - \mu}{\sigma}$

$-1.28 = \frac{x - 0.78}{0.11}$

$-1.28 \times 0.11 = x - 0.78$

$-0.1408 = x - 0.78$

$-0.1408 + 0.78 = x$

$x = 0.6392$

The highest marks obtained by lowest of 10% is 0.6392×100

$= 64.92$

$= 64\%$

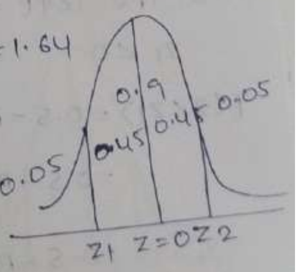
(iii) from the fig

$P(0 < Z < Z_1) = 0.45 = -Z_1 (\text{say}) = 1.64$

$P(0 < Z < Z_2) = 0.45 = Z_2 (\text{say})$

$= 1.64$

(from table)



$Z_1 = -1.64$, $Z_2 = 1.64$

$Z_1 = \frac{x_1 - \mu}{\sigma}$

$-1.64 = \frac{x_1 - 0.78}{0.11}$

$-1.64 \times 0.11 + 0.78 = x_1$

$x_1 = 0.5996$

$z_2 = \frac{x_2 - \mu}{\sigma}$

$1.64 = \frac{x_2 - 0.78}{0.11}$

$1.64 \times 0.11 + 0.78 = x_2$

$x_2 = 0.9604$

$\therefore x_1 = 0.5996 \times 100\%$

$= 59.96 \simeq 60$

$x_2 = 0.9604 \times 100$

$= 96.04 \simeq 96$

$\therefore$ The middle 90% of marks lies b/w 60 to 96

If x is a normal variate find the area

i) to the left of $z = -1.78$

ii) to the right of $z = -1.45$

iii) corresponding to $-0.8 \leq z \leq 1.53$

iv) to the left of $z = -2.52$ and to the right of $z = 1.83$

i) To the left of $z = -1.78$

$z_1 < 0$

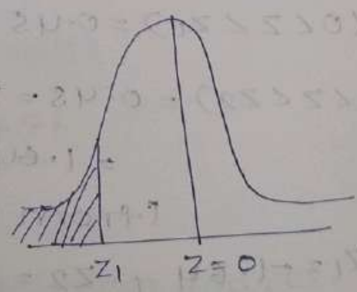
$P(z < z_1) = 0.5 - A(z_1)$

$= 0.5 - A(-1.78)$

$= 0.5 - A(1.78)$

$= 0.5 - 0.4625$

$= 0.0375$



ii) To the right of $z = -1.45$

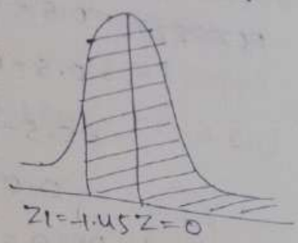
$z_1 = -1.45 < 0$

$P(Z > z_1) = 0.5 + A(z_1)$

$= 0.5 + A(-1.45)$

$= 0.5 + 0.4265$

$= 0.9265$



iii) corresponding to $0.8 \le z \le 1.53$

$z_1 = -0.8 < 0$

$z_2 = 1.53 > 0$

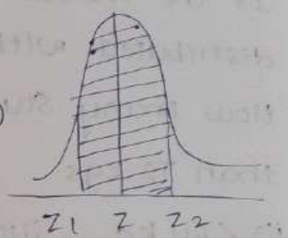
$P(z_1 < z < z_2) = A(z_2) + A(z_1)$

$= A(1.53) + A(-0.8)$

$= A(1.53) + A(0.8)$

$= 0.4870 + 0.2881$

$= 0.7251$



iv) To the left of $z = -2.52$ and to the right of $z = 1.83$

$z_1 < 0$

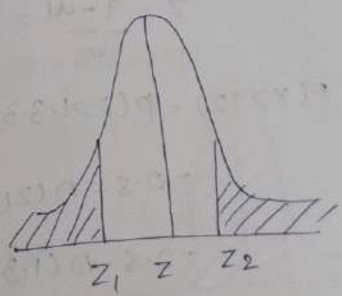
$P(Z < z_1) = 0.5 - A(z_1)$

$= 0.5 - A(-2.52)$

$= 0.5 - A(2.52)$

$= 0.5 - 0.4941$

$= 0.0059$



$Z_2 > 0$

$P(Z > Z_2) = 0.5 - A(Z_2)$

$= 0.5 - A(1.83)$

$= 0.5 - 0.4664$

$= 0.0336$

$\therefore$ Required Area $= P(Z < Z_1) + P(Z < Z_2)$

$= 0.0059 + 0.0336$

$= 0.0395$

If the masses of 300 students are normally distributed with mean 68 kg and S.D 3 kg. How many students have masses greater than 72 kgs.

ii) ≤ 64 kg iii) Between 65 and 71 inclusive.

Given, Mean $\mu = 68$ kg

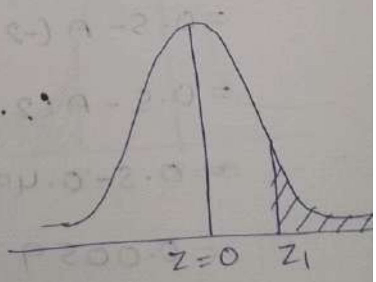
$\sigma = 3$

i) Greater than 72 kg i.e $P(X > 72)$

$x = 72$

$$Z = \frac{x - \mu}{\sigma} = \frac{72 - 68}{3} = 1.333 = Z_1 \text{ (say)}$$

$P(X > 72) = P(Z > 1.33)$



$= 0.5 - A(Z_1)$

$= 0.5 - A(1.33)$

$= 0.5 - 0.4032$

$= 0.0918$

No. of students masses > 72 kg are 300×0.0918

$= 27.54$

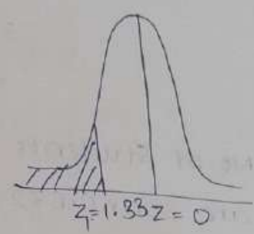
$= 28$ (app)

ii) less than or equal to 64 kgs $P(X \le 64)$

$x = 64$

$z = \frac{x - \mu}{\sigma} = \frac{64 - 68}{3}$

$= -1.33 = z_1$ (say)



$P(X \le 64) = P(z > -1.33)$

$= 0.5 - A(z_2)$

$= 0.5 - A(1.33)$

$= 0.5 - 0.4082$

$= 0.0918$

No. of students masses ≤ 64 kg are 300×0.0918

$= 27.54$

$\simeq 28$

iii) Between 65 and 71 inclusive i.e

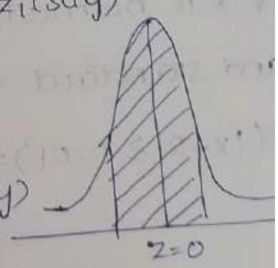
$P(65 \le X \le 71)$

$x_1 = 65$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{65 - 68}{3} = -1.0 = z_1$ (say)

$x_2 = 71$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{71 - 68}{3} = 1 = z_2$ (say)



$P(65 \le x \le 71) = P(-1 \le z \le 1)$

$= A(z_2) + A(z_1)$

$= A(1) + A(-1)$

$= 2A(1)$

$= 2(0.3413)$

$= 0.6826$

$\therefore$ No. of students of masses between 65 & 71 kg

are 300×0.6826

$= 204.78$

$\simeq 205$

Suppose 10% of the probability for a normal distribution is below 35 and 5% above 90. Find mean and standard deviation of distribution

- The Mean and standard deviation of a normal variable are 8 and 4 respectively. Find i) $P(5 \le x \le 10)$ ii) $P(x \ge 5)$

If x is normally distributed with mean 2 and standard deviation 0.1. Then find

$P(|x-2| \ge 0.01) =$

$P(|x-2| \geq 0.01) =$

Mean $= \mu = 2$

$S.D = \sigma = 0.1$

$|x-2| = 0.01$

$x-2 = \pm 0.01$

$x-2 = -0.01$ $\quad$ $x-2 = 0.01$

$x = -0.01+2$ $\quad$ $x = 0.01+2$

$x_1 = 1.99$ $\quad$ $x_2 = 2.01$

$$z = \frac{x_1-\mu}{\sigma} = \frac{1.99-2}{0.1} = -0.1 = z_1 (\text{say})$$

$$z = \frac{x_2-\mu}{\sigma} = \frac{2.01-2}{0.1} = 0.1 = z_2 (\text{say})$$

Here $z_1 < 0, z_2 > 0$

$\therefore P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2)$

$= P(-0.1 \leq z \leq 0.1)$

$= A(0.1) + A(-0.1)$

$= A(0.1) + A(0.1)$

$= 2A(0.1)$

$= 2 \times 0.0398$

$= 0.0796$

$P(|x-2| \geq 0.01) = 1 - P(|x-2| < 0.01)$

$= 1 - 0.0796$

$= 0.9204$

$\therefore P(|x-2| \geq 0.01) = 0.9204$

Normal approximation to the Binomial distribution:

The Normal distribution can be used to approximate the binomial distribution suppose the number of success x ranges from x_1 to x_2.

Then the probability of getting x_1 to x_2 success is given by

$$\sum_{r=x_1}^{x_2} nc_r p^r q^{n-r}$$

For large 'n' the calculation of binomial probabilities is very difficult

In such cases the binomial distribution can be replaced by Normal distribution and the required probability is computed

we consider two cases:

Case I: $p=q=\frac{1}{2}$

Even when n is not too large the binomi distribution can be approximated by normal distribution.

Mean of the binomial distribution is $\mu=$

and standard deviation $\sigma=\sqrt{npq}$

Hence for the corresponding normal distribution μ and σ are known.

We know that $z = \frac{x-\mu}{\sigma}$.

Let z_1 & z_2 be the values of z corresponding to x_1 & x_2 respectively. Then

$$p(x_1 \le x \le x_2) = p(z_1 \le z \le z_2)$$

$$= \int_{z_1}^{z_2} \phi(z)\, dz$$

This can be determined by using normal tables.

case 2: when $p \neq q \neq \frac{1}{2}$

For large n we can approximate the binomial distribution by the normal distribution and calculate the probability

→ For any success x the real class interval is $\left(x-\frac{1}{2}, x+\frac{1}{2}\right)$

Here z_1 must correspond to the lower limit of x_1 class and z_2 to the upper limit of x_2 class. Hence, z_1

$$z_1 = \frac{\left(x_1-\frac{1}{2}\right)-\mu}{\sigma} = \frac{x_1-\frac{1}{2}-np}{\sqrt{npq}}$$

$$z_2 = \frac{\left(x_2+\frac{1}{2}\right)-\mu}{\sigma} = \frac{x_2+\frac{1}{2}-np}{\sqrt{npq}}$$

Hence the required probability

$$\int_{z_1}^{z_2} \phi(z)\,dz \text{ can be calculated}$$

by using normal tables.

Find the probability that out of 100 patients between 84 and 95 inclusive will survive a heart operation given that chances of survival is 0.9.

Given

$n = 100$

p = chance of survival = 0.9

$q = 1-p = 1-0.9 = 0.1$

$\therefore p = 0.9,\ q = 0.1,\ n = 100$

Mean $= \mu = np$

$= 100 \times 0.9 = 90$

$S.D = \sigma = \sqrt{npq} = \sqrt{90 \times 0.1} = \sqrt{9} = 3$

By Binomial distribution

$$\sum_{r=84}^{95} 100cr\,(0.9)^r (0.1)^{100-r}$$

The sum of all the probabilities which is difficult to avoid this. we replace Binomial distribution by Normal distribution

$x_1 = 84$

$$z_1 = \frac{(x_1 - 1/2) - \mu}{\sigma} = \frac{84 - \frac{1}{2} - 90}{3} = \frac{-13}{6}$$

$= -2.166$

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$x_2 = 95$

$$z_2 = \frac{(x_2 + 1/2) - \mu}{\sigma} = \frac{95 + \frac{1}{2} - 90}{3} = \frac{11}{6} = 1.833$$

Here $z_1 < 0$ & $z_2 > 0$

$$\therefore P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$$
$$= P(-2.166 \le z \le 1.833)$$
$$= A(z_2) + A(z_1)$$
$$= A(1.833) + A(-2.166)$$
$$= A(1.833) + A(2.166)$$
$$= 0.4664 + 0.4846$$
$$= 0.9610$$

8 coins are tossed together find the probability of getting 1 to 4 heads in a single toss.

Given, $n = 8$

p = probability of getting head $= 1/2$

$q = 1/2$

$\mu = np$

$= 8 \times \frac{1}{2} = 4$

S.D $\sigma = \sqrt{npq} = \sqrt{8 \times \frac{1}{4}} = \sqrt{2}$
$= 1.414$

$x_1 = 1$

$$z_1 = \frac{\left(x_1 - \frac{1}{2}\right) - \mu}{\sigma} = \frac{1 - \frac{1}{2} - 4}{1.414} = -2.474$$

$$z_2 = \frac{\left(x_2+\frac{1}{2}\right)-\mu}{\sigma} = \frac{4+\frac{1}{2}-4}{\sqrt{2}}$$

$= 0.3535$

Here $z_1 < 0$ & $z_2 > 0$

$\therefore P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$

$= P(-2.474 \le z \le 0.3535)$

$= A(z_2) + A(z_1)$

$= A(0.3535) + A(-2.474)$

$= A(0.3535) + A(2.474)$

$= 0.1368 + 0.4932$

$= 0.63$

Find the probability of getting an even number on faces 3 to 6 times in throwing in throwing 10 dice together

$n = 10$

p = probability of getting even number on faces $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$\mu = np$

$= 10\left(\frac{1}{2}\right) = 5$

$\sigma = \sqrt{npq} = \sqrt{10 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{2.5} = 1.581$

$x_1 = 3$

$$z_1 = \frac{\left(x_1 - \frac{1}{2}\right) - \mu}{\sigma}$$

$$= \frac{3 - \frac{1}{2} - 5}{1.581} = -1.581$$

$x_2 = 6$

$$z_2 = \frac{\left(x_2 + \frac{1}{2}\right) - \mu}{\sigma} = \frac{\left(6 + \frac{1}{2} - 5\right)}{1.58} = 0.9487$$

Here $z_1 < 0$ & $z_2 > 0$

$= A(1.5811) + A(0.9487)$

$= 0.4429 + 0.3264$

$= 0.7693$

Find the probability of guesswork a student can correctly answer 25 to 30 in a multiple choice quiz consisting of 80 questions. assume that in each question with 4 choices only 1 choice is correct and student has no knowledge of subject

$n = 80,\ p = 1/4,\ q = 3/4$

$\mu = np$

$= 80\left(\frac{1}{4}\right) = 20$

$S.D = \sigma = \sqrt{npq} = \sqrt{20 \cdot 80 \cdot 1/4} = \sqrt{15}$

$x_1 = 25$

$$z_1 = \frac{\left(x_1 - \frac{1}{2}\right) - \mu}{\sigma}$$

$$= \frac{25 - \frac{1}{2} - 20}{\sqrt{15}} = 1.1618$$

$$z_2 = \frac{\left(x_2 + \frac{1}{2}\right) - \mu}{\sigma} = \frac{30 + \frac{1}{2} - 20}{\sqrt{15}} = 2.7110$$

Here $z_1 > 0$ & $z_2 > 0$

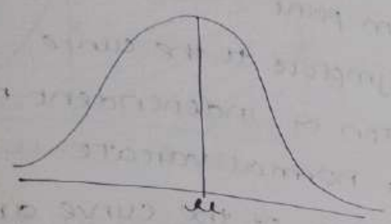
$$P(x_1 \le x \le z_2) = |A(z_2) - A(z_1)|$$

$$= A(2.7110) - A(1.1618)$$

$$= 0.4966 - 0.3770$$

$$= 0.1196$$

Shape of the Normal distribution:
The shape of the Normal distribution is bell shaped curve and which is symmetrical about x-axis:

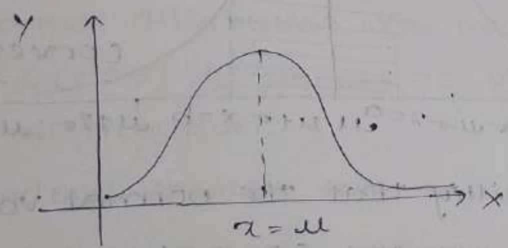


The total area bounded by the curve and the x-axis is 1.

i.e $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Chief characteristics of Normal distribution:

1) The graph of the normal distribution $y=f(x)$ in the XY-plane is known as normal curve



2) The curve is a bell shaped curve and symmetrical about the x-axis line $x=\mu$ and the two tails on the right and left sides of the mean(μ) extends to infinity.

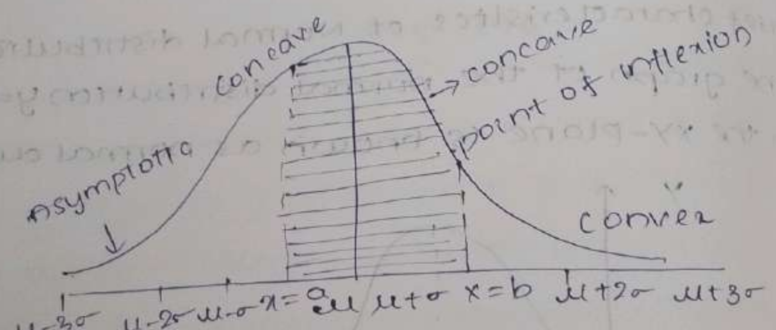
3) Area under the normal curve represents the total population

4) Mean, median and mode of distribution coincide at $x=\mu$ so the normal curve has only one maximum point

5) x-axis is an asymptote to the curve

6) Linear combination of independent normal variates is also a normal variate

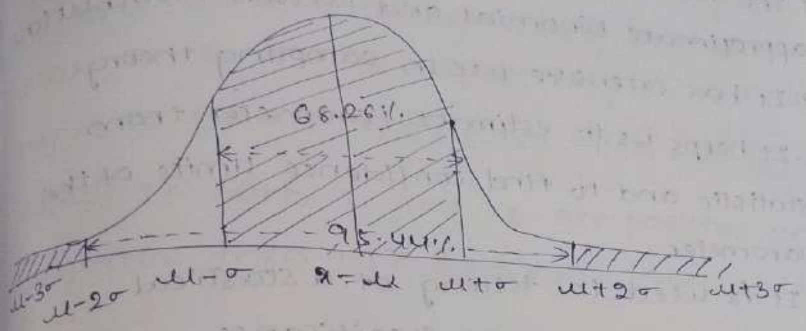
7) The points of inflexion of the curve are at $x=\mu\pm\sigma$ and the curve changes from concave to convex at $x=\mu+\sigma$ and $x=\mu-\sigma$



8) The probability that the normal variate x with mean μ and S.D. σ lies between x_1 and x_2 is given by

$$P(x_1 \le X \le x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

4) Area under the normal curve is distribution as follows



i) Area of normal curve between $\mu-\sigma$ and $\mu+\sigma$ is 68.26%.

ii) Area of normal curve between $\mu-2\sigma$ and $\mu+2\sigma$ is 95.44%.

iii) Area of normal curve between $\mu-3\sigma$ and $\mu+3\sigma$ is 99.73%.

Standard normal distribution:

The normal distribution with mean(μ) = 0 and S.D(σ) = 1 is known as standard normal distribution.

The random variable that follows a distribution is denoted by z and z can be defined as

$z = \frac{x-\mu}{\sigma}$ has a standard normal deviation with mean = 0 and S.D = 1

i.e mean $= E(z) = E\left(\frac{x-\mu}{\sigma}\right) = E\left(\frac{x}{\sigma}\right) - E\left(\frac{\mu}{\sigma}\right) =$

$\frac{1}{\sigma}E(x) - \frac{\mu}{\sigma} = \frac{1}{\sigma}\mu - \frac{\mu}{\sigma} = 0$

Uses of Normal distribution:

1. The Normal distribution can be used to approximate Binomial and poisson distribution.
2. It has extensive use in sampling theory.
3. It helps us to estimate parameter from statistic and to find confidence limits of the parameter.
4. It is used in testing of a stastical hypothesis and tests of significance.

Area under the normal curve:

By taking $z = \frac{x-\mu}{\sigma}$, the standard normal curve is formed by

The total area under the curve is devided in to two equal parts from $z=0$, left handside area and right hand side area to $z=0$ i.e 0.5.

The area between the ordinate $z=0$ and any other ordinate can be noted from the table of normal curve.

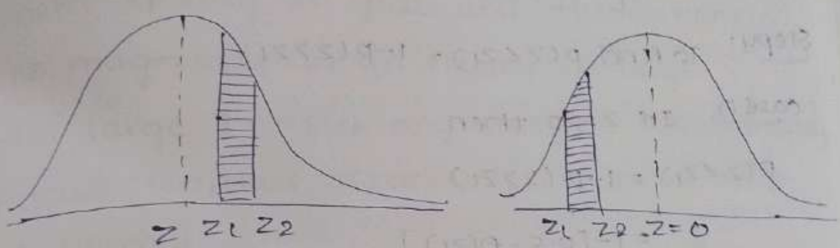
To find the probability of a normal curve

The probability that the normal variate x with mean μ and standard deviation σ lies between two specific values x_1 and x_2 with $x_1 \leq x_2$ can be obtained using area under the standard normal curve as follows.

step1: perform the change of scale $z = \frac{x-\mu}{\sigma}$ and find z_1 and z_2 corresponding to the values of x_1 and x_2 respectively.

step2: To find $P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$

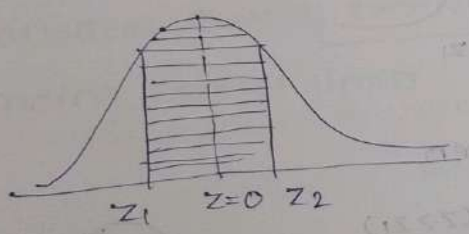
case(i) If both z_1 and z_2 are positive or negative then $P(x_1 \le x \le x_2) = |A(z_2) - A(z_1)|$



case(ii)

If $z_1 < 0$ and $z_2 > 0$ then

$P(x_1 \le x \le x_2) = A(z_2) + A(z_1)$

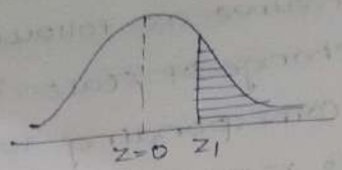


Step 3:

To find $P(z > z_1)$

case(i) If $z_1 > 0$ then

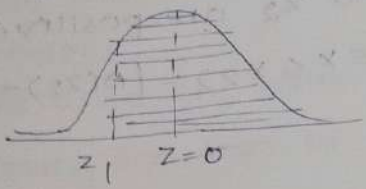
$P(z > z_1) = 0.5 - A(z_1)$



case(ii)

If $z_1 < 0$ then

$P(Z > z_1) = 0.5 + A(z_1)$



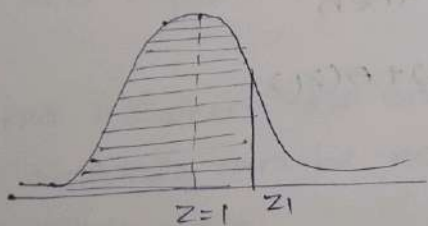
Step4: To find $P(Z < z_1) = 1 - P(Z > z_1)$

case(i): If $z_1 > 0$ then

$P(Z < z_1) = 1 - P(Z > z_1)$

$= 1 - [0.5 - A(z_1)]$

$= 0.5 + A(z_1)$



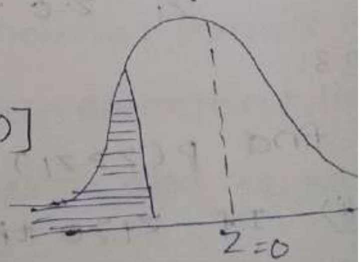
case(ii)

If $z_1 < 0$ then

$P(Z < z_1) = 1 - P(Z > z_1)$

$= 1 - [0.5 + A(z_1)]$

$= 0.5 - A(z_1)$



Application of the Normal distribution:

1) Normal distribution is used to oblained data from psychological, physical and Briological measurements approximately.

2) It is used in I.Q scores, heights and weights of individuals

3) Normal distribution is a limiting case of Binomial distribution. so It is applicable in kinetic theory of gases and fluctuations in the magnitude of an electric current

4) For large samples any statisitc approximately follows Normal distribution with the help of normal curve

5) Normal curve is used to find confidence limits of the populaiton paramelers.

6) Normal distribution find, large application in Statistical Quality control in industry for finding control limits.

Sampling distribution:

population:

A finite collection of objects like things, animals, plants group of people all possible outcomes from some complicated engineering system or biological or numerical data is called population.

Finite population:

The population which can be countable is known as finite population.

Ex: No. of students in a class ...

the no. of alphabets in english

the goods manufactured in factory....

Infinite population:

The population which cannot be countable is known as infinite population.

Ex: stars in the sky....

population size:

The no. of observations in a population is known as population size

It is denoted by N.

Sample:

A finite subset of a population is known as sample

sample size:
The no of observations in a sample are known as sample size and it is denoted by 'n'.

large samples:
If the sample size $n \geq 30$. then such a sample is said to be large samples.

small samples:
If the sample size $n < 30$. Then the sample is said to be small sample.

parameter:
The Mean, Median, Mode, S.D, variance measures of the population are called parameter

population mean (μ), population S.D(σ)
population variance (σ^2)

Statistic:-
The measures obtained from the sample of the population are called as stastic.

sample mean ($\bar{x}$)
sample variance (s^2)
sample S.D (s)
Sample proportion (p) are the statistics.

Sampling distribution:
The probability distribution of a stastic obtained from a large no. of samples

drawn from a specific population.

Sample mean:

If $x_1, x_2, x_3 \cdots x_n$ represents a set of random sample of sample size n. Then the sample mean is defined and denoted as

$$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$$

Sample Variance:

If $x_1, x_2 \cdots x_n$ represents a set of random samples of sample size n. Then the sample variance is defined and denoted as

$$\boxed{S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Here S^2 is defined to be the average of the squares of the deviations of the observations from their mean.

Sample S.D:

The sampling distribution of S.D is the +ve square root of sample variance.

$$\boxed{S = \sqrt{\sum \frac{(x_i - \bar{x})^2}{n-1}}}$$

standard error of statistic:

The standard error of the statistic is the S.D of the sampling distribution of the statistic.

→ It is used for accessing the difference between the expected value and observed value.

→ It also enable us to determine the confidence limits within which the parameters are expected to live.

central limit theorm:

If $\bar{x}$ be the mean of sample size n drawn from a population with mean μ and S.D σ then the standardized sample mean is given by

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

is a random variable whose distribution function approaches that of the standard Normal distribution i.e

$N(z; 0, 1)$ as $n \to \infty$

Formulas for standard Error:

i) S.E of $\bar{x} = \frac{\sigma}{\sqrt{n}}$

ii) S.E of $P = \sqrt{\frac{PQ}{n}}$

iii) S.E of $s = \frac{\sigma}{\sqrt{2n}}$

iv) S.E of $(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

where $\bar{x}_1, \bar{x}_2$ are sample means of random samples, sizes n_1, n_2 drawn from two populations with S.D σ_1, σ_2 respectively.

v) SE of $(P_1 - P_2) = \sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$

where P_1, P_2 are sample proportions of random samples of sizes n_1, n_2 draw from two populations with proportions P_1, P_2 respectively.

vi) S.E of $(S_1 - S_2) = \sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$

For a finite population of population size n when the sample is drawn without replacement

i) standard error of sample mean $\bar{x}$

$$\bar{x} = \frac{\sigma}{\sqrt{n}} \left[\sqrt{\frac{N-n}{N-1}} \right]$$

(ii) standard error of sample proportion

$$P = \left(\sqrt{\frac{PQ}{n}}\right)\sqrt{\frac{N-n}{n-1}}$$

For an infinite population the sample is drawn with replacement

i) standard error $\bar{x} = \frac{\sigma}{\sqrt{n}}$

standard error of sample proportion

$$P = \sqrt{\frac{PQ}{n}}$$

NOTE: Here $\frac{N-n}{n-1}$ is called finite population correction factor

sampling with replacement:

If each element of a population may be selected more than once. Then it is called sampling with replacement (infinite population)

sampling without replacement:

If each element cannot be selected more than once. Then it is called sampling without replacement

NOTE: If N is the size of the population and n is the sample size

i) The No. of samples with replacement is N^n

ii) No. of samples without replacement is $^{N}C_{n}$

iii) If $x_1, x_2, \ldots x_n$ are the population values and N is the population size then

i) population mean $\mu = \frac{\sum_{i=1}^{n} x_i}{N}$

ii) population variance $\sigma^2 = \frac{\sum_{i=1}^{n} x_i^2}{N} - \mu^2$

Sampling distribution of Mean (σ known)

Infinite population:

Suppose the samples are drawn from an infinite population i.e (with replacement)

Then

i) Mean of the sampling distribution of Mean is

$$\mu_{\bar{x}} = \frac{\mu + \mu + \mu + \ldots + \mu \text{ times}}{n}$$

$$= \frac{n\mu}{n} = \mu$$

$$\boxed{\mu_{\bar{x}} = \mu}$$

ii) variance of the sampling distribution of Means is

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

iii) standard deviation of the sampling distribution of means is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

finite population:

suppose the samples are drawn from a finite population i.e (without replacement) Then i) Mean of the sampling distribution of means is $\mu_{\bar{x}} = \mu$

ii) variance of the sampling distribution of mean is $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$

iii) S.D of the sampling distribution of mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\left[\sqrt{\frac{N-n}{N-1}}\right]$$

Find the value of finite population correction factor for $n=10$, $N=1000$

population size $N = 1000$.

sample size $n = 10$

$\therefore$ correction factor $= \frac{1000-10}{1000-1}$

$= 0.9909$

$n = 5$, $N = 200$

population size $N = 200$

sample size $n = 5$.

$\therefore$ correction factor $= \frac{200-5}{200-1}$

$= 0.9798$

A population consist of 5 numbers 2,3,6,8,11 consider all possible samples of size 2 can be drawn with replacement from this population & without replacement find

i) Mean of the population

ii) S.D of the population

iii) Mean of the sampling distribution of means

iv) S.D of the sampling distribution of means

population values 2,3,6,8,11

population size N=5

i) Mean of the population

$$\mu = \frac{\Sigma x_i}{N}$$

$$\mu = \frac{2+3+6+8+11}{5}$$

$$\boxed{\mu = 6}$$

ii) Variance of the population

$$\sigma^2 = \frac{\Sigma x_i^2}{N} - \mu^2$$

$$= \frac{4+9+36+64+121}{5} - 36$$

$$= 10.8$$

$$\sigma = \sqrt{10.8} = 3.2863$$

S.D of population $\boxed{\sigma = 3.2863}$

To find mean of sampling distribution of means

population size N=5

sample size n=2

No. of samples in a finite population are $N^n = 5^2 = 25$

Listing of all possible samples with size 2 from the population values are as follows

(2,2) (2,3) (2,6) (2,8) (2,11)

(3,2) (3,3) (3,6) (3,8) (3,11)

(6,2) (6,3) (6,6) (6,8) (6,11)

(8,2) (8,3) (8,6) (8,8) (8,11)

(11,2) (11,3) (11,6) (11,8) (11,11)

The means of all 25 samples are as follows

2	2.5	4	5	6.5
2.5	3	4.5	5.5	7
4	4.5	6	7	8.5
5	5.5	7	8	9.5
6.5	7	8.5	9.5	11

This is called sampling distribution of means

(iii) The mean of the sampling distribution of means is

$$\mu_{\bar{x}} = \frac{2+2.5+4+5+6.5+2.5+3+4.5+5.5+7+4+4.5+6+7+8.5+5+5.5+7+8+9.5+6.5+7+8.5+9.5+11}{25}$$

$$= \frac{150}{25} = 6$$

$$\boxed{\mu_{\bar{x}} = 6}$$

$$\sigma_{\bar{x}}^2 = \frac{(2-6)^2+(2.5-6)^2+(4-6)^2+(5-6)^2+(6.5-6)^2+(2.5-6)^2+(3-6)^2+(4.5-6)^2+(5.5-6)^2+(7-6)^2+(4-6)^2+(4.5-6)^2+(6-6)^2+(7-6)^2+(8.5-6)^2+(5-6)^2+(5.5-6)^2+(7-6)^2+(8-6)^2+(9.5-6)^2+(6.5-6)^2+(7-6)^2+(8.5-6)^2+(9.5-6)^2+(11-6)^2}{25}$$

$$= \frac{16+12.25+4+1+0.25+12.25+9+2.25+0.25+1+4+2.25+1+6.25+1+0.25+1+4+12.25+0.25+1+6.25+12.25+25}{25}$$

$= 5.4$

S.D of sampling distribution of means

$\sigma_{\bar{x}} = \sqrt{5.4} = 2.323$

verification!

S.D of sampling distribution of means (infinite population)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{3.29}{\sqrt{2}} = 2.323$$

ii) without replacement (finite population)

population values

2, 3, 6, 8, 11

$N=5$

i) population mean $(\mu) = \frac{2+3+6+8+11}{5}$

$= \frac{30}{5} = 6$

$\therefore \boxed{\mu = 6}$

ii) variance of population $\sigma^2 = (2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2 / 5$

$= 16 + 9 + 0 + 4 + 25$

$\sigma^2 = 10.8$

$\sigma = \sqrt{10.8} = 3.2863 \simeq 3.29$

$\boxed{\sigma = 3.29}$

c) To find the mean of samples of size '2' in a finite population (i.e without replacement) is given by $NCn = 5C2 = 10$

Listing of all samples

(2,3) (2,6) (2,8) (2,11)

(3,6) (3,8) (3,11)

(6,8) (6,11)

(8,11)

The mean of above 10 samples are as follows

2.5 4 5 6.5

4.5 5.5 7

7 8.5

9.5

This is called sampling distribution of means.

$$\mu_{\bar{x}} = \frac{2.5+4+5+6.5+4.5+5.5+7+7+8.5+9.5}{10}$$

$$= \frac{60}{10} = 6$$

$$\boxed{\mu_{\bar{x}} = 6}$$

Verification: $\mu_{\bar{x}} = 6$

$\mu = 6$

$\therefore \mu_{\bar{x}} = \mu$

d) variance of sampling distribution of means

$$\sigma_{\bar{x}}^2 = \frac{(2.5-6)^2+(4-6)^2+(5-6)^2+(6.5-6)^2+(4.5-6)^2+(5.5-6)^2+(7-6)^2+(7-6)^2+(8.5-6)^2+(9.5-6)^2}{10}$$

$$= \frac{12.25+4+1+0.25+2.25+0.25+1+1+6.25+12.25}{10}$$

$$= \frac{40.5}{10} = 4.05$$

$\sigma_{\bar{x}}^2 = 4.05$

$\sigma_{\bar{x}} = \sqrt{4.05} = 2.0124$

SD of sampling distribution of means $= 2.012$

Verification:

In finite population the S.D of sampling distribution of means is,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\left[\sqrt{\frac{N-n}{N-1}}\right]$$

$$\sigma_{\bar{x}} = \frac{3.29}{\sqrt{2}}\left[\sqrt{\frac{5-2}{5-1}}\right]$$
$$= 2.014$$

A population consist of 5, 10, 13, 14, 18, 24. consider all possible samples of size 2 which can be drawn without replacement from the population. Find. i) Mean of the population.
ii) S.D of the population.
iii) Mean of the sampling distribution of means.
iv) S.D of the sampling distribution of means

population values 5, 10, 13, 14, 18, 24

$N = 6$

a) population mean $(\mu) = \frac{5+10+13+14+18+24}{6}$

$$\boxed{\mu = 14}$$

ii) variance of population $\sigma^2 = \frac{(5-14)^2+(10-14)^2+(13-14)^2+(14-14)^2+(18-14)^2+(24-14)^2}{6}$

$$= \frac{81+16+1+0+16+100}{6}$$

$$\sigma^2 = 35.666$$

$$\sigma = \sqrt{35.66}$$

$$\boxed{\sigma = 5.972}$$

iii) To find the mean of samples of size 2 in a infinite population (i.e without replacement) is given by $NCn = 6C_2 = 15$

5, 10, 13, 14, 18, 24

Listing of all samples

(5,10) (5,13) (5,14) (5,18) (5,24)

(10,13) (10,14) (10,18) (10,24)

(13,14) (13,18) (13,24)

(14,18) (14,24)

(18,24)

The mean of above 15 samples are as follow

7.5 9 9.5 11.5 14.5

11.5 12 14 17

13.5 15.5 18.5

16 19

21

This is called sampling distribution of means

$$\mu_{\bar{x}} = \frac{7.5+9+9.5+11.5+14.5+11.5+12+14+17+13.5+15.5+18.5+16+19+21}{15}$$

$\boxed{\mu_{\bar{x}} = 14}$

verification:

$\mu_{\bar{x}} = 14$

$\mu = 14$

$\boxed{\mu_{\bar{x}} = \mu}$

iv) variance of sampling distribution of means

$$\sigma_{\bar{x}}^2 = \frac{(7.5-14)^2+(9-14)^2+(9.5-14)^2+(11.5-14)^2+(14.5-14)^2+(11.5-14)^2+(12-14)^2+(14-14)^2+(17-14)^2+(13.5-14)^2+(15.5-14)^2+(18.5-14)^2+(16-14)^2+(19-14)^2+(21-14)^2}{15}$$

$$= \frac{42.25+25+20.25+6.25+0.25+6.25+4+9+0.25+2.25+20.25+4+25+49}{15}$$

$$\sigma_{\bar{x}}^2 = 14.266$$

$$\sigma_{\bar{x}} = \sqrt{14.266} = 3.777$$

S.D of sampling distribution of means = 3.777

Verification:

Infinite population ~~the~~ S.D of sampling distribution of means is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\left[\sqrt{\frac{N-n}{N-1}}\right]$$

$$= \frac{5.972}{\sqrt{2}}\left[\sqrt{\frac{6-2}{6-1}}\right]$$

$$= 3.777$$

Samples of size 2 are taken from the population 1,2,3,4,5,6

i) with replacement

ii) without replacement

find a) population mean b) S.D of the population

c) Mean of the sampling distribution of means

d) S.D of the sampling distribution of means.

i) population values : 1,2,3,4,5,6

population size N = 6

a) Mean of population

$$\mu = \frac{\Sigma x_i}{N}$$

$$= \frac{1+2+3+4+5+6}{6} = 3.5$$

b) variance of the population

$$\sigma^2 = \frac{\Sigma x_i^2}{N} - \mu^2$$

$$= \frac{1+4+9+16+25+36}{6} - 12.25$$

$$\sigma^2 = 2.916$$

$$\sigma = \sqrt{2.916} = 1.707$$

To find mean of sampling distribution of means

No. of samples in infinite population are

$N^n = 6^2 = 36$

Listing of all possible samples with size 2

From population values are as follows.

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

The mean of all 36 samples are as follows

1	1.5	2	2.5	3	3.5
1.5	2	2.5	3	3.5	4
2	2.5	3	3.5	4	4.5
2.5	3	3.5	4	4.5	5
3	3.5	4	4.5	5	5.5
3.5	4	4.5	5	5.5	6

This is called sampling distribution of means

c) mean of the sampling distribution of means

$$\mu_{\bar{x}} = \frac{1+1.5+2+2.5+3+3.5+1.5+2+2.5+3+3.5+4+2+2.5+3+3.5+4+4.5+2.5+3+3.5+4+4.5+5+3+3.5+4+4.5+5+5.5+3.5+4+4.5+5+5.5+6}{36}$$

$= 3.5$

$4+2.25+1+0.25+0+$

$0.5+0.25$

$[\sigma_{x^2} = (1-\overset{3.5}{\cancel{6}})^2 + (1.5-6)^2 + (2-6)^2 + (2.5-6)^2 + (3-6)^2 +$
$(3.5-6)^2 + (1.5-6)^2 + (2-6)^2 + (2.5-6)^2 + (3-6)^2 +$
$(3.5-6)^2 + (4-6)^2 + (4.5-6)^2 + (2.5-6)^2 + (3-6)^2 +$
$(3.5-6)^2 + (4-6)^2 + (4.5-6)^2 + (5-6)^2 + (3-6)^2 +$
$(3.5-6)^2 + (4-6)^2 + (4.5-6)^2 + (5-6)^2 + (5.5-6)^2 +$
$(3.5-6)^2 + (4-6)^2 + (4.5-6)^2 + (5-6)^2 + (5.5-6)^2 +$
$(6-6)^2]$

$\sigma_{x^2} = (1-3.5)^2 + (1.5-3.5)^2 + (2-3.5)^2 + (2.5-3.5)^2 + (3-3.5)^2 +$
$(3.5-3.5)^2 + (1.5-3.5)^2 + (2-3.5)^2 + (2.5-3.5)^2 + (3-3.5)^2 +$
$(3.5-3.5)^2 + (4-3.5)^2 + (4.5-3.5)^2 + (2.5-3.5)^2 + (3-3.5)^2 +$
$(3.5-3.5)^2 + (4-3.5)^2 + (4.5-3.5)^2 + (5-3.5)^2 + (3-3.5)^2 +$
$(3.5-3.5)^2 + (4-3.5)^2 + (4.5-3.5)^2 + (5-3.5)^2 + (5.5-3.5)^2 +$
$(3.5-3.5)^2 + (4-3.5)^2 + (4.5-3.5)^2 + (5-3.5)^2 + (5.5-3.5)^2 +$
$(6-3.5)^2$
$\overline{\qquad 36 \qquad}$

$= \dfrac{13.75 + \overset{7}{\cancel{28}}.75 + 4.75 + 4.75 + 7.75 + 13.75}{36}$

$= 2.041 \quad 1.45$

$\sigma = \sqrt{2.041} = 1.43\cancel{8} \quad \sigma = 1.207.$

verification

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

$= \dfrac{1.707}{\sqrt{2}} = 1.207.$

A population consist of 6 numbers 4, 8, 12, 16, 20, 24 consider all possible samples of size 2 that can be drawn without replacement from this population find i) mean of the sampling distribution of means ii) S.D of the sampling distribution of means.

without replacement (finite population)

population values 4, 8, 12, 16, 20, 24

$N = 6$

a) population mean(μ) $= \frac{4+8+12+16+20+24}{6}$

$\boxed{\mu = 14}$

variance of population $\sigma^2 = (4-14)^2 + (8-14)^2 + (12-14)^2 + (16-14)^2 + (20-14)^2 + (24-14)^2$

$100+36+4+4+36+100 / 6$

$\sigma^2 = 46.6$

$\sigma = \sqrt{46.6} = 6.826$

$\boxed{\sigma = 6.83}$

To find the mean of samples of size 2

in finite population $NCn = 6C2$

$= 15$

4, 8, 12, 16, 20, 24

(4, 8) (4, 12) (4, 16) (4, 20) (4, 24)

(8, 12) (8, 16) (8, 20) (8, 24)

(12, 16) (12, 20) (12, 24)

(16, 20) (16, 24)

(20, 24)

The mean of above 15 samples are

6 8 10 12 14

10 12 14 16

14 16 18

18 20

22

i) $\mu_{\bar{x}} = \dfrac{6+8+10+12+14+10+12+14+16+14+16+18+18+20+22}{15}$

$\boxed{\mu_{\bar{x}} = 14}$

ii) Variance of sampling distribution of means

$= \dfrac{(6-14)^2+(8-14)^2+(10-14)^2+(12-14)^2+(14-14)^2+(10-14)^2+(12-14)^2+(14-14)^2+(16-14)^2+(14-14)^2+(16-14)^2+(18-14)^2+(18-18)^2+(20-18)^2+(22-14)^2}{\cancel{6}\ 15}$

$= \dfrac{64+36+16+4+16+4+4+4+16+16+36+64}{\cancel{6}\ 15} = 18.66$

$\sigma = \sqrt{18.66} = 4.319$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\left[\sqrt{\frac{N-n}{N-1}}\right]$$

$$= \frac{6.83}{\sqrt{2}}\left[\sqrt{\frac{6-2}{6-1}}\right]$$

$$= 4.319$$