mood-book

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Unit-II

Sets and Relations

* Definition: A set is a well-defined collection or class of distinct objects.

Example: 1. The odd positive integers less than 6

2. the vowels in english Alphabet

* Definition: the objects in a set are called its elements (or) Members. the elements in the set Must be distinct and distinguishable

Generally, capital letters A, B, C, X, Y, Z etc are used to denote sets, set symbol is { } and the elements by small letters a, b, c, x, y, z etc.

1. Finite set: A set is said to be finite if it contains finite number of different elements

Ex: $A = \{1, 2, 5, 7\}$

$B = \{x \mid x \text{ is a country in the world}\}$

2. Infinite set: A set is said to be infinite if it contains infinite distinct elements

Ex: $N = \{1, 2, 3, 4, 5, \ldots\}$

$A = \{a \mid a \text{ is the point in the plane}\}$

3. Singleton set: A set which contains only one element is called a singleton set

Ex: $A = \{x \mid 2 < x < 4, x \text{ is an integer}\} = \{3\}$

$B = \{0\}$

4. Null set : A set which contains no element is called an Empty (null or void) set. This is denoted by ϕ

Ex : 'set of all integers who square is 3 $= \phi$

A set which is not a nullset is called non empty set.

5. Equality of sets : Two sets A and B are said to be equal if every element of A is an element of B and also Every element of B is an element of A. The equality of two sets A and B is denoted by $A = B$

Ex : $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4\}$

6. Equivalent sets : If the elements of one set can be put into one to one correspondence with the elements of another set, then the two sets are called equivalent sets.

The symbol $\sim$ (or) $=$ is used to denote equivalent sets

Ex : Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$ be two sets the elements of A can be put into one-to-one correspondence with those of B. Then $A \sim B$

7. Subset : Let A, B be two non-empty sets. The set A is subset of B (or A is contained in B) iff every element of A is an element of B

i.e $A \subseteq B$ iff $x \in A \Rightarrow x \in B$

Here B is called the superset of A if $A \subseteq B$

Example : Let $A = \{1, 5, 7\}$, $B = \{1, 3, 5, 7\}$ then

$A \subseteq B$

8. Proper Subset : The set A is a proper subset of the set B (or) A is properly contained in B iff

(1) every element of A is also an element of B i.e $A \subseteq B$

(2) There is atleast one element in the set B, which is not in A i.e $A \neq B$.

If A is a proper subset of B, We write $A \subset B$

Ex: Let $A = \{1, 5, 7\}$, $B = \{1, 3, 5, 7\}$ then $A \subset B$

9. power set : If S is any set, then the family of all the subsets of S is called the power set of S and is denoted by P(S) i.e $P(S) = \{A \mid A \subseteq S\}$

clearly ϕ and S are both Members of P(S)

Note : If A is a finite set of n elements then the power set of A P(A) contains 2^n elements.

Ex: Let $A = \{a\}$ then $P(A) = \{\phi, \{a\}\}$

Let $A = \{a, b\}$ then $P(A) = \{a\}, \{b\}, \{a, b\}, \{\phi\}$

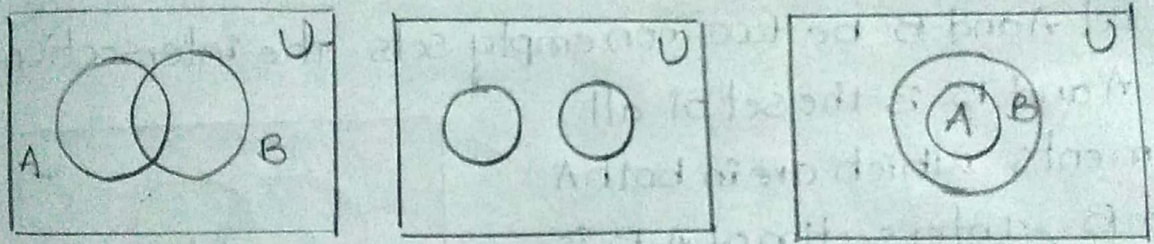
10. Universal set : In applications of set theory all the sets under consideration are said to be the subset of the Fixed set. This set is called the universal set (or) universal discourse. This set is usually denoted by U

Ex: All the people in the world constitute the universal set in any study of the population.

VENN-EULER DIAGRAMS:

A device known as Venn-Euler diagram (or) simply venn diagram is a pictorial representation of sets. In venn diagrams, A universal set U is represented by the interior of rectangle. each subset of U is represented by the circle inside the rectangle.

If few elements are common in both A and B then the sets A and B are represented by Fig. If the set A and B are disjoint (i.e) they have no common elements then circles are represented by Fig(ii) and if A is subset of B all the elements in A are also in B the circles A and B represented by the fig(iii)



(i) (ii) (iii)

Example: Write down the following sets:

(i) $A = \{x \mid x^2 = 9\}$ (ii) $B = \{x \mid x^2+4 = 0\}$

Sol (i) $A = \{-3, 3\}$

(ii) $x^2+4 = 0 \Rightarrow x^2 = -4 \Rightarrow x = \pm 2i$

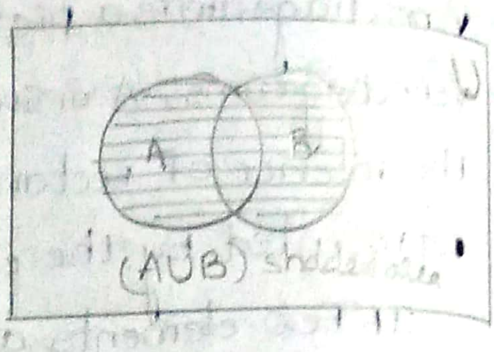
$B = \{-2i, 2i\}$

Operations on sets and properties of set Algebra:

In this section we shall define certain basic set operations on sets so you need to yield new sets with the given data.

① Union of Sets :

Let A, B be any two nonempty sets. The union of A and B is the set of all elements which are either in A or in B (or) in the both A and B. The union of A, B is denoted by $A \cup B$, we read it as A union B.



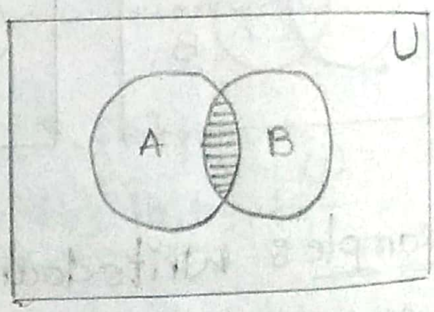
Symbolically, $A \cup B = \{x \mid x \in A \text{ (or) } x \in B\}$

Ex: 1. Let $A = \{1, 2, 3\}$, $B = \{2, 3, 6\}$

Then $A \cup B = \{1, 2, 3, 6\}$

② Intersection of sets :

Let A and B be two non empty sets. The intersection of A and B is the set of all elements which are in both A and B. Intersection of A, B is denoted by $A \cap B$, we read it as A intersection B.



Symbolically, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Ex: Let $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{5, 6, 7, 8\}$

$A \cap B = \{5, 6\}$

Properties of union of sets and intersection of sets :

① $A \cup B = B \cup A$; $A \cap B = B \cap A$ (commutative law)

② $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$ (associative law)

③ $A \cup A = A$, $A \cap A = A$ (Idempotent law)

④ $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$ (Distributive law)

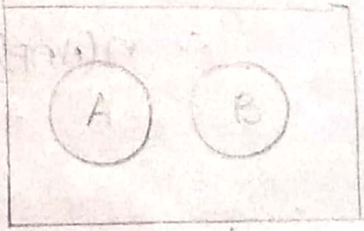
$A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

⑤ $A\cup\phi = A$, $A\cap\phi = \phi$

⑥ $A\cup U = U$, $A\cap U = A$

Disjoint sets:

Let A and B be two non-empty sets - these two sets A and B are said to be disjoint if they have no common elements i.e $A\cap B = \phi$



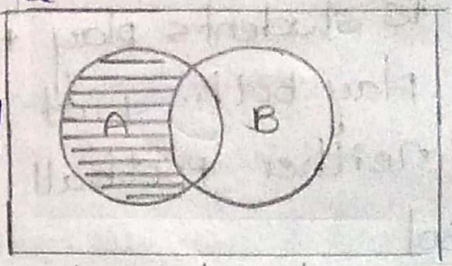
$A\cap B = \phi$

Ex: If $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7\}$ then $A\cap B = \phi$, A, B are disjoints.

Difference of two sets:

If A, B are any sets, then difference of A and B is the set of elements which belong to A and does not belong to B. The difference of A and B is defined by $A-B$ (or) $A\sim B$

We read it as A difference B



A-B (shaded area)

Ex: $A = \{2, 4, 6, 8\}$, $B = \{6, 8, 10, 12\}$

$A-B = \{2, 4\}$; $B-A = \{10, 12\}$

$A-B \neq B-A$

Complement of a set:

Let A be any set. The complement of A defined as the set of that are in the universal set but not in A. It is denoted by $U-A$ (or) A' (or) A^c

Symbolically, $A' = U-A = \{x \mid x\in U \text{ and } x\notin A\}$

Ex: If $\{1, 2, 3, 4, \ldots\}$ is universal set, $A = \{2, 4, 6, 8 \ldots\}$

$U-A = \{1, 3, 5, 7\}$

$U^c = \phi$, $\phi^c = U$

Q. Among 60 students in a class 45 pass in first semister examination, 30 pass in 2nd semister Exam. If 12 didn't pass in either semister how many passed in Both semisters.

Sol

$$n(A) = 45$$
$$n(B) = 30$$
$$n(A \cup B) = 60 - 12 = 48$$

$$\therefore n(A \cap B) = n(A) + n(B) - n(A \cup B)$$
$$= 45 + 30 - 48$$
$$= 27$$

$\therefore$ Hence the no. of students who passed Both Semistar is 27

Q) In a class of 50 students. 20 stud. play Football, 16 students play Hockey it is found that 10 students play both. Find the no of students who play neither football nor Hockey.

Sol

$$n(A) = 20; \; n(A \cap B) = 10; \; n(B) = 16$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 20 + 16 - 10$$
$$= 26$$

$$n(A \cup B)' = 50 - 26 = 24$$

$\therefore$ The no. of students who play neither football nor Hockey is 24.

complementary properties :

1. $A \cup A^c = U$
2. $A \cap A^c = \phi$
3. $(A^c)^c = A$
4. $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$ (Demorgan's laws)

Symmetric difference of sets :

Let A, B be two non-empty sets. then the symmetric difference of A, B denoted by $A \Delta B$, is defined as the set containing elements which either belongs to A or B but not to bc $A \oplus B = (A \cup B) - (A \cap B)$

Ex: Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ then

$A \oplus B = (A \cup B) - (A \cap B) = \{1, 2, 3, 4, 5\} - \{3\} = \{1, 2, 4, 5\}$

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Relations :-

Let A, B are two sets. A subset $A \times 1$ is called a relation or binary relation from A to B.

NOTE If $R \subseteq A \times B$, then R is a relation on from A to B.

If $(a, b) \in R$ is also written as a R b and we read as "a relates b."

Examples of Relations

1) $S = \{(1, 2), (3, 4), (5, 6)\}$
2) $R = \{(x, y) / x + y = 10\}$ set builder form

Domain and Range of a relation

Let S be a binary relation. The domain of the relation S is defined as the set of all first elements of ordered pairs that belong to S thus $D(S) = \{x / (\exists y), (x, y) \in S\}$

The range of the relation S is defined as the set of all the second elements of the ordered pairs that belong to S and is denoted by R or R(S)

thus $R(S) = \{ y / (\exists x) (x,y) \in S \}$

1] Let $A = \{2,3,4\}$ and $B = \{3,4,5,6,7\}$ find domain and range also defines a relation R from A to B by $(a,b) \in R$ if a divides b

Sol $R \subseteq A \times B$

$\{(2,3),(2,4),(2,5),(2,6),(2,7),(3,3),(3,4),(3,5),(3,6),(3,7)$

$(4,3),(4,4),(4,5),(4,6),(4,7)\}$

$R = \{(2,4),(2,6),(3,3),(3,6),(4,4)\}$

Domain of $R = \{2,3,4\}$

Range of $R = \{3,4,5,6,7\}$

Inverse of a Relation

Let R be a relation from A to B then Inverse of relation R from B to A is denoted by R^{-1} and its is defined as $R^{-1} = \{(b,a) / (a,b) \in R\}$

Ex If $R = \{(2,4),(2,6),(3,3),(3,6),(4,4)\}$ is a relation from A to B Then $R^{-1} = \{(3,3),(4,2),(4,4),(6,2),(6,3)\}$ is inverse relation from B to A.

Operations on Relations

1] Let R, S be two relations on $A = \{1,2,3\}$ and $R = \{(1,1),(1,2)$ $(2,3),(3,1),(3,3)\}$ and $S = \{(1,2),(1,3),(2,1),(3,3)\}$ then find the following i) $R \cup S$ ii) $R \cap S$ iii) $R - S$ iv) R^C v) $R \circ S$ vi) R^2

i) $R \cup S = \{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,3)\}$

ii) $R \cap S = \{(1,2),(3,3)\}$

iii) $R - S = \{(1,1),(2,3),(3,1)\}$

iv) $R^c = \{(1,3),(2,1),(2,2),(3,2)\}$

v) $R \circ S = \{(1,2),(1,3),(1,1),(2,3),(3,2),(3,3)\}$

vi) $R^2 = R \circ R$

$= \{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)\}$

Properties of a Relation

A relation R on a set A is said to be

i] **Reflexive** on A if $(a,a) \in R \ \forall a \in A$

Ex Let $A = \{1,2,3\}$ and $R = \{(1,1),(1,3),(2,2),(2,3),(3,1),(3,3)\}$

then R is reflexive because $(1,1)(2,2),(3,3) \in R$

ii] **Symmetric** on A if $(a,b) \in R$ then $(b,a) \in R \ \forall a,b \in A$

Ex Let $A = \{1,2,3\}$ and $R = \{(1,2),(2,1),(3,2),(2,3),(4,3),(3,4)\}$

R is symmetric

iii] **Transitive** on A if $(a,b) \in R, (b,c) \in R$ then $(a,c) \in R \ \forall$ $a,b,c \in A$

Ex Let $A = \{1,2,3,4\}$ and $S = \{(1,1),(2,2),(3,3),(2,3),(3,1)$ $(2,1)\}$ S is transitive

iv] **Irreflexive** on A if $(a,a) \in R \ \forall a \in A$

Ex Let $A = \{1,2,3\}$ and $S = \{(1,1),(2,2),(2,3)\}$

S is irreflexive

v] **Antisymmetric** on A if $(a,b) \in R$ and $(b,a) \in R$ then $x=y$ $\forall a,b \in A$

Ex Let $A = \{1,2,3,4\}$ and $R = \{(1,2),(2,3),(3,4)\}$

R is antisymmetric

Ex i) Given $S = \{1, 2, \ldots, 10\}$ and a relation R on S where $R = \{(x,y) / x+y = 10\}$

2) Let $A = \{2,3,4\}$ $B = \{3,4,5,6,7\}$ define a relation R from A to B if a divides b.

Representation of Relations

Relations can be represented by two methods

i] Matrix method ii] Directed graph

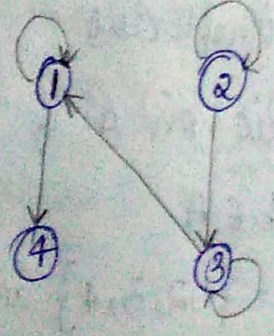
i] A binary relation R from a set A with n elements to a set B with m elements is presented by $n \times m$ matrix, called relation matrix, denoted by $M_R = [a_{ij}]$ where

$$a_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ element } A \text{ is related to } j^{th} \text{ element of } B \text{ ie } (a_i, a_j) \in R \\ 0 & \text{otherwise } (a_i, a_j) \notin R \end{cases}$$

2] Digraph A relation can also be represented pictorially by drawing its digraph. The elements of X are represented by points (or) circles called nodes (or) vertices. An arrow is drawn from the vertex x_i to vertex x_j, iff $x_i R x_j$. This is called an edge.

Ex Let $A = \{1,2,3\}$ $B = \{a,b,c,d\}$ the relations R from A to B is given by $R = \{(1,1), (1,4), (2,2), (2,3), (3,1) (3,3)\}$ Write relation matrix and graph.

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



Equivalence Relations

A Relation R on a set A is said to be an equivalence relation on A if,

(i) R is reflexive on A

(ii) R is symmetric on A

(iii) R is transitive on A

Eg:

1) Let $A = \{1, 2, 3, 4\}$ & $R = \{(1,1), (1,4), (4,1) (4,4), (2,2), (2,3), (3,2), (3,3)\}$ prove that R is an equivalence relation.

Sol: If R is Reflexive then $(a,a) \in R \; \forall \; a \in X$

$(1,1)(2,2)(3,3)(4,4) \in R$

R is Reflexive

If R is symmetric if $(a,b) \in R$ then

$(b,a) \in R \; \forall \; a,b \in X$

$(1,4)(4,1)(2,3)(3,2) \in R$

R is symmetric

If R is transitive iff $(a,b) \in R$ & $(b,c) \in R$

then $(a,c) \in R \;\; \forall \; a,b,c \; \forall x$

$\therefore$ R is transitive

(2) Let $X = \{1, 2, \ldots 7\}$ & $R = \{(x,y) / x-y$ is divisible by 3$\}$ show that R is equivalence relation.

Sol: $R = \{(1,7) (1,4), (2,5), (3,6), (4,7), (4,1) (5,2), (6,3), (7,1) (7,4)\}$ $(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)(7,7)\}$

If R is reflexive then $(a,a) \in R \; \forall \; a \in X$

$(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)(7,7) \in R$

R is reflexive

If R is symmetric, if $(a,b) \in R$ then $(b,a) \in R \ \forall a,b \in x$

$(1,7)(7,1)(1,4)(4,1)(2,5)(5,2)(3,6)(6,3)(4,7)(7,4) \in R$

R is symmetric

If R is transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R \ \forall a,b,c \in x$

$\therefore$ R is transitive

Transitive closure of a Relation(R):-

It is the smallest transitive relation containing R. We denote transitive closure of R by R^+. Let x be any finite set containing 'n' elements & R be a relation in x. The relation $R^+ = R \cup R^2 \cup R^3 \cup \ldots R^n$ in x is called the transitive closure of R in X

Transitive closures of relations have important applications in areas like networks, fault detection etc.

Eg:

(1) Let $x = \{1,2,3,4\}$ & $R = \{(1,2)(2,3)(3,4)\}$ be a relation on X. Find R^+

Sol given $R = \{(1,2)(2,3)(3,4)\}$

$R^2 = \{(1,3),(2,4)\}$

$R^2 o R = R^3 = \{(1,4)\}$

$R^4 = \phi$

$R^+ = R \cup R^2 \cup R^3 \cup R^4$

$= \{(1,2),(2,3),(3,4)(1,3)(2,4)(1,4)\}$

(2) Let $x = \{(1,2)(2,3)(3,3)\}$ on the set $A = \{1,2,3\}$ obtain transitive closure of R & write its Matrix.

Sol: $R = \{(1,2)(2,3)(3,3)\}$

$R^2 = \{(1,3)(2,3)(3,3)\}$

$R^3 = R^2 o R = \{(1,3)(2,3)(3,3)\} o \{(1,2)(2,3)(3,3)\}$

$= \{(1,3)(2,3)(3,3)\}$

$R^+ = R \cup R^2 \cup R^3$

$= \{(1,2)(2,3)(3,3)(1,3)(2,3)\}$

Equivalence class :

The equivalence class of x is denoted by $[x]$

$[x] = \{y \mid y \in A \ \& \ (x,y) \in R\}$

1, Let R be the equivalence relation on the set $A = \{1,2,3,4,5\}$ Where $R = \{(1,1)(2,2)(3,3)(4,4)(5,5)(1,2)(2,1)(4,5)(5,4)\}$ Find the partition of A.

Sol Equivalence class of $[1] = \{1,2\}$

$[2] = \{1,2\}$

$[3] = \{3\}$

$[4] = \{4,5\}$

$[5] = \{4,5\}$

Partitions of $A = \{\{1,2\}\{3\}\{4,5\}\}$

2. If $P = \{\{1,3\},\{2,4\}\}$ is a partition set of $A = \{1,2,3,4,5\}$. Determine corresponding equivalence relation.

Sol $R = \{\{1,3\} \times \{1,3\}\}, \{2,4,5\} \times \{2,4,5\}\}$

$= \{(1,1)(1,3)(3,1)(3,3)(2,2)(2,4)(2,5)(4,2)(4,4)(4,5)(5,2)(5,4)(5,5)\}$

(3) Let R be an equivalence relation on set $A = \{1, 2, 3, 4, 5, 6\}$ where $R = \{(1,1)(1,5)(2,2)(2,3)(2,6)(3,2)(3,3),(3,6)(4,4)(5,1)(5,5)(6,2)(6,3)(6,6)\}$ Find the partition of A induced by R i.e equivalence class of R

<u>Sol</u> Partition of A induced by R

$P = \{\{1,5\}, \{2,3,6\}, \{3,6\}, \{4\}\}$

(ii) equivalence class of $[1] = \{1,5\}$

$[2] = \{2,3,6\}$

$[3] = \{3,6\}$

$[4] = \{4\}$

$[5] = \{5,1\}$

$[6] = \{3,6\}$

Compatability Relations :-

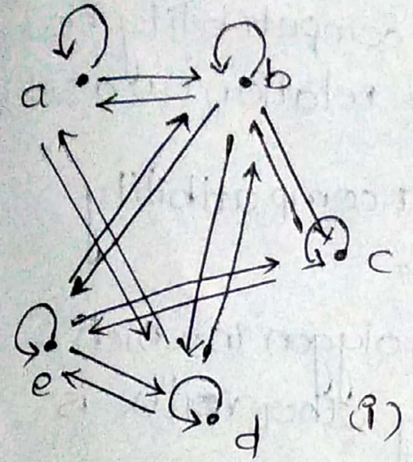
A Relation R on X is said to be a compatability relation if it is both reflexive and symmetric.

Let $X = \{$ball, bed, dog, let, egg$\}$ & let the relation R be given by $R = \{(x,y) | x,y \in X \wedge xRy\}$ if x & y contains some common letter.

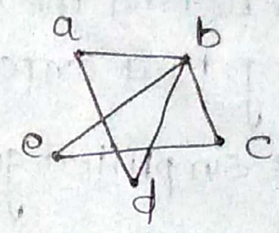
compatiability relation is denoted by the symbol $\approx$. Note that ball $\approx$ bed, ball $\approx$ let, ball $\not\approx$ egg.

Let X be a set and $\approx$ be a compatability relation on X. A subset $A \subseteq X$ is called a Maximal compatibility block if any element of A is compatable to every other element of A and no element of X-A is compatible to all the element of A. compatibility relation are useful in solving certain Minimization problems of switching theory.

let us denote "ball" by a 'bed' by b 'dog' by c, 'let' by d and 'egg' by e. the graph of ≈ is given in below.



(i)



(ii)

since ≈ is a compatibility relation, it is not necessary to draw the loops at each element i.e the elements are Mutually compitible. Also the sets {a,b,d} and {b,c,e} are covering of X. The set {b,c,e} also has elements compatable to each other.

The relation Matrix of a compatibility relation is symmetric and has its diagonal element unity. Therefore it is sufficient to give only the elements of the lower triangular part of the relation Matrix.

Relation Matrix (Mr) for above figure is

$$M_r = \begin{array}{c} \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & & & \\ 0 & 1 & & \\ 1 & 1 & 0 & \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Maximal compatibility block

Let x be a set and ≈ is a compatibility relation on X. A subset $A \subseteq X$ is called a Maximal compatibility block if any element of A is compatible to every other element of A and no element of X−A is compatible to all the elements of A.

procedure to find the Maximal compatibility blocks corresponding to a compatibility relation on a set X

(1) Draw the simplified graph of a compatibility relation.

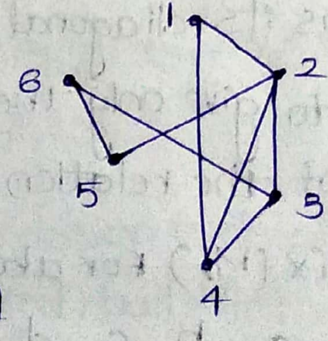
(2) Select largest polygon i.e a polygon in which any vertex is connected to every other vertex is connected to every other vertex

(3) Any element of the set which is related only to itself from a Maximal compatiblity block

(4) Similarly, Any two elements which are compatible to one another and which does not cover in set also form a compatibility block

Eg: (1) Find the Maximal compatibility block of

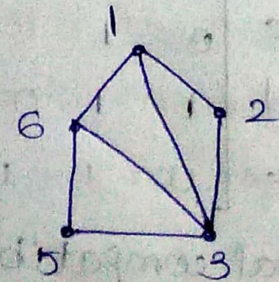
$$\begin{array}{ccccc} 1 \\ 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{array}$$



{2, 3, 4}, {5, 6}
{1, 4, 2}

(2)

$$\begin{array}{cccc} 1 \\ 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{array}$$



{1, 2, 3}, {1, 3, 6}, (3, 5, 6}

partial ordering Relations :

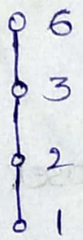
A relation R on a set P is called a partial order relation or a partial ordering in P iff is reflexive, antisymmetric and transitive, we denote a partial ordering by the symbol $\leq$.

poset (or) partially ordered set : A set P on which a partial ordering $\leq$ is defined is called a poset and it denoted by $(P, \leq)$ or $[P, \leq]$

Let $(P, \leq)$ be a poset. elements a, b in P are said to be comparable under $\leq$ if either $a \leq b$ or $b \leq a$ otherwise they are incomparable

Totally ordered set : Let $(P, \leq)$ be a poset if every pair of elements of A are comparable then $(P, \leq)$ is called a totally ordered set or a chain

Eg :



Harse Diagram (OR) Poset diagram :

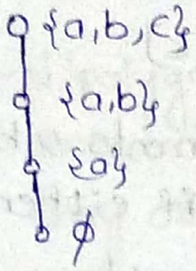
A partial ordering $\leq$ on a set P can be represented by means of a diagram known as Harse diagram (or) poset diagram of $(P, \leq)$. The procedure of for drawing Harse diagram for a poset P as follows:

1) Each element is represented by a small circle
2) The circle for $x \in P$ is drawn below the circle for $y \in P$ if $x < y$
3) A line drawn between x and y if y covers x and if $x < y$ but y does not cover x, then x and y are not connected directly by a single line.

For a totally ordered set $(P, \leq)$, the Hasse diagram consists of circles, one below the other thus poset is called chain in this.

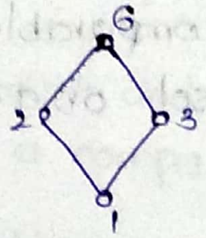
Eg: (1) consider $P = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}\}$ $(P, \leq)$ be poset where $\subseteq$ is set inclusion then the Hasse diagram of $(P, \subseteq)$

Sol:



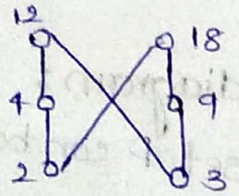
(2) let $D_6 = \{1, 2, 3, 6\}$. Draw the Hasse diagram of $(D_6, 1)$

Sol:



(3) Let $\{2, 3, 4, 9, 12, 18\}$ be a poset

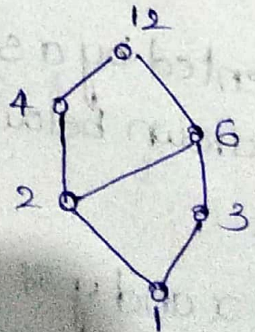
Sol:



(4) Draw Hasse diagram of poset $(D_{12}, 1)$

Sol:

$D_{12} = \{1, 2, 3, 4, 6, 12\}$

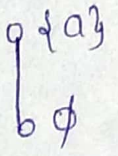


4) Let A be any finite set and $P(A)$ be the power set of A; $\subseteq$ be the inclusion relation on the elements of $P(A)$. Draw the hasse diagrams of $(P(A), \subseteq)$ for

(i) $A = \{a\}$ (ii) $A = \{a, b\}$ (iii) $A = \{a, b, c\}$

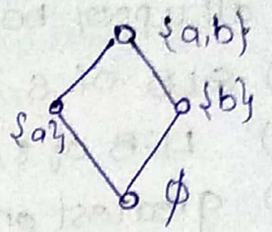
Sol: (i) Given $A = \{a\}$

$P(A) = \{\phi, \{a\}\}$



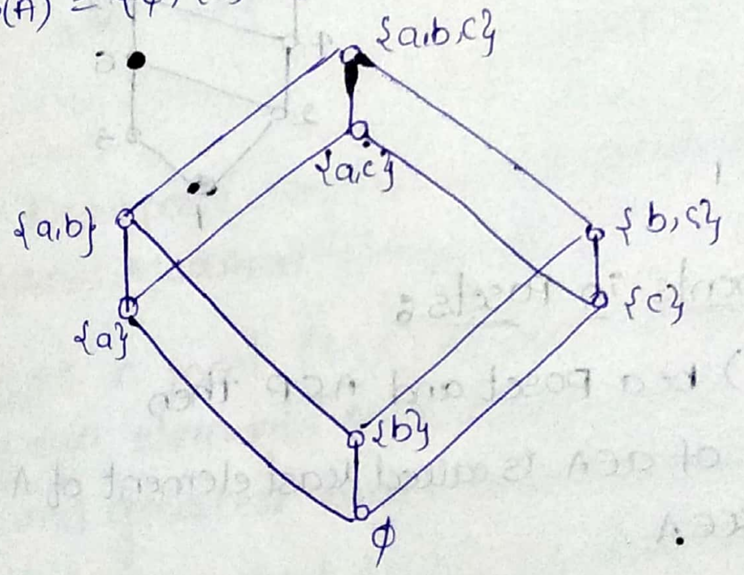
(ii) $A = \{a, b\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$



(iii) $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a b\} \{b, c\}, \{c, a\}, \{a, b, c\}\}$



Greatest lower bound (GLB):

Let $(P, \leq)$ be a poset and $A \leq P$, then $a \in A$ is called a lower bound of A, if $a \leq x \; \forall x \in A$ and if there are no lower bounds of A which are greater than a, then a is called a greatest lower bound (GLB) of A.

Least upper bound (LUB):

let $(P, \leq)$ be a poset, $A \leq P$, then $a \in A$ is called an upper bound of A if $x \leq a \; \forall x \in A$ and if there are no upper bounds of A which are lesser than a is called least upper bound (LUB) of A

Eg(1): Let $D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$ and relation divides $\times$ be a partial ordering on D_{24} draw the Hasse diagram of $(D_{24}, |)$ and find the following

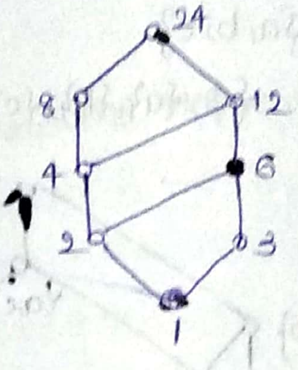
(i) all lower bounds of 8, 12

(ii) all upper bounds of 8, 12

(iii) GLB of 8, 12

(iv) LUB of 8, 12

(v) greatest and least element if exists



Sol

(i) 1, 2, 4

(ii) 24

(iii) 4

(iv) 24

(v) 24, 1

Special Elements in Posets:

Let $(P, \leq)$ be a poset and $A \subseteq P$ then

a) an element of $a \in A$ is called least element of A if $a \leq x \ \forall \ x \in A$

b) an element $a \in A$ is called greatest element of A if $x \leq a \ \forall \ x \in A$

NOTE:

① If least element of A exists then it is unique

② If greatest element of A exists then it is unique

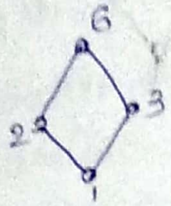
③ An element $a \in A$ is said to be a Minimal element of A if $\exists$ no x in A such that $x < a$

④ An element $a \in A$ is said to be Maximal element of A if $\exists$ no x in A such that $a < x$

Note: (1) Minimal / Maximal elements are not using

Eg (1) consider the poset $(D_6, |)$

$D_6 = \{1, 2, 3, 6\}$



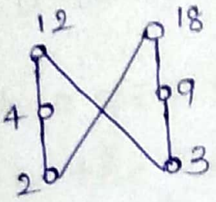
Minimal element = 1

Maximal element = 6

least element = 1

greatest element = 6

(2) Let $X = \{2, 3, 4, 9, 12, 18\}$



Minimal = 2, 3

maximal = 12, 18

least = does not exist

greatest = "

(3) Draw poset diagram and determine least, great, Minimal, Maximal elements.

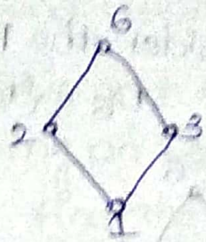
Lattice: A poset $(L, \leq)$ is said to be lattice if every two elements in L have least upper bound LUB and greatest lower bound bound (GLB). the GLB of a subset $\{a, b\} \subseteq L$ will be denoted by $a \wedge b$ (meet of a, b) and the LUB of a subset $\{a, b\} \subseteq L$ will be denoted by $a \vee b$ (join of a, b)

NOTE: Every lattice is a poset but all posets are not lattices.

Eg: $(D_6, |)$ is a lattice

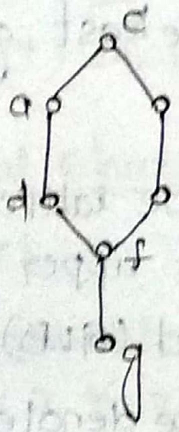
Sol: $(D_6 = \{1, 2, 3, 6\}, |)$ is a poset then $(D_6, |)$ is a lattice because every pair of elements of D_6 have GLB and LUB in D_6

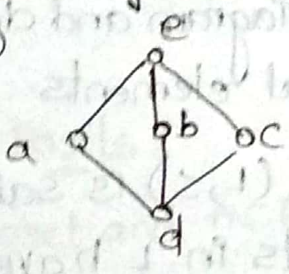
G.L.B	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

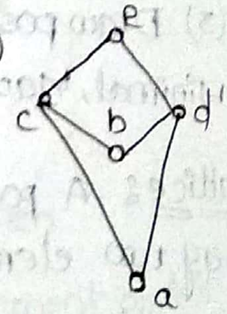


L.U.B	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

(2) Which of the following posets are lattices.

(i)


(ii)


(iii)


Sol: (i) Poset is a lattice because every pair of elements have LUB and GLB in poset

(ii) Lattice

(iii) NOT a lattice because $a \wedge b$ does not exists

Properties of Lattices:

If $\{\alpha, \leq\}$ is a lattice and $\wedge$ and $\vee$ are the meet and join operations defined on it. then $\forall\, a, b, c \in \alpha$ we have.

(i) Idempotent property

$a \wedge a = a,\ a \vee a = a,\ \forall\ a \in L$

(ii) commutative property

$a \wedge b = b \wedge a,\ a \vee b = b \vee a\ \forall\ a, b \in L$

(iii) Associative property

$a \wedge (a \vee b)$
$a \vee (a \wedge b)$

$(a \wedge b) \wedge c = a \wedge (b \wedge c),$

$(a \vee b) \vee c = a \vee (b \vee c)\ \forall\ a, b, c \in L.$

(iv) Absorption property

$a \wedge (a \vee b) = a;\ a \vee (a \wedge b) = a\ \forall\ a, b \in L$

* Let A be a given finite set and P(A) its power set. Let $\subseteq$ be the inclusion relation on the elements of P(A). Draw the Harse diagram of $\langle P(A); \subseteq \rangle$ for

(a) $A = \{a\}$ (b) $A = \{a, b\}$ (c) $A = \{a, b, c\}$

(d) $A = \{a, b, c, d\}$

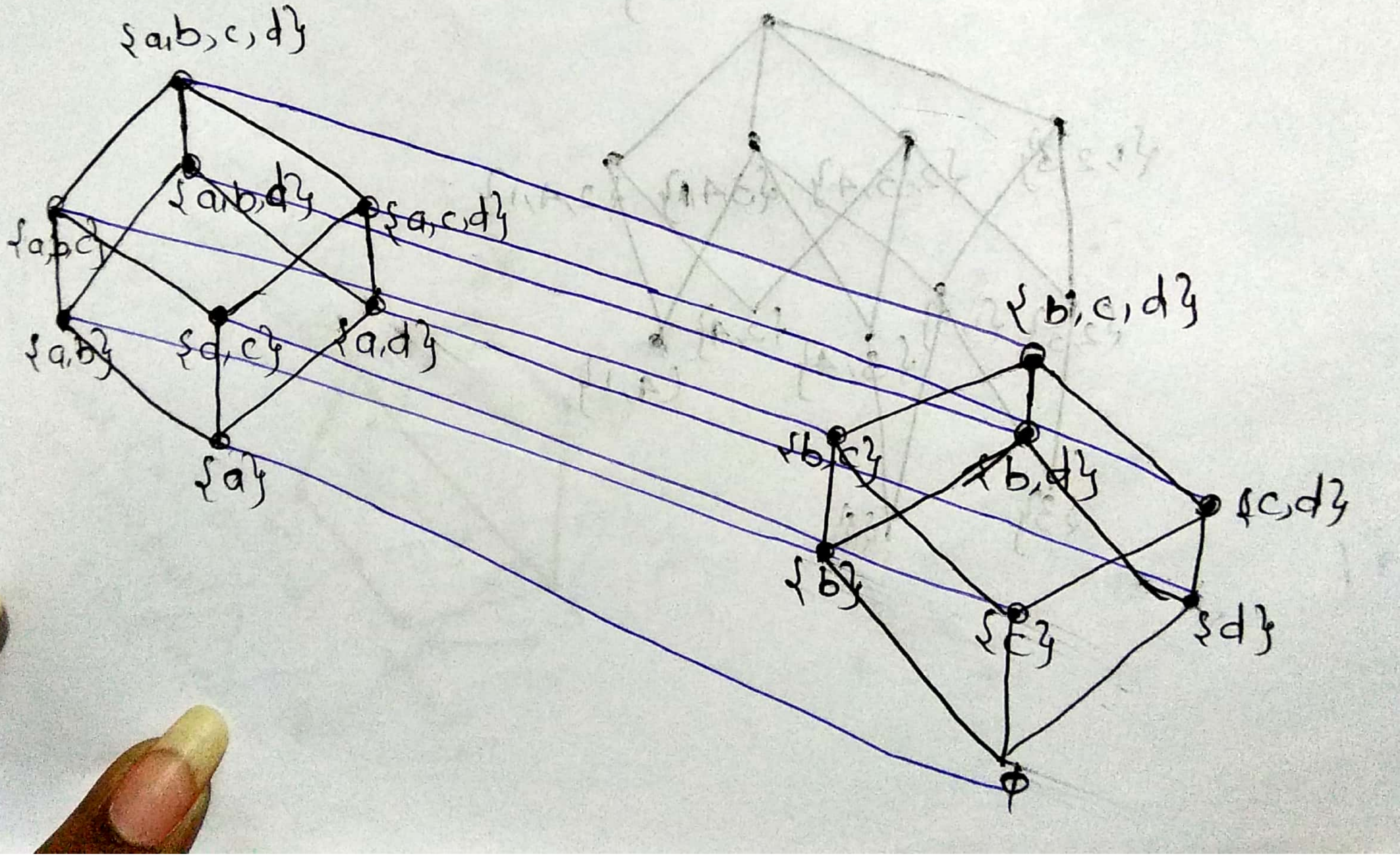
sol: Refer sol^n of no. of 5 for (a)(b) and (c)

(d) $A = \{a, b, c, d\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{ab\}\{bc\}\{cd\}, \{da\}\{a, c\}\{bd\}$
$\{abc\}\{bcd\}\{cda\}\ \{abd\}, \{a, b, c, d\}\}$

$A = \{a, b, c, d\}$ then $\langle P(A), \subseteq \rangle$

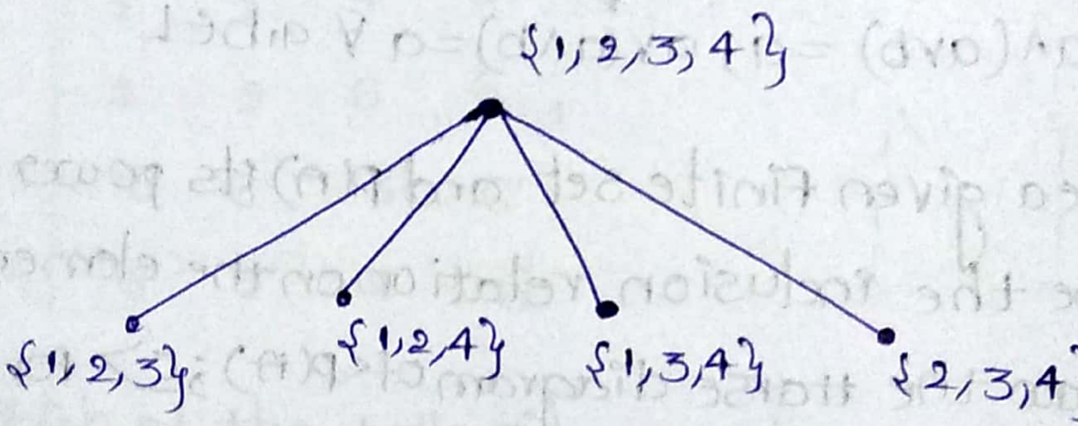
$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{c,d\}, \{d,a\}$
$\{a,c\}, \{b,d\}, \{a,b,c\}, \{b,c,d\}, \{c,d,a\}, \{a,b,d\} \{a,b,c,d\}\}$



(2) If $A = \{1, 2, 3, 4\}$ Draw the Hasse diagram of $(P(A), \subseteq)$

Sol: given $A = \{1, 2, 3, 4\}$

then $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\} \{2,3\} \{3,4\}, \{4,1\} \{1,3\} \{2,4\} \{1,2,3\}, \{2,3,4\} \{3,4,1\} \{1,2,4\} \{1,2,3,4\}\}$



$A = \{1, 2, 3, 4\}$ then $\langle P(A), \subseteq \rangle$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{4,1\}, \{1,3\}, \{2,4\}$
$\{1,2,3\}, \{2,3,4\}, \{3,4,1\}, \{2,4,1\}, \{1,2,3,4\}\}$