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# UNIT-4

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# UNIT-4

## Elementary combinatorics

**Combinatorics:-**

combinatorics is an important part of discrete mathematics that solves counting problems without actually enumerating all possible cases.

Combinatorics deals with counting the no. of ways of arranging or choosing objects from a finite set according to the certain specified rules.

for arrangement purpose use permutations
" selection purpose use combinations.

**Permutations:-** An ordered arrangement of r elements of a set (finite) containing n distinct elements is called an r-permutation of n elements and is denoted by $P(n,r)$ (or) $^nP_r$ where $r \leq n$

$$^nP_r = P(n,r) = n(n-1)(n-2)\ldots(n-r+1)$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

P denotes permutation

$$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

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## Combinations:-

An ordered selection of $r$ elements of a set containing $n$ distinct elements is called an $r$-combination of $n$ elements and is denoted by $C(n,r)$ (or) $^nC_r$ (or) $\binom{n}{r}$

$$\boxed{\begin{array}{l} C(n,r) = \dfrac{n!}{r!(n-r)!} \\[2ex] C(n,n) = 1 \left( \because \dfrac{n!}{n!(n-n)!} \Rightarrow \dfrac{\cancel{n}!}{\cancel{n}!\,(0)!} = 1 \right) \end{array}}$$

$\because 0! = 1$

## PERMUTATIONS:- (Arrangements)

① Number of permutations of $n$ distinct objects (without duplication):-

The number of different arrangements (permutations) of $n$ distinct objects, taken all at a time is

$$\boxed{P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1!} = n!}$$

② Number of permutations of $r$ objects among $n$ distinct objects:-

Suppose we are given $n$ distinct objects and wish to arrange $r$ ~~of the~~ objects denoted by

$$\boxed{^nP_r \text{ (or) } P(n,r) = \frac{n!}{(n-r)!}}$$

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## 3 Number of Permutations of n objects (with duplicate):-

It is required to find the number of permutations that can be formed from a collection of n objects of which $n_1$ are of one type, $n_2$ are of a second type, ...... $n_k$ are of $k^{th}$ type with $n_1+n_2+\dots+n_k=n$, then the number of permutations of n objects are

$$\boxed{\frac{n!}{n_1! * n_2! * \dots * n_k!}}$$

## 4 Circular Permutation:-

Permutations in a circle are called circular permutation the tot. no. of ways of arranging the n persons in a circle $=\boxed{(n-1)!}$

**Ex:-1**:- How many ways are there to sit 10 boys & 10 girls around a circular table?

**A**:- Here 10 Boys and 10 Girls sit around a circular table.

Total no. of persons $= 10(B)+10(G)=20$

$$\boxed{n=20}$$

Total no. of ways of circular permutations are $=(n-1)!$

$=(20-1)!$

$=19!$

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**Ex: (2)** How many ways 6 men and 6 women can be seated in a row.

**Ans:-** $n!$ ways

$tot = \underset{(M)}{6} + \underset{(W)}{6} = 12$

$= 12!$ ways.

**Ex: (3)** How many ways ~~are~~ 3 persons sit around a round table?

**Sol:-** Number of persons = 3

$\therefore n = 3$

total no. of ways of arranging 3 persons around a round is $(n-1)! = (3-1)! = 2! = 2$

**Ex: (4)** How many different arrangements of letters in the word BOUGHT?

**Sol:-** The given word BOUGHT contains 6 letters that are distinct (without duplication)

The total no. of arrangements of letters in the word

BOUGHT $= P(n,n) = n!$

$= 6!$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$ ways.

=•=

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**Ex:-** (5) How many different strings of length 4 can be formed using the letters of the word PROBLEM?

**A:-** The given word "PROBLEM" has 7 letters ∴

$n = 7$

The no. of different strings of length 4 can be formed by using the letters of the word PROBLEM =

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$^7P_4 \text{ (or) } P(7,4) = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7\times6\times5\times4\times\cancel{3!}}{\cancel{3!}}$$

$= 840$ ways

**Ex:-** (6) How many different strings of length four can be formed using the letters from FLOWER

**As:** $^6P_4 = \frac{6!}{(6-4)!} = \frac{6\times5\times4\times3\times\cancel{2!}}{\cancel{2!}}$

$= 360$ ways.

**Ex:-** (7) How many words of 3 distinct letters can be formed from the letters of the word PASCAL

**Ans:-** The given word PASCAL contains 6 Letters

$\therefore n = 6$

the no. of ways (permutations) of 3 distinct letters can be formed by using the letters of the word PASCAL

$$\Rightarrow P(n,r) = P(6,3) = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6\times5\times4\times\cancel{3!}}{\cancel{3!}}$$

$= 120$ ways

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Ex:-(8) Find the no. of permutations of the letters of the word DIFFICULT

Ans:- The given word DIFFICULT has 9 letters, out of which, 2 are I, 2 are F, 1 is D, 1 is C, 1 is U, 1 is L and 1 is T

∴ Tot no. of permutations of the letters of the word DIFFICULT

$$= \frac{9!}{2! * 2! * 1! * 1! * 1! * 1! * 1!} \quad \left| = \frac{n!}{n_1! * n_2 * n_3 * n_4 * n_5 * n_6} \right.$$

$= 90,720$

Ex:-(9) find the no. of permutations of the letters of the word ENGINEERING?

The given word ENGINEERING has 11 letters, out of which, 3 are E,
3 are N,
2 are I,
2 are G,
1 is R.

∴ Tot no. of permutations of the letters of the word ENGINEERING

$$= \frac{n!}{n_1! * n_2! * n_3! * n_4! * n_5!}$$

$$= \frac{11!}{3! * 3! * 2! * 2! * 1!}$$

$$= \frac{11 * 10 \times 9 \times \overset{8\times}{7} \times \cancel{6} \times 5 \times \cancel{4} \times \cancel{3!}}{\cancel{3!} \times \cancel{3} \times 2! \times \cancel{2} \times \cancel{2} \times 1}$$

$= 2,\overset{10}{\cancel{77}},200/-$

110
× 9
990
× 8
7920
7
9040
5
2,70,200

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Ex:- 10 Find the no. of permutations of the letters of the word SUCCESS

No. of letters in the word SUCCESS = 7

$\therefore n = 7$

$$= \frac{n!}{n_1! \times n_2! \cdots n_k!}$$

Out of 7 letters, 3 are S
2 are C
U is 1 and E is 1

$\therefore$ The number of permutations of the letters of the word

$$SUCCESS = \frac{n!}{n_1! * n_2! * n_3! * n_4!}$$

$$= \frac{7!}{3! * 2! * 1! * 1!}$$

$$= \frac{7 \times 6 \times 5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} * \cancel{2}}$$

$\therefore$ ~~No of letters in the word SUCCESS~~

$= 420$ ways.

Ex:- 11 Find the number of permutations of the letters of the word "STRUCTURES"?

Ans:- shortcut

$$\frac{10!}{2! * 2! * 2! * 2! * 1! * 1! * 1! * 8!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times \cancel{6}^{3} \times 5 \times \cancel{4} \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2} \times \cancel{2} \times \cancel{2}}$$

$= 2,26,800$ ~~10~~ ways

630
6
3780
5
18900
12
226800

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**Ex (12) :-** find the no. of permutations of the letters of the word MATHEMATICS?

$$= \frac{n!}{n_1! * n_2! * n_3! * n_4! * n_5! * n_6! * n_7!}$$

$$= \frac{11!}{\underset{(M)}{2!} * \underset{(A)}{2!} * \underset{(T)}{2!} * \underset{H}{1!} * \underset{E}{1!} * \underset{I}{1!} * \underset{C}{1!} * \underset{S}{1!}}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2} \times \cancel{2}}$$

= 49,89,600 ways.

720
1
5040
9
453600
11
4989600

**Ex:- (13)** find the no. of permutations of the letters of the word MASSASAUGA. In how many of these all four A's together? How many of them begins with S

The given word "MASSASAUGA" has 10 letters.

4 – A
3 – S
1 – M, U & G

$$= \frac{10!}{4! \times 3! * 1! * 1! * 1!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times \cancel{6} \times \cancel{5} \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times 2 \times 1}$$

= 25,200 Ways

(1) (AAAA), S, S, S, M, U, G → (7)

$$\frac{n!}{n_1! * n_2! * \cdots n_k!} \qquad \frac{7!}{\underset{(AAAA)}{1!} * \underset{S}{3!} * \underset{M}{1!} * \underset{U}{1!} * \underset{G}{1!}} = \frac{7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

= 840 ways

(2) out of (10) S _ _ _ _ _ _ _ _ _ → (9)

$$= \frac{9!}{\underset{(3-2's)\,2's}{2!} * \underset{4A's}{4!} * \underset{1M}{1!} * \underset{1U}{1!} * \underset{1G}{1!}} = 7560 \text{ ways}$$

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## Combinations :- (refer definition @ 2nd page)

## Example problems on Combinations :- (without repetition)

1. How many committees of 5 with a given chair person can be selected from 12 persons?

Sol :- Total number of persons = 12

⇒ Each committee consisting of 5 persons, among them, one person is chair person

⇒ The chair person can be selected among 12 person = 12 ways

⇒ The remaining 4 members/persons in the committee can be selected in $^{11}C_4$ ways

∴ Total no. of ways = $12 * {}^{11}C_4$   $\left[\because {}^nC_r = \frac{n!}{(n-r)!\,r!}\right]$

$$= 12 * \frac{11!}{4!(11-4)!}$$

$$= 12 * \frac{11!}{4!\,7!}$$

$$= 12 * \frac{11 \times 10 \times \cancel{9}^{3} \times \cancel{8} \times \cancel{7!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{7!}}$$

$$= 3960 \text{ ways}$$

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(2) Find the number of committees of 5 that can be selected from 7 men and 5 women. If the committee ~~to~~ consists of atleast one man and atleast one women.?

Sol:- Tot number of persons = 7 M + 5 W

= 12 persons.

⇒ Each committee consists of 5 persons

⇒ No. of committee of 5 that can be selected among 12 persons = $\boxed{{}^{12}C_5}$

⇒ Among these possible committees, No. of committees to select 5 men = $\boxed{{}^{7}C_5}$ &

No. of committes formed to select 5 women = $\boxed{{}^{5}C_5}$

∴ Total no of committees formed with atleast one men and one women = $\boxed{{}^{12}C_5 - {}^{7}C_5 - {}^{5}C_5}$

$= 792 - 21 - 1$

$= 770$ ways

(3) At a certain college hostel, the housing office has decided to appoint one Male & 1 female residential advisor for each floor. How many different pairs of advisors can be selected for a 7 floor building from 12 M and 15 F candidates.

Sol:- No. of floors = 7

No. of Male candidates = 12

No. of female candidates = 15

∴ Housing office decided to appoint one Male & 1 female advisor for each floor.

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from 12 male candidates, 7 male candidates for 7 floors can be selected in $\boxed{{}^{12}C_7 \text{ ways}}$

$${}^{12}C_7 = {}^{12}C_7 = \frac{12!}{7!\,(12-7)!} = \frac{12!}{7! \times 5!}$$

$$= \frac{12 \times 11 \times \cancel{10} \times \cancel{9}^{3} \times \cancel{8}^{2} \times \cancel{7!}}{\cancel{7!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 792 \text{ ways.}$$

from 15 female candidates, 7 female candidates for 7 floors can be selected in ${}^{15}C_7$ ways i.e., $C(15,7)$

$$C(15,7) = {}^{15}C_7 = \frac{15!}{7!\,(15-7)!}$$

$$= \frac{15!}{7! \times 8!}$$

$$= \frac{15 \times \cancel{14}^{2} \times 13 \times \cancel{12}^{2} \times 11 \times \cancel{10} \times \cancel{9}^{3} \times \cancel{8!}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{8!}}$$

$$= 6435 \text{ ways}$$

No. of pairs of advisors can be selected for a 7 floor building from 12 M & 15 female candidates $= 792 \times 6435$ ways

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(4) A certain Question paper contains 2 parts A & B, each containing 4Q's. How many diff ways a student can answer 5 Questions by selecting atleast 2 Q's from each part?

Sol:- Case (1): A student can select atleast 3 Questions from Part-A & 2 Questions from part B

3 Questions from part-A can be selected $4C_3$ ways

2 Questions from part-B can be selected $4C_2$ ways

$\therefore$ No. of ways $= 4C_3 * 4C_2$

$= 24$ ways.

Case (2):- A student can select 2 Q from part A & 3Q from P-B

2 Questions are selected from part-A in $4C_2$ ways

3 Questions from part-B can be selected in $4C_3$ ways

$\therefore$ No. of ways $= 4C_2 * 4C_3$

$= 24$ ways

$\therefore$ Tot no. of ways, a student can answer 5Q's by selecting atleast 2 Questions from each part

$= 24 + 24$

$= 48$ ways

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H/W
(5) A certain Question paper contains 3 parts A, B, C with 4 Q's in Part-A, 5 Questions in Part-B, 6 Questions in Part-C. It is required to answer 7 Questions. selecting 2 Questions from each part. In how many different ways can a student selecting his Seven Questions for answering?

Sol: Case (1):- 2Q part A — $4C_2$ ways
2Q part B — $5C_2$ ways
3Q part C — $6C_3$ ways

$\therefore$ Tot no. of ways $= 4C_2 * 5C_2 * 6C_3$
$= 6 \times 10 \times 20$
$= 1200$ ways

Case (2):- 2Q from P-A — $4C_2$ ways
3Q from P-B — $5C_3$ ways
2Q from P-C — $6C_2$ ways

$\therefore$ Tot no. of ways $= \cancel{6} * 10 * 15 = 900$ ways

III^ly Case (3): 3Q from P-A — $4C_3$ ways
2Q from P-B — $5C_2$ ways
2Q from P-C — $6C_2$ ways

$\therefore$ Tot no. of ways $= \cancel{60}\ 4 * 10 * 15 = 600$ ways

$\therefore$ Tot no of ways, a student select his 7 Q's for answering

$= 1200 + 900 + 600$

$= 2700$ ways

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## Combinations with repetition:-

⇒ Suppose we wish to select, a combination of $r$ objects with repetition from a set of $n$ distinct objects. The number of such selections is given by.

$$\boxed{C(n+r-1, r) = C(r+n-1, n-1)}$$

⇒ The following are the other interpretations of this number.

(i) $C(n+r-1, r) = C(r+n-1, n-1)$ represents the number of ways in which $r$ identical objects can be distributed among $n$ distinct containers.

(ii) $C(n+r-1, r) = C(r+n-1, n-1)$ represents the number of non-negative integer solutions of the equation.

Note:- A non-negative integer soln of the equation $x_1 + x_2 + \cdots + x_n = r$ is an $n$-tuple, where $x_1, x_2, x_3 \ldots x_n$ are non-negative integers whose sum is $r$.

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# Rough

from ($n$ distinct) objects
not same

↓

we have to select
same
($r$ = identical) objects
with repitition

↓

how many no. of such selection are

$C(n+r-1, r) = C(r+n-1, n-1)$

↓ these 2 formulae gives same results

used for 2 diff applns

① from n district containers we have to distribute $r$ - identical objects

$C(n+r-1, r)$

② we have to find no. of non-negtive integer sol^ns

$C(n+r-1, r)$

Note: A non-neg integer soln of the eqn

$C \leftarrow x_1$.

= · =

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Ex(1): In how many ways can we distribute 10 Identical Marbles among 6 distinct containers? (case (1) category from defn.)

Sol:- 10 Identical marbles $\Rightarrow r = 10$

6 distinct containers $\Rightarrow n = 6$

No. of such selections $= C(n+r-1, r)$

$= C(10+6-1, 10)$

$= C(15, 10)$

$= {}^{15}C_{10}$

$= \dfrac{15!}{10!(15-10)!}$

$= \dfrac{15!}{10! \times 5!}$

$= \dfrac{\cancel{15}^{3} \times \cancel{14}^{7} \times 13 \times \cancel{12}^{3} \times 11 \times \cancel{10!}}{\cancel{10!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$= 3003$ ways

Ex(2). find the no. of non-negative integer soln of the eqns

$x_1 + x_2 + x_3 + x_4 + x_5 = 8$ (case (2) type) from defn.)

Here $r = 8$

No. of non negative integers $n = 5$

$= C(n+r-1, r)$

$= C(5+8-1, 8)$

$= C(12, 8)$

$= {}^{12}C_8$

$= \dfrac{12!}{8!(12-8)!} \Rightarrow \dfrac{12!}{8! * 4!} \Rightarrow \dfrac{\cancel{12} \times 11 \times \cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$= \dfrac{55 \times 9}{1} = 495$ ways

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# Basics of Counting:-

there are 2 basic rules of counting.

1. Sum Rule
2. Product Rule.

## Sum Rule:-

⇒ Suppose 2 tasks $T_1$ and $T_2$ are to be performed. If the task $T_1$ can be performed in $m$ different ways & the $T_2$ can be performed in $n$ diff ways. If the 2 tasks cannot be performed simultaneously, then one of the 2 tasks ($T_1$ or $T_2$) can be performed in $m+n$ ways

⇒ Generally, if $T_1, T_2, T_3, .., T_K$ are $K$ tasks such that no two of these tasks can be performed at the same time & if the task $t_i$ can be performed in $n_i$ diff ways, then one of the $k$ tasks ($T_1$ or $T_2$ or $T_3$ or .... or $T_K$) can be performed in $n_1+n_2+n_3+.....+n_K$ ways

Rough:- formula for 2 tasks

$$\text{Task 1} \to T_1 \to m \qquad \text{Task 2} \to T_2 \to n$$

$T_1$ or $T_2$ = $m+n$ ways.

Generic formula:-

$T_1, T_2, T_3, \ldots, T_n$

$\downarrow\ \downarrow\ \downarrow \ \ldots \downarrow$

$n_1$ ways, $n_2$ ways, $n_3$ .... $n_K$

$\therefore n_1+n_2+n_3+\ldots\ldots+n_K$ ways

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Ex① :- Suppose there are 16 boys & 18 girls in a class. How many ways to select one of these students either a boy or a girl as class representative?

Sol :- No. of Boys = 16
No. of Girls = 18

Task① :- Number of ways for selecting a boy as a class (CR) = $m$ = 16 ways.

Task② :- No. of ways for selecting a Girl as a CR = $n$ ways = 18 ways

$T_1$ & $T_2$ can't be performed simultaneously.

No. of ways selecting a boy (or) a girl as a CR

= 16 + 18
= 34 ways

—∘—

Ex② Suppose a hostel library has 12 books on Mathematics, 10 books on physics, 16 books on computer science, & 11 books on Electronics. How many ways a student wishes to choose one of these books for study.

Sol :- Number of Mathematics Books = 12
" " physics Books = 10
" " Computer Science " = 16
" " Electronics Books = 11

No. of ways for selecting Maths Book for study = 12
" " " " " physics " " " = 10
" " " " " Computer Science " " = 16
" " " " " Electronics Book " " = 11

∴ No. of ways for selecting either M or P or C or E is
12 + 10 + 16 + 11
= 49 ways.

—∘—

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Ex③ :- find out how many no. of ways for selecting a prime no. less than 10 and even number less than 10.

| prime <10 | even <10 |
|---|---|
| 2 | 2 |
| 3 | 4 |
| 5 | 6 |
| 7 | 8 |

Sol :- No. of ways for selecting a prime No(<10)
Task ① :-
: 2, 3, 5, 7
= 4 ways.

Task ② :- No. of ways for selecting an even no's (<10)
: 2, 4, 6, 8
= 4 ways

Number of ways for selecting a prime number or even number = 4 + 4 − 1 (2 is common in both prime & even; same number can't be selected 2 times)
= 7 ways

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## (2) Product rule:-

Suppose that 2 tasks $T_1$ and $T_2$ are to be performed one after the other. If $T_1$ can be performed in $n_1$ ~~ways~~ different ways and for each of these ways, $T_2$ can be performed in $n_2$ different ways, then both of these tasks can be performed in $n_1 * n_2$ different ways.

### General rule:-

More generally, suppose that k tasks $T_1, T_2, T_3, \ldots T_k$ are to be performed in a sequence. If $T_1$ can be performed in $n_1$ different ways and for each of these ways $T_2$ can be performed in $n_2$ diff ways & for each of $n_1 * n_2$ diff ways of performing $T_1$ and $T_2$ in that order, $T_3$ can be performed in $n_3$ different ways & so on....., then the sequence of tasks $T_1, T_2, T_3, \ldots, T_k$ can be performed in $n_1 * n_2 * n_3 * \ldots * n_k$ different ways.

Exp:- Rough

Task 1    Task 2

$T_1 \rightarrow n_1$    $T_2 \rightarrow n_2$

$n_1 * n_2$ diff ways

General rule

k tasks :- $T_1, T_2, T_3, \ldots T_k$

$\downarrow \; \downarrow \; \downarrow \; \ldots \; \downarrow$

$n_1, n_2, n_3 \ldots n_k$

∴ Tot no. of tasks performed in sequence is

$n_1 * n_2 * n_3 * \ldots * n_k$ ways

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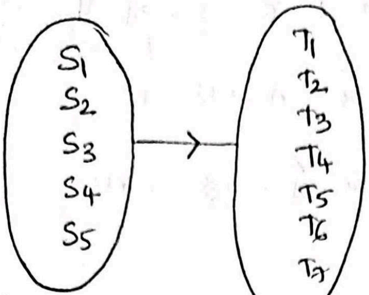
**Ex 1**: Suppose a person has 5 shirts & 7 ties. How many ways a person can choose a shirt and Tie?

**Sol**:-




for shirt $(S_1)$ → Among 7 ties he can choose one Tie → 7 ways
" $S_2$ → " " " " " 1 Tie → 7 ways
" $S_3$ → " " " " " → 7 ways
" $S_4$ → " " " " " " → 7 ways
" $S_5$ → " " " " " " → 7 ways

35 ways
$n_1 * n_2$
$7 * 5 = 35$ ways

**Ex 2**:- How many ways to construct sequence of 5 letters in which first 3 letters are English letters and the next 2 letters are single digit no., If no letter or digit can be repeated

**Sol**:

**Case 1**:- Repitition not allowed:-

$\underbrace{\_\ \_\ \_}_{\text{English letter (26)}}\ \underbrace{\_\ \_}_{\text{single digit no. (0 to 9) 10}}$

26 25 24 10 9

Total possible no. of ways $= 26 \times 25 \times 24 \times 10 \times 9$

**Case 2**:- Repitition allowed:-

26 26 26 10 10

∴ Tot no. of Possible ways $= 26 \times 26 \times 26 \times 10 \times 10$ ways

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## The principle of Inclusion-Exclusion:-

The principle of Inclusion-Exclusion can be stated in terms of sets as follows.

Let $A_1$ and $A_2$ be two sets. Let $T_1$ be the task of choosing an element from $A_1$ and $T_2$ be the task of choosing another element from $A_2$.

There are $n(A_1)$ ways to do Task $T_1$ and $n(A_2)$ ways to do Task $T_2$. then the no of ways to do either Task $T_1$ or task $T_2$ is

$$\boxed{n(A_1 \cup A_2) = n(A_1) + n(A_2) - n(A_1 \cap A_2)}$$

where

$n(A_1 \cup A_2)$ is the no. of ways to do either Tasks $T_1$ or $T_2$

$n(A_1 \cap A_2)$ is the no. of ways to do both Tasks $T_1$ and $T_2$

Let us take 3 tasks $T_1, T_2$ and $T_3$. Let $A_1, A_2, A_3$ are 3 sets, then the no. of ways to do tasks $T_1$ or $T_2$ or $T_3$ is

$$\boxed{n(A_1 \cup A_2 \cup A_3) = n(A_1) + n(A_2) + n(A_3) - n(A_1 \cap A_2) - n(A_2 \cap A_3) - n(A_3 \cap A_1) + n(A_1 \cap A_2 \cap A_3)}$$

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**Example:-** consider set of integers from 1 to 250 find out.

(i) How many of these numbers are divisible by 3 or 5

(ii) " " " " by 5 or 7

(iii) " " " " by 3 or 7

(iv) " " " " by 3 or 5 or 7

(v) " " " are divisible by 3 or 5 but not by 7

(vi) " " " " " by 5 or 7 but not by 3

(vii) " " " " " by 3 or 7 but not by 5

(viii) How many of these integers b/w 1 & 250 not divisible by 3, 5 & 7.

**Sol:-** Let $A_1, A_2, A_3$ are 3 sets consisting of integers that are divisible by 3, 5 and 7 resp from 1 to 250.

$$n(A_1) = \frac{250}{3} = 83$$

$$n(A_2) = \frac{250}{5} = 50$$

$$n(A_3) = \frac{250}{7} = 35$$

$$n(A_1 \cap A_2) = \frac{250}{3\times5} = 16$$

$$n(A_2 \cap A_3) = \frac{250}{5\times7} = 7$$

$$n(A_1 \cap A_3) = \frac{250}{3\times7} = 11$$

$$n(A_1 \cap A_2 \cap A_3) = \frac{250}{3\times5\times7} = 2$$

$= 2$

Let solve all the 8 Questions

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(i) How many no. of ~~ways~~ integers that are divisible by 3 or 5 from 1 to 250.

$n(A_1 \cup A_2) = n(A_1) + n(A_2) - n(A_1 \cap A_2)$

$= 83 + 50 - 16$

$= 133 - 16$

$= 117$

(ii) How many no. of integers that are divisible by 5 or 7 from 1 to 250.

$n(A_2 \cup A_3) = n(A_2) + n(A_3) - n(A_2 \cap A_3)$

$= 50 + 35 - 7$

$= 85 - 7$

$= 78$

(iii) How many no. of integers that are divisible by 3 or 7 from 1 to 250

$n(A_1 \cup A_3) = n(A_1) + n(A_3) - n(A_1 \cap A_3)$

$= 83 + 35 - 11$

$= 118 - 11$

$= 107$

(iv) How many no. of integers that are divisible by 3 or 5 or 7 from 1 to 250.

$n(A_1 \cup A_2 \cup A_3) = n(A_1) + n(A_2) + n(A_3) - n(A_1 \cap A_2) - n(A_2 \cap A_3) - n(A_1 \cap A_3) + n(A_1 \cap A_2 \cap A_3)$

$= 83 + 50 + 35 - 16 - 7 - 11 + 2$

$= 136.$

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(v) How many no. of integers that are divisible by 3 or 5 but not by 7 from 1 to 250.

Sol:- No. of integers that are divisible by 3 or 5 but not by 7

is $= n(A_1 \cup A_2 \cup A_3) - n(A_3)$

$= 136 - 35$

$= 101$

(vi) How many no. of integers that are divisible by 5 or 7 but not by 3 from 1 to 250.

Sol:- No. of integers that are divisible by 5 or 7 but not by 3

is $= n(A_1 \cup A_2 \cup A_3) - n(A_1)$

$= 136 - 83 = 53$

(vii) How many no. of integers that are divisible by 3 or 7 but not by 5 from 1 to 250.

Sol:- No. of integers that are divisible by 3 or 7 but not by 5

is $= n(A_1 \cup A_2 \cup A_3) - n(A_2)$

$= 136 - 50$

$= 86$

(viii) How many no. of integers that are not divisible by 3 or 5 or 7 from 1 to 250

$n(\overset{\text{compliment}}{\overline{A_1 \cup A_2 \cup A_3}})$ = Tot no. of integers $- n(A_1 \cup A_2 \cup A_3)$.

$= 250 - 136$

$= 114$

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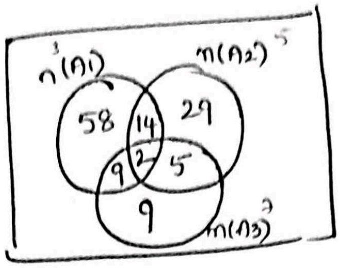
## Method ②: Using venn diagram:-



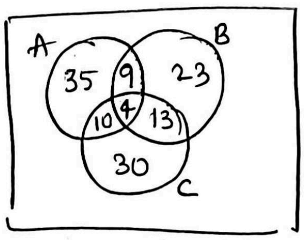

= ◦ =

**Ex②**:- Given 150 students
3 cooldrinks A, B, C.

58 students drink A
49 " " B
57 " " C
14 " " A and C
13 " " A and B
17 " " B and C
4 " " A, B and C

**Q**: How many drink none?

**Method②**:-




No. of students who drink ...

$= 150 - 35 - 10 - 4 - 9 - 30 - 13 - 23$

$= 26$

**Method①**:-

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

who drink A or B or C

$= 58 + 49 + 57 - 14 - 13 - 17 - 4$

$= 124.$

who drink none

$n(\overline{A \cup B \cup C}) = 150 - 124 = 26$

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# Binomial Theorem

A Binomial theorem describes the algebraic expansion of powers of a Binomial with two variables.

$$\boxed{(x+y)^n = {}^nC_0 \cdot x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 \cdot x^{n-2} y^2 + \dots + {}^nC_{n-1} \cdot x^1 y^{n-1} + {}^nC_n x^0 y^n}$$

It can be written as

$$\boxed{(x+y)^n = \sum_{r=0}^{\infty} {}^nC_r \cdot x^{n-r} \cdot y^r}$$

$$(1+x)^n = \underbrace{{}^nC_0}_{\downarrow} \cdot 1^n x^0 + {}^nC_1 \cdot 1^{n-1} x^1 + {}^nC_1 \cdot 1^{n-2} x^2 + \dots + {}^nC_n \cdot 1^1 \cdot x^{n-1} + {}^nC_n \cdot 1^0 \cdot x^n$$

$$= 1 \cdot x^0 + \frac{n!}{(n-1)!\,1!} \cdot x + \frac{n!}{(n-2)!\,2!} x^2 + \frac{n!}{(n-3)!\,3!} x^3 + \dots\dots$$

$$= 1 \cdot 1 + \frac{n(n-1)!}{(n-1)!} \cdot x + \frac{n(n-1)(n-2)}{(n-2)!\,2!} x^2 + \frac{n(n-1)(n-2)(n-3)!}{(n-3)!\,3!} \cdot x^3 + \dots$$

$$\boxed{(1+x)^n = 1 + n \cdot x + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots}$$

$$\boxed{(1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r \cdot x^r}$$

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1. find out the coefficient of $x^9 . y^3$ in the expansion of $(x+2y)^{12}$

Sol: we know that, Binomial theorem

$$\boxed{(x+y)^n = \sum_{r=0}^{\infty} n_{c_r} . x^{n-r} . y^r}$$

According to the Binomial theorem,

$x = x, \ y = 2y, \ n = 12$.

$$(x+2y)^{12} = \sum_{r=0}^{\infty} 12_{c_r} . x^{12-r} . (2y)^r$$

$$= \sum_{r=0}^{\infty} 12_{c_r} . x^{12-r} . 2^r . y^r$$

$$= \sum_{r=0}^{\infty} 12_{c_r} . 2^r . x^{12-r} . y^r \longrightarrow (1)$$

we have to find out $x^9 . y^3$

$x^9 = x^{12-r}$

$9 = 12 - r$

$12 - r = 9$

$r = 12 - 9$

$\boxed{r = 3}$

$y^3 = y^r$

$\boxed{r = 3}$

r value substituted in eqn (1)

$= 12_{c_3} . 2^3 . x^{12-3} . y^3$

$= \underbrace{12_{c_3} . 2^3}_{\Downarrow} . x^9 . y^3$

$= 12_{c_3} . 2^3$

$= \frac{\overset{2}{\cancel{12}} \times 11 \times 10}{1 \times \cancel{2} \times \cancel{3}} * 2^3 \Rightarrow 220 * 2^3$

$= 220 \times 8$

$= 1760.$

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2. find out the coefficient of $x^5 \cdot y^2$ in the expansion of $(2x-3y)^7$.

we know that, Binomial theorem

$$\boxed{(x+y)^n = \sum_{r=0}^{\infty} {}^nC_r \cdot x^{n-r} \cdot y^r}$$

According to the above B.Theorem.

$x = 2x,\ y = -3y \quad n = 7$

Substituting those values in the above eqn

$$(2x-3y)^7 = \sum_{r=0}^{\infty} {}^7C_r \cdot (2x)^{7-r} \cdot (-3y)^r$$

$$= \sum_{r=0}^{\infty} {}^7C_r (2)^{7-r} (x)^{7-r} \cdot (-3)^r \cdot y^r$$

$$= \sum_{r=0}^{\infty} {}^7C_r (2)^{7-r} \cdot (-3)^r \cdot (x)^{7-r} \cdot (y)^r \longrightarrow (1)$$

we have to find out the coefficient of $x^5 \cdot y^2$

| | |
|---|---|
| $x^{7-r} = x^5$ | $y^r = y^2$ |
| $7-r = 5$ | $\therefore \boxed{r=2}$ |
| $r = 7-5$ | |
| $\boxed{r=2}$ | |

$\therefore$ r value is substituted in eqn (1)

$$= {}^7C_2 \cdot (2)^{7-2} \cdot (-3)^2 \cdot x^{7-2} \cdot y^2$$

$$= \underbrace{{}^7C_2 \cdot (2)^5 \cdot 9}_{} \cdot x^5 \cdot y^2$$

$$= \frac{7 \times \cancel{6}^{3}}{\cancel{2} \times 1} \cdot 32 \cdot 9 \cdot x^5 \cdot y^2$$

$$= 288 \times 21 \cdot x^5 \cdot y^2$$

$$= 6048\, x^5 \cdot y^5$$

$\therefore$ The coefficient of $x^5 \cdot y^5$ in $(2x-3y)^7$ is 6048.

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3. find out the coefficient of $x^5$ in the expansion of $(1-2x)^{-7}$.

we know that

$$\boxed{(1-x)^{-n} = \sum_{r=0}^{\infty} {}^{n+r-1}C_r * x^r}$$

According to the above formula

$\cancel{+}x = \cancel{+}2x$ | $-n = -7$

$\therefore x = 2x$ | $\therefore n = 7$

Substituting these values in the above formula.

$$(1-2x)^{-7} = \sum_{r=0}^{\infty} {}^{7+r-1}C_r \cdot (2x)^r$$

$$= \sum_{r=0}^{\infty} {}^{6+r}C_r \cdot 2^r \cdot x^r \longrightarrow 1$$

we have to find out the coefficient of $x^5$

$$x^r = x^5$$

$$\boxed{\therefore r = 5}$$

$r$ value is substituted in eqn 1

$$= \sum_{r=0}^{\infty} {}^{6+5}C_5 \cdot 2^5 \cdot x^5$$

$$= {}^{11}C_5 \cdot 2^5 \cdot x^5$$

$$= 14784 \cdot x^5$$

$\therefore$ The coefficient of $x^5$ in the expansion of $(1-2x)^{-7}$ is 14784

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4) find out the coefficient of $x^{27}$ is the expansion of $(x^4+x^5+x^6+\dots)^5$

Hint $(x+y)^n$ ✗
(or)
$(1+x)^n$ ✓

Sol: Given that

$$(x^4+x^5+x^6+\dots)^5$$

$$\Rightarrow (x^4(1+x+x^2+\dots))^5$$

$$=(x^4)^5(1+x+x^2+\dots)^5$$

$$=x^{20}((1-x)^{-1})^5 \quad [\because (1-x)^{-1}=1+x+x^2+\dots]$$

$$=x^{20}(1-x)^{-5}$$

we know that

$$\boxed{(1-x)^{-n}=\sum_{r=0}^{\infty} {}^{n+r-1}C_r \cdot x^r} \longrightarrow (1)$$

$-x=-x$, $\boxed{x=x}$ | $-n=-5$, $\boxed{n=5}$

$x, n$ values substituted in eqn (1)

$$(1-x)^{-5}=\sum_{r=0}^{\infty} {}^{5+r-1}C_r \cdot (x)^r$$

$$x^{20}\cdot(1-x)^{-5}=x^{20}\cdot\sum_{r=0}^{\infty} {}^{4+r}C_r \cdot x^r$$

$$=\sum_{r=0}^{\infty} {}^{4+r}C_r \cdot x^{20}\cdot x^r$$

$$=\sum_{r=0}^{\infty} {}^{4+r}C_r \cdot x^{20+r} \longrightarrow (2)$$

we have to find out the coefficient of $x^{27}$

$$x^{27}=x^{20+r}$$

$$27=20+r$$

$$\boxed{r=7}$$

$r$ value substitute in eqn (2)

$$=\sum_{r=0}^{\infty} {}^{4+7}C_7 \cdot x^{27} \Rightarrow {}^{11}C_7 \cdot x^{27} \Rightarrow 330 \cdot x^{27}$$

$\therefore$ coefficient of $x^{27}$ is 330.

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H/w

(5) find out the coefficient of $x^0$ in the expansion of

$$\left(3x^2-\left(\frac{2}{x}\right)\right)^{15}.$$

H/w.

(6) $x^{12}$ in $x^3(1-2x)^{10}$

(7) $x^{10}$ in $(x^3-5x)/(1-x)$

6.Ans:- $^{10}C_9(-2)^9$ (Refer 27pg)

7Ans:- $(x^3-5x)$ 2 $(1+x+x^2+x^3+..)$

$1-5=-4$

Sol:- we know that Binomial theorem is

$$\boxed{(x+y)^n=\sum_{r=0}^{\infty} {}^nC_r \cdot x^{n-r} \cdot y^r}$$

In the given problem $\left(3x^2-\left(\frac{2}{x}\right)\right)^{15}$

where $x=3x^2$ $y=\frac{-2}{x}$, $n=15$

Substituting these values in the above formula

$$\left(3x^2-\left(\frac{2}{x}\right)\right)^{15}=\sum_{r=0}^{\infty} {}^{15}C_r \cdot (3x^2)^{15-r} \cdot \left(\frac{-2}{x}\right)^r$$

$$=\sum_{r=0}^{\infty} {}^{15}C_r (3)^{15-r} \cdot (x^2)^{15-r} \cdot (-2)^{-r} \cdot \left(\frac{1}{x}\right)^r$$

$$=\sum_{r=0}^{\infty} {}^{15}C_r (3)^{15-r} \cdot (-2)^r \cdot (x)^{30-2r} \cdot (x)^{-r} \longrightarrow (1)$$

we have to find out $x^0$ value

$$(x)^{30-2r} \cdot (x)^{-r} = x^0$$

$$x^{30-3r}=x^0$$

$$30-3r=0$$

$$3r=30$$

$$r=30/3 = 10 \quad \therefore r=10$$

$\therefore$ r value substituted in eqn (1)

$$={}^{15}C_{10} \cdot (3)^{15-10} \cdot (-2)^{10} \cdot x^{30-3(10)}$$

$$=\underbrace{{}^{15}C_{10} \cdot (3)^5 \cdot (-2)^{10}}\cdot x^0$$

$\therefore$ The coefficient of $x^0$ in the expansion of $\left(3x^2-\left(\frac{2}{x}\right)\right)^{15}$ is

$$=-747242496.$$

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## Multinomial theorem :-

Multinomial Theorem is a generalization of the Binomial theorem with more than 2 variables.

$$\boxed{(x_1+x_2+\cdots+x_k)^n = \sum_{n_1,n_2\cdots n_k} \frac{n!}{n_1! * n_2! * \cdots * n_k!} \cdot (x_1)^{n_1} * (x_2)^{n_2} * \cdots * x_k^{n_k}}$$

where $n_i \leq n$ and $n_1+n_2+\cdots+n_k = n$

(or) $\binom{n}{n_1, n_2, \ldots n_k}$

① Compute the following.

ⓐ $\binom{7}{2,3,2}$

Sol:- It is in the form of $\binom{n}{n_1, n_2, n_3}$ where $n_1+n_2+n_3 = n$.

By applying the multinomial theorem.

$$\frac{n!}{n_1! * n_2! * n_3!} = \frac{7!}{2! * 3! * 2!} = 210$$

ⓑ $\binom{8}{4, 2, 2, 0}$

Sol:- $\frac{8!}{4! * 2! * 2! * 0!} = 420$

ⓒ $\binom{12}{5\ 3\ 2\ 2}$

$\Rightarrow \frac{12!}{5! * 3! * 2! * 2!} \Rightarrow 166320$

ⓓ $\binom{4}{1\ 1\ 2}$

Sol:- $\frac{4!}{1! * 1! * 2!} = \frac{24}{2} = 12$

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② Determine the coefficient of

(i) $xyz^2$ in the expansion of $(2x-y-z)^4$

(ii) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$

(i) Sol:- we know

$$(x_1+x_2+x_3+\dots+x_n)^k = \frac{k!}{n_1!*n_2!*n_3!*\dots*n_k!}\cdot(x_1)^{n_1}*(x_2)^{n_2}*\dots(x_k)^{n_k}$$

Given expression is $(2x-y-z)^4$

where $n=4, x_1=2x, x_2=-y, x_3=-z$

Apply multinomial theorem..

$$\boxed{\frac{n!}{n_1!*n_2!*n_3!}\cdot(x_1)^{n_1}\cdot(x_2)^{n_2}\cdot(x_3)^{n_3} = \frac{4!}{n_1!*n_2!*n_3!} * (2x)^{n_1}*(-y)^{n_2}*(-z)^{n_3}} \quad \text{①}$$

we have to findout the coef $xyz^2 = x_1^{n_1}\cdot x_2^{n_2}\cdot x_3^{n_3}$

$n_1=1, n_2=1, n_3=2$

$n_1+n_2+n_3=4$

Substitute $n_1, n_2, n_3$ values in eqn ①

$$=\frac{4!}{1!*1!*2!}*(2x)^1*(-y)^1*(-z)^2$$

$$=\frac{\cancel{24}^{12}}{\cancel{2}}\cdot 2x^1*(-y)^1*(z)^2$$

$$=-24xyz^2$$

$\therefore$ $-24$ is the coefficient of $xyz^2$ in the expansion of $(2x-y-z)^4$.

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(11) Given expansion is $(a+2b-3c+2d+5)^{16}$

where $n=16, x_1=a, x_2=2b, x_3=-3c, x_4=2d, x_5=5$

$$\frac{16!}{n_1! * n_2! * n_3! * n_4! * n_5!} \cdot a^{n_1} \cdot 2b^{n_2} \cdot (-3c)^{n_3} \cdot (2d)^{n_4} \cdot (5)^{n_5} \longrightarrow (1)$$

$n_1+n_2+n_3+n_4+n_5 = n = 16$

we have to find out the coef of $a^2b^3c^2d^5$

$\Rightarrow a^{n_1} \cdot (b)^{n_2} \cdot (c)^{n_3} \cdot d^{n_4}$

$\Rightarrow n_1=2, n_2=3, n_3=2, n_4=5$

$n_1+n_2+n_3+n_4+n_5 = n$

$2+3+2+5+n_5 = 16$

$n_5 = 16-(2+3+2+5)$

$\boxed{n_5 = 4}$

Substitute $n_1, n_2, n_3, n_4, n_5$ values in eqn (1)

$$= \frac{16!}{2! * 3! * 2! * 5! * 4!} * a^2 * (2b)^3 * (-3c)^2 * (2d)^5 * (5)^4$$

$$= \frac{16!}{2! * 3! * 2! * 5! * 4!} * 2^3 * (-3)^2 * 2^5 * 5^4 * a^2 * b^3 * c^2 * d^5$$

$$= \underbrace{\frac{16!}{2!\,3!\,2!\,5!\,4!}\, 72 * 2^5 * 5^4}\, a^2b^3c^2d^5$$

is the coef of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$

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③ Find the coef of $x^{11}y^4z^2$ in the expansion of $(2x^3-3xy^2+z^2)^6$

Sol: The given expansion is $(2x^3-3xy^2+z^2)^6$

where $n=6, x_1=2x^3, x_2=-3xy^2, x_3=z^2$

By applying the multinomial theorem.

$$\frac{n!}{n_1!*n_2!*n_3!}*(x_1)^{n_1}*(x_2)^{n_2}*(x_3)^{n_3}$$

$$= \frac{6!}{n_1!*n_2!*n_3!}*(2x^3)^{n_1}*(-3xy^2)^{n_2}*(z^2)^{n_3}$$

$$= \text{"} * (2)^{n_1}(x^3)^{n_1} * (-3)^{n_2}(x)^{n_2}(y^2)^{n_2}(z^2)^{n_3}$$

$$= \text{"} * (2)^{n_1}(-3)^{n_2}(x)^{3n_1}.(x)^{n_2}.(y)^{2n_2}.z^{2n_3}$$

$$= \text{"} * (2)^{n_1}.(-3)^{n_2}.(x)^{3n_1+n_2}.(y)^{2n_2}.(z)^{2n_3} \longrightarrow ①$$

Now we have to findout the coef of $x^{11}y^4z^2$

Comparing $x,y,z$ terms in eqn ①

| | | |
|---|---|---|
| $x^{3n_1+n_2}=x^{11}$ | $y^{2n_2}=y^4$ | $z^{2n_3}=z^2$ |
| $\boxed{3n_1+n_2=11}$ | $2n_2=4$ | $2n_3=2$ |
| | $\boxed{n_2=2}$ | $\boxed{n_3=1}$ |

$n_1+n_2+n_3=n$

$n_1+2+1=6$

$\boxed{n_1=3}$

Substitute $n_1,n_2,n_3$ values in eqn ①

$$= \frac{6!}{3!2!1!}*2^3*(-3)^2*x^{3(3)+2}*y^{2*2}.z^{2*1}$$

$$= \frac{6!}{3!2!1!}*72.x^{11}y^4z^2$$

$\therefore$ coef of $x^{11}y^4z^2$ is $\frac{6!}{3!2!1!}*72$ i.e. 4320

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m-pigeons
n-pigeon holes
$m > n$

# Pigeon-Hole Principle:-

⇒ If $m$ pigeons occupy $n$ pigeon-holes, where $m > n$, then atleast one Pigeon-hole must contain 2 or more pigeons.

(OR)

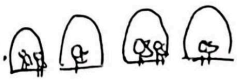
⇒ If $m$ pigeons occupy $n$ pigeon holes and if $m > n$, then 2 or more pigeons occupy the same pigeon hole.

this stmt is known as "Pigeon-hole principle"

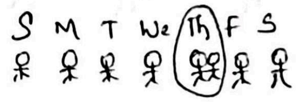
⇒ To apply the pigeon-hole principle, we must decide which objects will play the roles of pigeons & which objects will play the roles of pigeon holes.

## Applications:-

① If 6 pigeon occupy 4 Pigeon holes, then atleast one Pigeon-hole contain 2 or more pigeons in it.



② If 8 children are born on the same week, then 2 or more children are born on the same day of the week.




=o=

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## Generalized Pigeon-hole principle:-

$k$ = pigeons
$n$ - pigeon holes

If $k$-pigeons are assigned to $n$ pigeon holes, then one of the pigeon hole must contain atleast $\left[\frac{k-1}{n}\right]+1$ pigeons

**Ex:-** Suppose there are 26 students and 7 cars to transport them. S/T atleast one car must have 4 or more passengers.

**Sol:-** Here we have to find which object will play role of Pigeons & " " " " " " Pigeon holes

Here Pigeons = 26 ie $k=26$
Pigeon hole = 7 ie $n=7$

Apply Generalized pigeon hole principle $\left[\frac{k-1}{n}\right]+1$

$$=\left[\frac{26-1}{7}\right]+1$$
$$\doteq 3+1$$
$$= 4$$

∴ One car must have 4 or more passengers

**Ex(2):-** Prove that If any 30 peoples are selected, then we may choose a subset of 5, so that all 5 were born on the same day of the week.

**Sol:-** $k = 30$ pigeons.
$n = 7$ days of a week.

Apply Generalized pigeon hole principle $\left[\frac{k-1}{n}\right]+1$

$$=\left[\frac{30-1}{7}\right]+1$$
$$= 4+1 = 5$$

∴ 5 people were born on the same day of the week.

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Ex(3): Prove that if 30 dictionaries in a library contain a total of 61327 pages. then atleast one of the dictionary must have 2045 pages.

Ans :- $61327 = k$

$k = 61327$ (pigeons)

$n = 30$ (d)

$= \left[\frac{k-1}{n}\right] + 1$

$= \left[\frac{61327-1}{30}\right] + 1$

$= 2045$ pages.