mood-book

Unit-2

Regular Expression ⇒ IS a string That Describes the whole
Regular language. set of strings of a language according
to certain syntax Rules.

⇒ used to Describe the Regular language.

⇒ morphology of a languages.
to describe a languages.
⇒ Notations of

⇒ They are used as powerful tools in search
Engine.

1, 8, 11, 13, ~~17~~,
18, 25, 29, 32,
34, 40, 41, 44,
45, 47, 50, 59, 60
18/3.

Regular Sets:

S → Regular Sets.

All FSM is Regular.

All Regular languages accepted by FSM.

1. Set of words over S.

2. If A and B are regular set over S.
AUB and AB are also regular.

3. If S → is a Regular Then S^* also regular.

* It's Closed under Union, concatenation and kleen
closure operations.

* Let $\Sigma = \{a, b\} \Rightarrow \{a, aa, bb, b, bbb \ldots\}$

* Let $\Sigma = \{0\} \Rightarrow \{0, 00, 0000 \ldots\}$

* Let $\Sigma : \{0,1\}$ ⇒ then set of strings are $\{01, 10\}$

Regular Expression:

The language accepted by FA are regular language and these languages are easily Described by Simple Expression Called Regular Expressions.

Properties of Regular Expressions.

1. $L(R_1) + L(R_2) = L(R_1) \cup L(R_2)$

2. $L(R_1) L(R_2) = R_1 R_2$

3. $L(R_1)^* = R_1^*$

Applications of R.E.

* used to check Correctness of Inputs.

* Lexical Analysis.

* used to Define the Expression and languages.

* Define the languages accepted by FA.

Ex: Find regular Expression for the Following.

(a) A language consists of all the word over {a,b} ending in b.

(b) ending in bb

(c) Starting with a and ending in b.

(d) bb as substring

(e) ending in aab.

(a) $(a+b)^* b$

2-2-19.

(b) $(a+b)^* bb$

3, 8, 11, 35,

(c) $a(a+b)^* b$

40, 44, 47, 48

(d) $(a+b)^* bb$ (or) $bb(a+b)^*$ (or) $(a+b)^*$

50, 58, ~~66~~

16/528 →

(e) $(a+b)^* aab$

Ex: Obtain a Regular Expression to accept a language consisting of strings of a's and b's of even length.

$(ab)^*$ aa

ab

Obtain Regular Expression to accept strings of a's and b's ending with b and has no substring aa.

$(b+ab)^*$

$(ab+b)^*$.

Ex: No two consecutive zero's.

$(1+01)^*$ (or) $(01+1)^*$

Ex: Obtain a regular expression to accept strings a's and b's of length ≤ 10.

$L = (\epsilon + a + b)^{10}$.

Ex: a's and b's starting with 'a' and ending with 'b'.

$a(a+b)^* b$.

Hierarchy of Evaluation of Regular Expression.

1. parenthesis
2. Kleen closure
3. Concatenation
4. union.

Consider the Regular Expression $(a+b)^* aab$ and Describe the All words

starting with a (or) b used more times and ending with aab.

Arden's Theorem

Checking the equivalence of two Regular Expressions.

Let P and Q be the Two Regular Expressions over the Input set Σ. The Regular Expression R is given as,

$R = Q + RP$ which has unique solution is $R = QP^*$.

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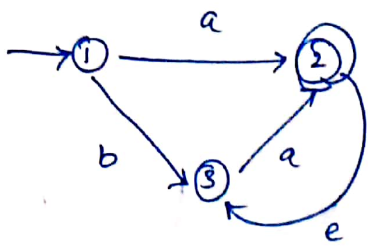
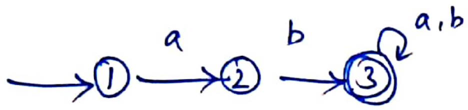
Construct NFA for the Regular Expression $a+ba^*$

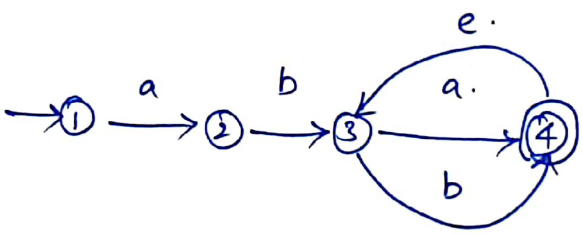
Sol.



Obtain an NFA which accepts strings of a's and b's starting with string ab.

$ab(a+b)^*$

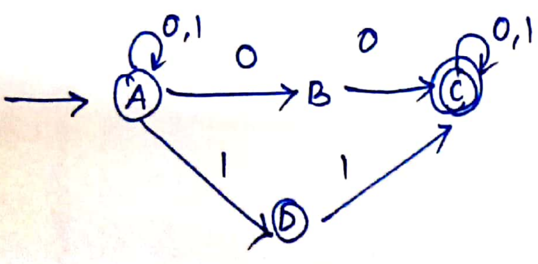




① $(a+b)^* aa (a+b)^*$

② $(a+b)(a+b)^*$.

$(0+1)^*(00+11)(0+1)^*$.



Identity Rules:

* R consists of P and Q are equivalent, if and only if P represents the same set of strings as Q does.

* For showing this equivalence of Regular Expressions we need to show identities of Regular Expression.

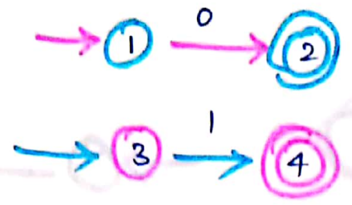
1. $\epsilon \cdot R = R \cdot \epsilon = R$
2. $\epsilon^* = \epsilon$ | ϵ - is Null string
3. $(\phi)^* = \epsilon$ | ϕ is empty string.
4. $\phi \cdot R = R \cdot \phi = \phi$
5. $\phi + R = R$
6. $R + R = R$
7. $R \cdot R^* = R^* \cdot R = R^*$
8. $(R^*)^* = R^*$
9. $\epsilon + RR^* = R^*$
10. $(P+Q)R = PR + QR$
11. $(P+Q)^* = (P^* Q^*) = (P^* + Q^*)^*$
12. $R^*(\epsilon + R) = (\epsilon + R)R^* = R^*$
13. $(R+\epsilon)^* = R^*$
14. $\epsilon + R^* = R^*$
15. $(PQ)^* P = P(QP)^*$
16. $R^* R + R = R^* R$

$(0+1)^*(00+11)(0+1)^*$

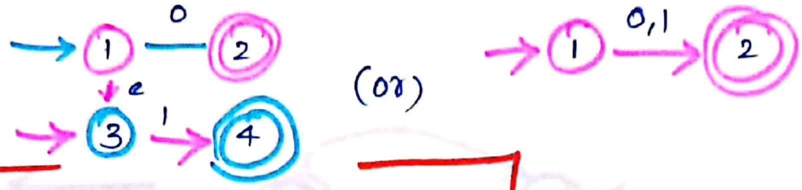
$(0+1)^*$

0

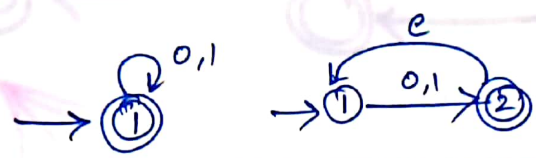
1



0+1



$(0+1)^*$



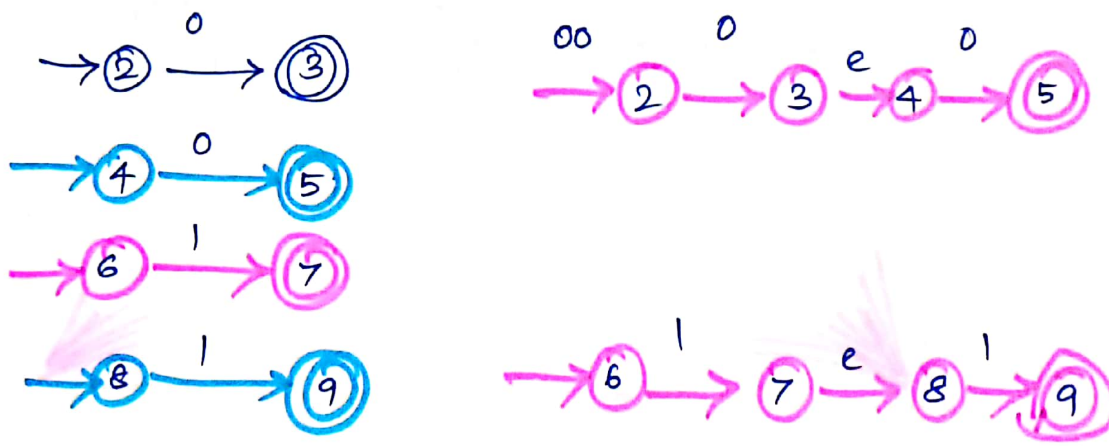
$(00+11)$

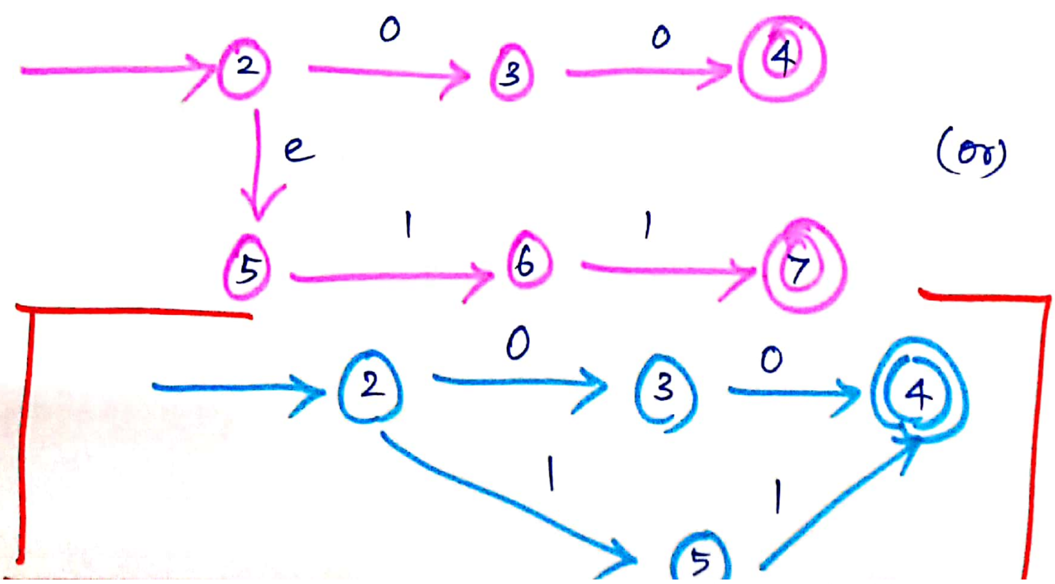
0

0

1

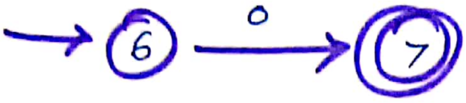
1



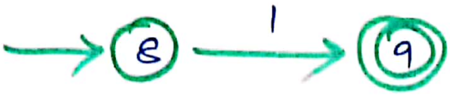


$(0+1)^*$

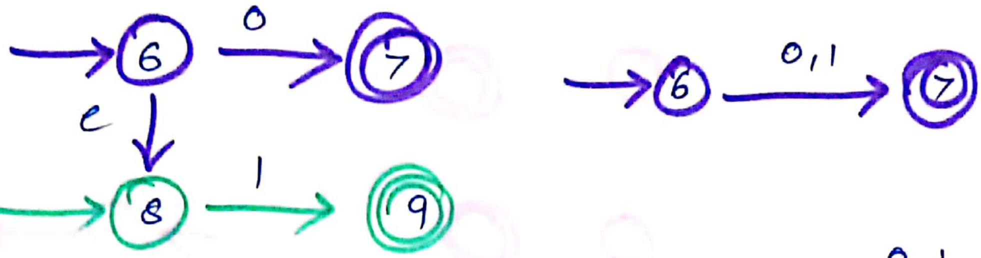
0



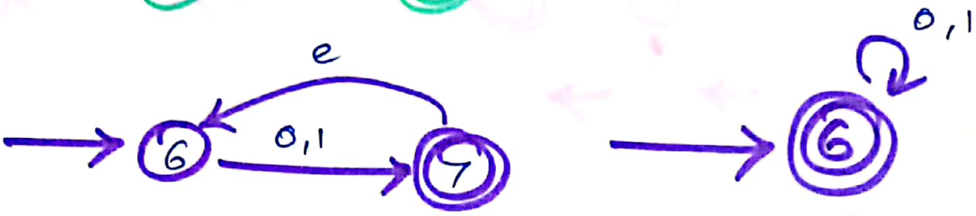
1

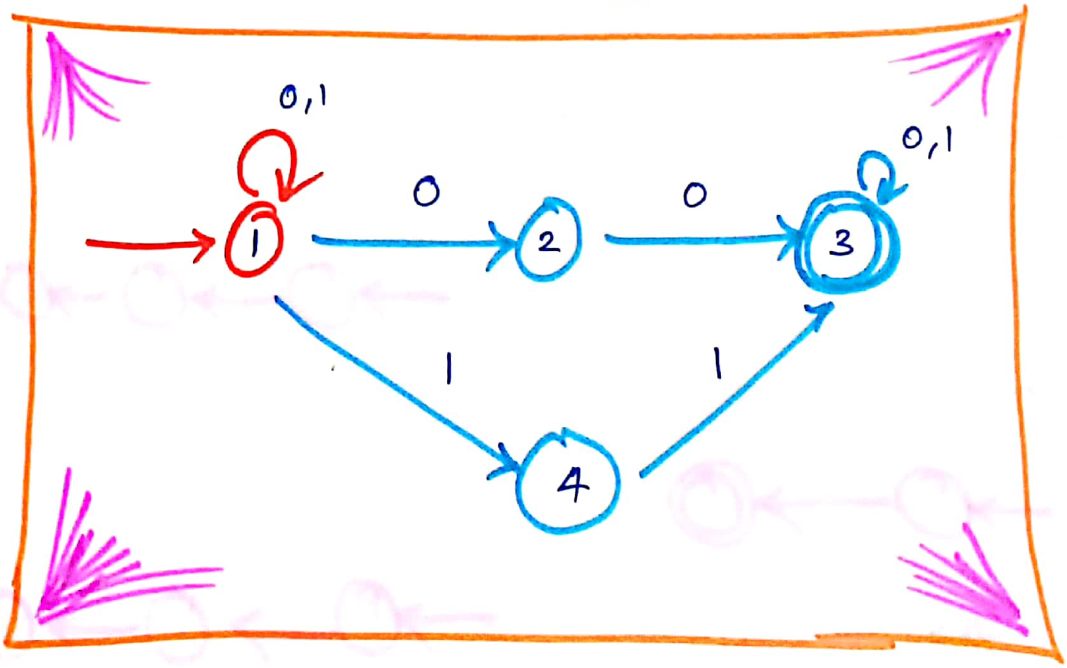


$(0+1)$



$(0+1)^*$

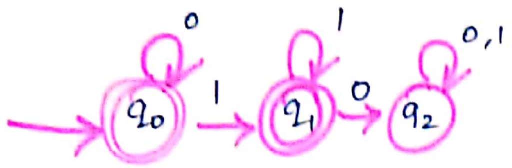




Construct NFA for the $(01+2^*)0$.

" for the RE $(0+1)^*(00+11)(01)(0+1)^*$

Finite Automata To Regular Expressions.



Step1: Form the equation.

$$q_0 = q_0 0 + \epsilon \quad \text{—} (1)$$

$$q_1 = q_0 1 + q_1 1 \quad \text{—} (2)$$

$$q_2 = q_1 0 + q_2 0 + q_2 1 \quad \text{—} (3)$$

$$= q_1 0 + q_2(0+1)$$

$$q_0 = \epsilon + q_0 0$$

$$\boxed{R = Q + RP}$$

$$R = QP^*$$

$$= \epsilon.(0)^*$$

$|\because \epsilon . R^* = \epsilon \cdot R = R.$

$$\boxed{q_0 = 0^*} \text{—} (4)$$

This value into,

$$R = Q + RP$$

q_1

$$q_1 = \underline{0^* 1} + q_1 1$$

$$\boxed{q_1 = 0^* 1 (1)^*} \text{—} (5).$$

$R = Q + RP$

$R = q_1$

$P = 1$

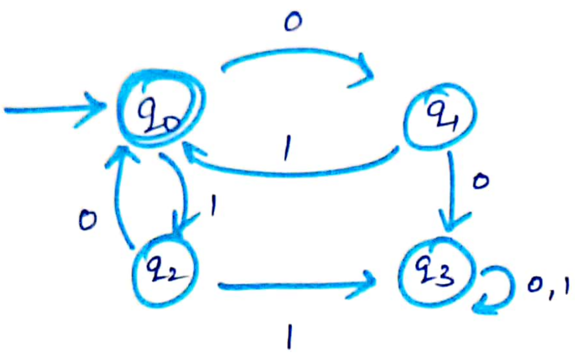
(4), (5) applied into eq (2),

$q_1 = q_0 1 + q_1 1$

$= 0^* 1 + 0^* 1 . 1^*$

$\boxed{r = 0^* + 0^* 1^*}$ $\Rightarrow 1.1^* = 1^*$

Construct R.E for the DFA given in below figure.



$q_0 = q_1 1 + q_2 0 + \epsilon$ —①

$q_1 = q_0 0$ —②

$q_2 = q_0 1$ —③

$q_3 = q_1 0 + q_2 1 + q_3 0 + q_3 1$

$q_3 = q_1 0 + q_2 1 + q_3 (0+1)$ —④

$q_0 = q_1 1 + q_2 0 + \epsilon$

$= q_0 01 + q_0 10 + \epsilon$

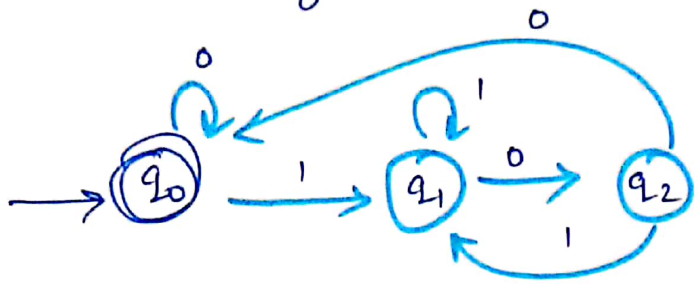
Sub ②, ③ in ①

$\underset{R}{q_0} = \underset{R'}{q_0} \underset{P}{(01+10)} + \underset{Q}{\epsilon}$ $\quad \because R = Q + RP.$

$q_0 = \epsilon . (01+10)^*$ $\quad R = QP^*$

$\boxed{q_0 = (01+10)^*}$ $\quad \epsilon . R^* = R^*$

✓ Find out Regular Expression from given DFA.



✓ $q_0 = q_0 0 + q_2 0 + \epsilon$ — ①

$q_1 = q_0 1 + q_1 1 + q_2 1$ — ②

$q_2 = q_1 0$ — ③

Apply. $q_1 = q$

Apply ③ in ②.

$q_1 = q_0 1 + q_1 1 + q_1 0 \cdot 1$

$q_1 = \underset{Q}{q_0 1} + \underset{R}{q_1} \underset{P}{(1+01)}$

$\downarrow$

$R = Q + RP.$

$Q = q_0 1$

$P = 1+01$

$R = QP^*$

$\boxed{q_1 = q_0 1 (1+01)^*}$ — ④

$q_0 = q_0 0 + q_1 0 . 0 + \epsilon$

Sub ③ in ① $q_0 = q_0 0 + q_1 00 + \epsilon$

Sub ④ in eq. $q_0 = q_0 0 + q_0 (1 (1+01)^*) 00 + \epsilon$

R R P Q.

$$q_0 = q_0(0+1(1+01)^*00)+\epsilon.$$

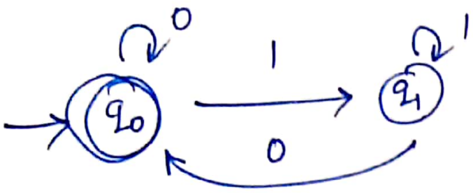
Again $R = Q + RP$

$$R = QP^*$$

$$= \epsilon.(0+1(1+01)^*00)^*$$

$$\boxed{q_0 = (0+1(1+01)^*00)^*.}$$

$\epsilon . R^* = R^*$



$$q_0 = q_0 0 + q_1 0 + \epsilon \quad \text{—①}$$

$$q_1 = q_0 1 + q_1 1 \quad \text{—②}$$

R Q. R P

$$q_1 = q_0 1 + q_1 1$$

$$R = Q + RP.$$

$$R = QP^*$$

$$= q_0 1 + 1$$

$$\boxed{q_1 = q_0 1 1^*} \longrightarrow ③$$

$$q_0 = q_0 0 + q_0 1 1^* 0 + \epsilon.$$

$$q_0 = q_0(0+11^*0)+\epsilon$$

R R P Q

$$R = QP^*.$$

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$R = QP^*$

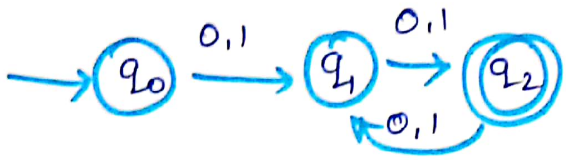
$q_0 = \epsilon(0+11^*0)^*$

$= \epsilon \cdot R^* = R.$

$\boxed{q_0 = (0+\underline{11}^*0)^*}$ $\quad R \cdot R^* = R^+ \quad R \cdot R^* = R^+$

$\boxed{q_0 = (0+1^*0)^*}$

Construct the Regular Expression for following DFA.



$q_0 = \epsilon$ —①

$q_1 = q_0 0 + q_0 1 + q_2(0+1)$

$q_1 = q_0(0+1) + q_2(0+1)$ —②

$q_2 = q_1(0+1)$ —③

Sub ① in ②,

$q_1 = \epsilon \cdot (0+1) + q_2(0+1)$

$q_1 = (0+1) + q_2(0+1)$

Solve ③, $q_2 = q_1(0+1)$

Sub q_1 value $\overset{R}{q_2} = \overset{Q.}{(0+1)} + \overset{R}{q_2}(0+1)\overset{P}{(0+1)}$

$=$

$$q_2 = q_1 (0+1)$$

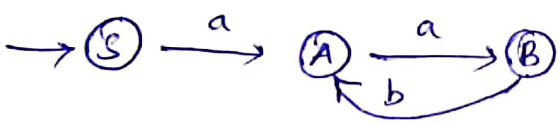
$$\overset{R}{q_2} = \overset{Q}{(0+1)} + \overset{R}{q_2} \overset{P}{(0+1) \cdot (0+1)}$$

$$= QP^*$$

$$= (0+1)((0+1)(0+1))^*$$

$$\boxed{q_2 = (0+1)((0+1)(0+1))^*}$$

ALGEBRAIC LAWS FOR REGULAR EXPRESSIONS:



Pumping lemma

Pumping lemma for Regular Sets.

* Pumping (generating) many Substrings from a given a String.

long String $\Rightarrow$ Substrings.

* It gives necessary conditions (S) to prove a set of Strings is not regular.

* M $\Rightarrow$ Tuples n states.

L consists of $Z \in L$, such that,

1. $|z| > n$ $z = uvw$, $v \neq \epsilon$.

2. $uv^iw \in L$ for $i \geq 0$.
3. $|uw| \leq n$.

Applications of Pumping Lemma.

$i=2$

$L = \{ a^n b^n \quad n \geq 1 \}$ is not regular.

a aaa bb.

$n \geq 1 \quad = 1, 2, 3 \ldots$

$n = 1$

$a^1 b^1 \Rightarrow ab$.

aabb

$\underset{u}{a}\,\underset{v}{a}\,\underbrace{bb}_{w}$

$n = 2$

$= a^2 b^2$

$=$ aabb.

$uv^iw \in L$. $i = 1$

$= a\,a^1\,bb$

$=$ aabb.